Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/43-1.2.2.6-P-x-d-x-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [145]. This is test number [43].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (145)	0.00 (0)
Mathematica	100.00 (145)	0.00 (0)
Maple	98.62 (143)	1.38 (2)
Mupad	98.62 (143)	1.38 (2)
Giac	95.86 (139)	4.14 (6)
Fricas	85.52 (124)	14.48 (21)
Sympy	55.17 (80)	44.83 (65)
Maxima	50.34 (73)	49.66 (72)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

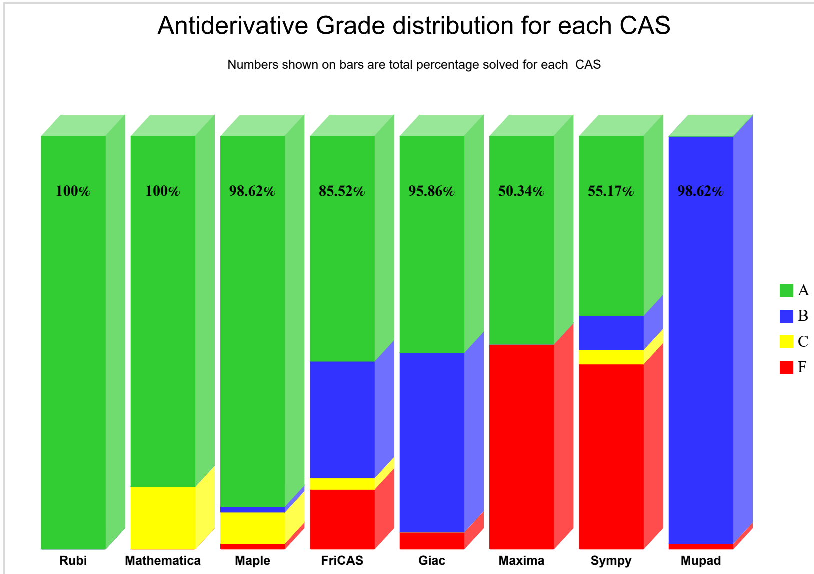
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

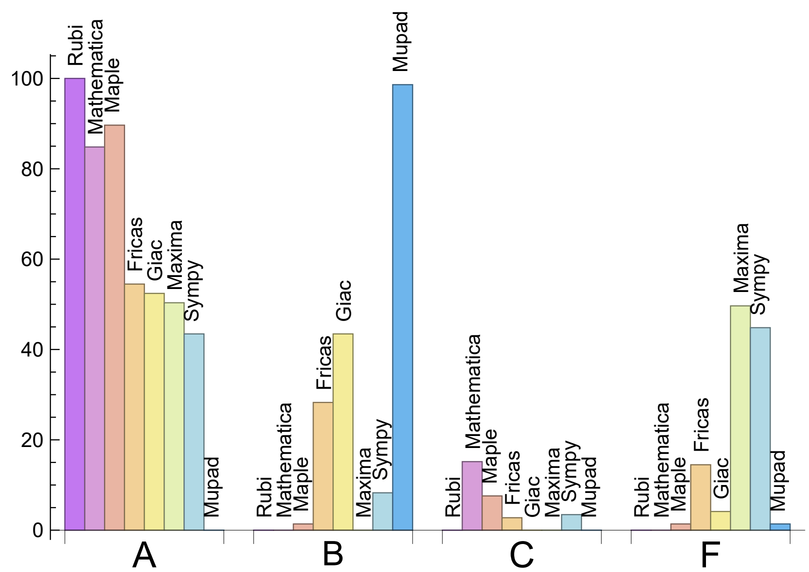
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	89.66	1.38	7.59	1.38
Mathematica	84.83	0.00	15.17	0.00
Fricas	54.48	28.28	2.76	14.48
Giac	52.41	43.45	0.00	4.14
Maxima	50.34	0.00	0.00	49.66
Sympy	43.45	8.28	3.45	44.83
Mupad	N/A	98.62	0.00	1.38

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	21	9.52 %	90.48 %	0.00 %
Giac	6	33.33 %	0.00 %	66.67 %
Maxima	72	77.78 %	0.00 %	22.22 %
Sympy	65	1.54 %	98.46 %	0.00 %
Mupad	2	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

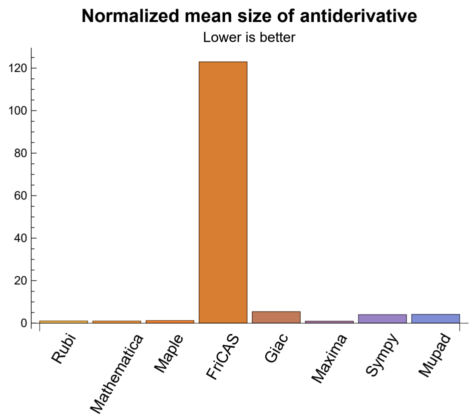
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

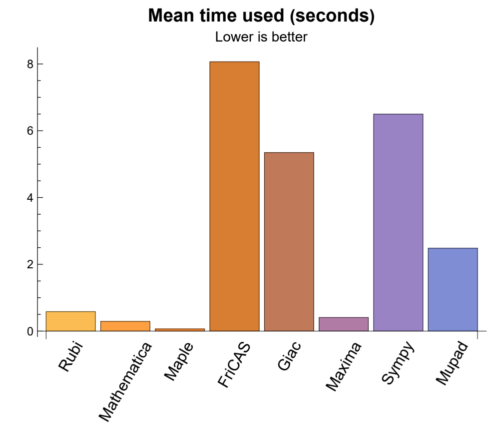
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.58	207.48	1.03	179.00	1.00
Mathematica	0.29	203.12	0.98	139.00	1.00
Maple	0.07	277.54	1.20	158.00	1.02
Maxima	0.41	101.92	0.93	65.00	0.88
Fricas	8.06	31149.27	122.99	143.00	1.38
Sympy	6.50	1048.58	3.99	71.00	0.98
Giac	5.35	1805.65	5.40	225.00	1.18
Mupad	2.48	1025.06	4.12	176.00	0.96

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
20
40
60
80
100
120
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {40}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

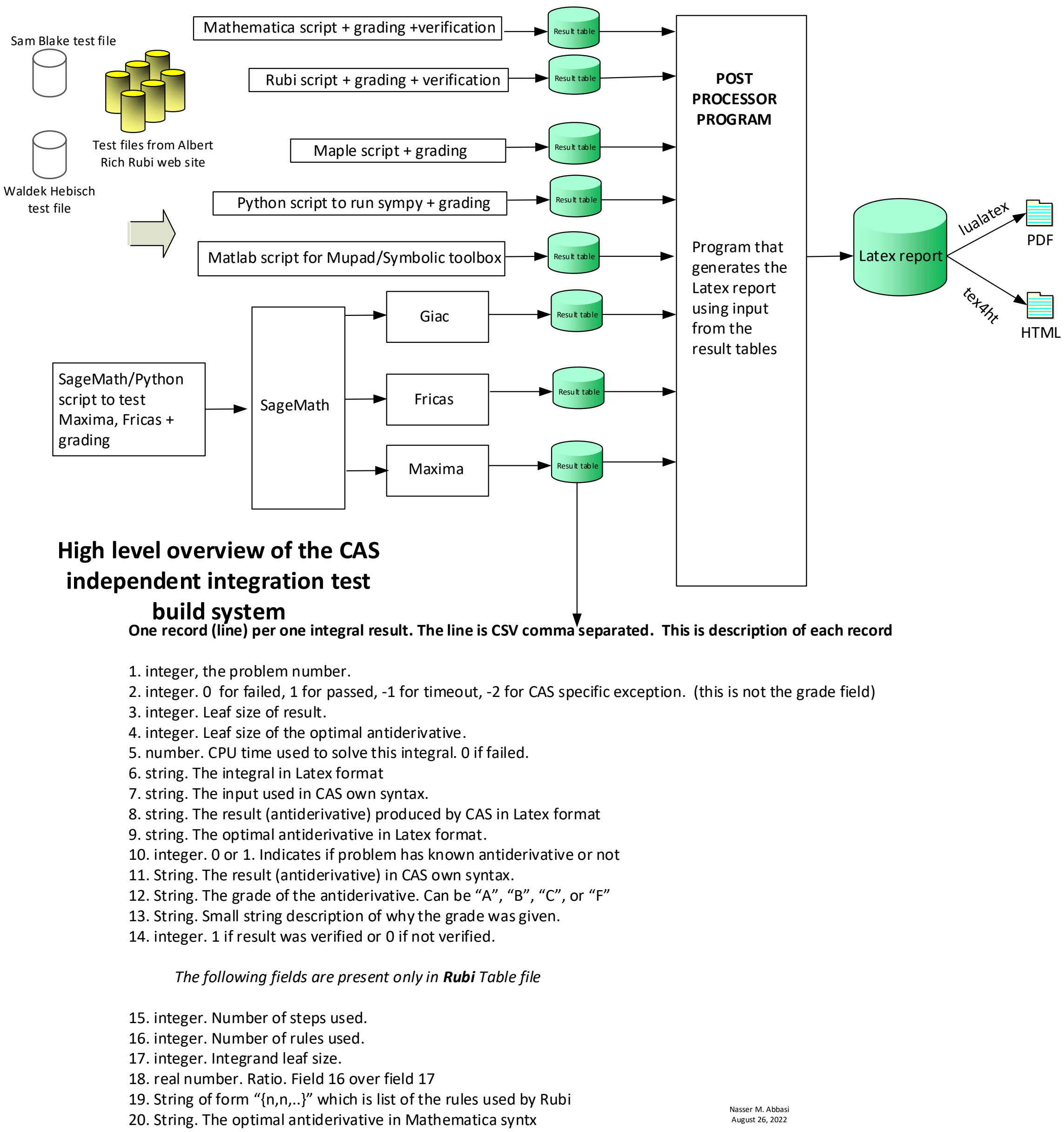
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { }

C grade: { 40, 41, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

B grade: { 37, 38 }

C grade: { 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

F grade: { 40, 41 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 37, 38, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { 37, 38, 39, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129 }

C grade: { 22, 23, 24, 25 }

F grade: { 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 126, 130 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 115, 118, 119, 123, 124 }

B grade: { 37, 38, 39, 110, 113, 114, 116, 117, 120, 121, 122, 131 }

C grade: { 132, 133, 134, 135, 142 }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 125, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 141, 143, 144, 145 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 132, 133, 134, 139, 140 }

B grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 141, 142, 143, 144, 145 }

C grade: { }

F grade: { 40, 41, 135, 136, 137, 138 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

C grade: { }

F grade: { 40, 41 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.84
time (sec)	N/A	0.054	0.011	0.074	0.275	0.356	0.009	3.787	0.036

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.84
time (sec)	N/A	0.037	0.009	0.073	0.287	0.377	0.009	4.002	0.030

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	61	59
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.88	0.86
time (sec)	N/A	0.024	0.009	0.041	0.274	0.378	0.010	6.323	0.029

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	60	55	55	63	60	57
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.97	0.92	0.88
time (sec)	N/A	0.027	0.011	0.013	0.281	0.366	0.054	5.962	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	55	62	58	57	56
N.S.	1	1.00	1.00	0.90	0.87	0.98	0.92	0.90	0.89
time (sec)	N/A	0.034	0.015	0.017	0.276	0.361	0.066	6.501	0.037

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	55	62	61	58	56
N.S.	1	1.00	0.92	0.92	0.87	0.98	0.97	0.92	0.89
time (sec)	N/A	0.036	0.025	0.014	0.281	0.373	0.149	4.623	0.035

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	55	56	62	63	56	55
N.S.	1	1.00	0.95	0.87	0.89	0.98	1.00	0.89	0.87
time (sec)	N/A	0.035	0.033	0.018	0.283	0.395	0.294	3.905	0.033

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	56	56	62	63	57	56
N.S.	1	1.00	0.98	0.89	0.89	0.98	1.00	0.90	0.89
time (sec)	N/A	0.034	0.020	0.017	0.283	0.372	1.082	3.908	0.048

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	56	62	66	57	56
N.S.	1	1.00	1.00	0.89	0.89	0.98	1.05	0.90	0.89
time (sec)	N/A	0.034	0.037	0.016	0.277	0.377	3.153	4.808	0.777

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	59	59	62	70	60	60
N.S.	1	1.00	1.00	0.87	0.87	0.91	1.03	0.88	0.88
time (sec)	N/A	0.031	0.032	0.013	0.275	0.382	9.680	4.202	0.790

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	168	154	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.06	0.97	0.89
time (sec)	N/A	0.148	0.029	0.077	0.281	0.397	0.019	3.401	0.817

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	163	154	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.03	0.97	0.89
time (sec)	N/A	0.098	0.025	0.087	0.296	0.349	0.019	4.187	0.068

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	140	140	165	151	138
N.S.	1	1.00	1.00	0.90	0.91	0.91	1.07	0.98	0.90
time (sec)	N/A	0.074	0.020	0.085	0.279	0.395	0.020	6.004	0.070

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	150	149	138	138	156	149	135
N.S.	1	1.00	1.00	0.99	0.92	0.92	1.04	0.99	0.90
time (sec)	N/A	0.071	0.026	0.019	0.287	0.366	0.117	4.185	0.799

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	147	137	145	156	147	135
N.S.	1	1.00	1.00	1.01	0.94	1.00	1.08	1.01	0.93
time (sec)	N/A	0.082	0.061	0.022	0.326	0.368	0.133	3.769	0.797

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	139	146	139	145	153	148	135
N.S.	1	1.00	0.93	0.98	0.93	0.97	1.03	0.99	0.91
time (sec)	N/A	0.083	0.065	0.023	0.289	0.365	0.213	3.334	0.792

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	151	143	140	145	160	146	137
N.S.	1	1.00	1.01	0.96	0.94	0.97	1.07	0.98	0.92
time (sec)	N/A	0.090	0.052	0.021	0.270	0.365	0.397	3.884	0.059

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	130	139	139	145	153	142	134
N.S.	1	1.00	0.88	0.94	0.94	0.98	1.03	0.96	0.91
time (sec)	N/A	0.097	0.052	0.021	0.267	0.377	1.441	6.232	0.058

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	142	135	138	145	155	140	136
N.S.	1	1.00	0.99	0.94	0.97	1.01	1.08	0.98	0.95
time (sec)	N/A	0.097	0.053	0.023	0.276	0.358	4.467	5.548	0.054

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	136	140	145	158	141	136
N.S.	1	1.00	0.97	0.91	0.94	0.97	1.06	0.95	0.91
time (sec)	N/A	0.092	0.063	0.021	0.268	0.367	29.622	5.529	0.057

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	460	331	0	0	0	5305	2588
N.S.	1	1.00	1.36	0.98	0.00	0.00	0.00	15.65	7.63
time (sec)	N/A	1.215	0.386	0.073	0.000	0.000	0.000	6.581	0.958

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	377	419	0	1329593	0	3520	2696
N.S.	1	1.00	1.36	1.51	0.00	4782.71	0.00	12.66	9.70
time (sec)	N/A	0.317	0.274	0.062	0.000	61.877	0.000	7.757	1.527

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	360	267	0	861800	0	3844	1890
N.S.	1	1.00	1.33	0.99	0.00	3191.85	0.00	14.24	7.00
time (sec)	N/A	0.569	0.238	0.066	0.000	57.707	0.000	6.789	2.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	240	313	0	845032	0	2368	2500
N.S.	1	1.00	1.08	1.40	0.00	3789.38	0.00	10.62	11.21
time (sec)	N/A	0.163	0.234	0.048	0.000	14.292	0.000	5.187	1.885

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	242	0	578003	0	1712	2500
N.S.	1	1.00	1.11	1.15	0.00	2739.35	0.00	8.11	11.85
time (sec)	N/A	0.174	0.139	0.043	0.000	27.603	0.000	6.855	2.306

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	285	251	0	0	0	2337	2258
N.S.	1	1.00	1.24	1.10	0.00	0.00	0.00	10.21	9.86
time (sec)	N/A	0.188	0.293	0.059	0.000	0.000	0.000	6.435	1.493

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	315	325	0	0	0	3508	2588
N.S.	1	1.00	1.21	1.25	0.00	0.00	0.00	13.49	9.95
time (sec)	N/A	0.318	0.693	0.068	0.000	0.000	0.000	7.457	1.022

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	377	300	0	0	0	3354	2500
N.S.	1	1.00	1.31	1.04	0.00	0.00	0.00	11.65	8.68
time (sec)	N/A	0.335	0.619	0.082	0.000	0.000	0.000	6.262	1.172

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	444	550	0	0	0	5220	2500
N.S.	1	1.00	1.08	1.33	0.00	0.00	0.00	12.67	6.07
time (sec)	N/A	0.893	0.876	0.082	0.000	0.000	0.000	8.913	1.774

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	358	458	0	0	0	3229	2500
N.S.	1	1.00	1.03	1.32	0.00	0.00	0.00	9.31	7.20
time (sec)	N/A	0.434	0.572	0.058	0.000	0.000	0.000	5.333	1.613

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	456	0	0	0	4440	2500
N.S.	1	1.00	1.06	1.28	0.00	0.00	0.00	12.47	7.02
time (sec)	N/A	0.630	0.668	0.070	0.000	0.000	0.000	6.104	1.671

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	335	510	0	0	0	3014	2500
N.S.	1	1.00	1.06	1.61	0.00	0.00	0.00	9.51	7.89
time (sec)	N/A	0.294	0.801	0.157	0.000	0.000	0.000	8.337	1.595

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	393	579	0	0	0	5158	2500
N.S.	1	1.00	1.07	1.57	0.00	0.00	0.00	14.02	6.79
time (sec)	N/A	0.597	0.796	0.141	0.000	0.000	0.000	7.624	1.675

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	458	566	0	0	0	6024	2500
N.S.	1	1.00	1.14	1.40	0.00	0.00	0.00	14.95	6.20
time (sec)	N/A	0.649	0.931	0.104	0.000	0.000	0.000	7.385	1.838

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	559	667	0	0	0	9016	2500
N.S.	1	1.00	1.09	1.30	0.00	0.00	0.00	17.54	4.86
time (sec)	N/A	1.073	1.259	0.135	0.000	0.000	0.000	9.025	2.468

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	655	778	0	0	0	6939	2500
N.S.	1	1.00	1.23	1.46	0.00	0.00	0.00	12.99	4.68
time (sec)	N/A	1.363	1.539	0.142	0.000	0.000	0.000	8.191	2.773

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	611	3898	47658	7808	2500
N.S.	1	1.00	0.74	13.83	1.53	9.77	119.44	19.57	6.27
time (sec)	N/A	0.281	2.106	0.036	0.427	0.427	3.363	3.716	3.280

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	187	2187	344	1603	16323	3203	1314
N.S.	1	1.00	0.72	8.41	1.32	6.17	62.78	12.32	5.05
time (sec)	N/A	0.156	0.897	0.022	0.335	0.410	1.468	5.223	1.810

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	92	136	155	444	3628	914	527
N.S.	1	1.00	0.67	0.99	1.13	3.24	26.48	6.67	3.85
time (sec)	N/A	0.063	0.279	0.022	0.306	0.380	0.584	6.065	1.075

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	438	0	0	32	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.09	0.00	0.00	-0.00
time (sec)	N/A	0.432	1.150	0.018	0.000	0.359	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	670	242	0	0	57	0	0	-1
N.S.	1	0.98	0.35	0.00	0.00	0.08	0.00	0.00	-0.00
time (sec)	N/A	1.583	1.182	0.006	0.000	0.361	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	456	0	0	0	4440	2500
N.S.	1	1.00	1.06	1.28	0.00	0.00	0.00	12.47	7.02
time (sec)	N/A	0.623	0.676	0.042	0.000	0.000	0.000	7.202	0.004

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	456	0	0	0	4440	2500
N.S.	1	1.00	1.06	1.28	0.00	0.00	0.00	12.47	7.02
time (sec)	N/A	0.270	0.104	0.053	0.000	0.000	0.000	7.457	1.551

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	456	0	0	0	4440	2500
N.S.	1	1.00	1.06	1.28	0.00	0.00	0.00	12.47	7.02
time (sec)	N/A	0.264	0.102	0.053	0.000	0.000	0.000	8.155	1.474

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	456	0	0	0	4440	2500
N.S.	1	1.00	1.06	1.28	0.00	0.00	0.00	12.47	7.02
time (sec)	N/A	0.263	0.102	0.060	0.000	0.000	0.000	7.393	1.389

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	456	0	0	0	4440	2500
N.S.	1	1.00	1.06	1.28	0.00	0.00	0.00	12.47	7.02
time (sec)	N/A	0.257	0.101	0.051	0.000	0.000	0.000	7.193	1.414

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	260	330	0	900	0	306	2972
N.S.	1	1.00	0.95	1.21	0.00	3.30	0.00	1.12	10.89
time (sec)	N/A	0.550	0.133	0.112	0.000	0.619	0.000	6.447	1.604

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	193	224	0	677	0	214	2295
N.S.	1	1.00	0.95	1.10	0.00	3.33	0.00	1.05	11.31
time (sec)	N/A	0.288	0.096	0.094	0.000	0.564	0.000	4.283	1.626

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	136	146	0	473	0	141	1689
N.S.	1	1.00	0.94	1.01	0.00	3.28	0.00	0.98	11.73
time (sec)	N/A	0.181	0.067	0.109	0.000	0.494	0.000	3.646	1.300

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	101	0	318	0	99	1081
N.S.	1	1.00	0.97	0.98	0.00	3.09	0.00	0.96	10.50
time (sec)	N/A	0.121	0.045	0.062	0.000	0.458	0.000	4.479	1.830

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	178	99	0	309	0	97	2500
N.S.	1	1.00	1.84	1.02	0.00	3.19	0.00	1.00	25.77
time (sec)	N/A	0.134	0.085	0.050	0.000	0.625	0.000	6.102	8.881

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	203	132	0	399	0	135	2500
N.S.	1	1.00	1.72	1.12	0.00	3.38	0.00	1.14	21.19
time (sec)	N/A	0.188	0.096	0.054	0.000	0.600	0.000	3.389	7.857

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	314	203	0	609	0	212	2500
N.S.	1	1.00	1.80	1.17	0.00	3.50	0.00	1.22	14.37
time (sec)	N/A	0.266	0.218	0.074	0.000	0.912	0.000	4.700	9.917

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	416	294	0	834	0	313	2500
N.S.	1	1.00	1.70	1.20	0.00	3.42	0.00	1.28	10.25
time (sec)	N/A	0.376	0.216	0.079	0.000	1.642	0.000	6.809	13.829

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	456	453	0	15467	0	7243	2500
N.S.	1	1.00	1.24	1.23	0.00	41.92	0.00	19.63	6.78
time (sec)	N/A	3.115	0.323	0.057	0.000	40.747	0.000	6.118	4.912

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	365	333	0	9364	0	5461	2500
N.S.	1	1.00	1.29	1.18	0.00	33.21	0.00	19.37	8.87
time (sec)	N/A	2.418	0.310	0.056	0.000	8.230	0.000	5.952	3.359

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	258	224	0	5788	0	4086	2500
N.S.	1	1.00	1.18	1.02	0.00	26.43	0.00	18.66	11.42
time (sec)	N/A	0.442	0.204	0.043	0.000	4.058	0.000	7.595	3.360

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	253	220	0	5930	0	3988	2500
N.S.	1	1.00	1.19	1.03	0.00	27.84	0.00	18.72	11.74
time (sec)	N/A	0.572	0.194	0.062	0.000	1.683	0.000	8.507	3.515

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	284	244	0	9850	0	3813	2500
N.S.	1	1.00	1.06	0.91	0.00	36.89	0.00	14.28	9.36
time (sec)	N/A	0.699	0.221	0.071	0.000	11.021	0.000	7.057	4.763

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	394	360	0	15830	0	6718	2500
N.S.	1	1.00	1.20	1.09	0.00	48.12	0.00	20.42	7.60
time (sec)	N/A	1.299	0.358	0.075	0.000	42.362	0.000	5.530	6.247

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	309	432	0	2111	0	424	2500
N.S.	1	1.00	0.97	1.35	0.00	6.60	0.00	1.32	7.81
time (sec)	N/A	0.813	0.320	0.154	0.000	0.626	0.000	5.527	1.333

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	236	309	0	1455	0	279	2450
N.S.	1	1.00	1.00	1.31	0.00	6.17	0.00	1.18	10.38
time (sec)	N/A	0.294	0.230	0.102	0.000	0.462	0.000	5.877	1.811

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	175	228	0	970	0	195	1651
N.S.	1	1.00	1.06	1.38	0.00	5.88	0.00	1.18	10.01
time (sec)	N/A	0.190	0.165	0.082	0.000	0.412	0.000	3.724	2.717

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	139	0	650	0	140	342
N.S.	1	1.00	1.06	1.13	0.00	5.28	0.00	1.14	2.78
time (sec)	N/A	0.122	0.067	0.054	0.000	0.388	0.000	3.637	0.378

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	268	228	0	1103	0	227	2500
N.S.	1	1.00	1.61	1.37	0.00	6.64	0.00	1.37	15.06
time (sec)	N/A	0.266	0.289	0.073	0.000	1.016	0.000	3.872	11.849

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	403	316	0	1764	0	287	2500
N.S.	1	1.00	1.72	1.35	0.00	7.54	0.00	1.23	10.68
time (sec)	N/A	0.483	0.416	0.089	0.000	2.024	0.000	5.063	12.979

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	592	466	0	2567	0	535	2500
N.S.	1	1.00	1.80	1.42	0.00	7.80	0.00	1.63	7.60
time (sec)	N/A	0.779	0.777	0.112	0.000	4.313	0.000	5.454	21.016

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	648	682	0	18909	0	8957	2500
N.S.	1	1.00	1.18	1.24	0.00	34.38	0.00	16.29	4.55
time (sec)	N/A	9.873	1.304	0.098	0.000	75.856	0.000	7.932	4.104

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	511	522	0	12597	0	7496	2500
N.S.	1	1.00	1.17	1.20	0.00	28.89	0.00	17.19	5.73
time (sec)	N/A	3.601	0.960	0.072	0.000	22.844	0.000	7.058	2.648

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	414	415	0	8951	0	6208	2500
N.S.	1	1.00	1.14	1.15	0.00	24.73	0.00	17.15	6.91
time (sec)	N/A	1.737	0.676	0.063	0.000	12.980	0.000	6.066	6.543

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	382	387	0	8991	0	6356	2500
N.S.	1	1.00	1.10	1.12	0.00	25.99	0.00	18.37	7.23
time (sec)	N/A	1.259	0.663	0.060	0.000	11.408	0.000	6.986	6.552

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	444	438	0	13111	0	7182	2500
N.S.	1	1.00	1.11	1.10	0.00	32.86	0.00	18.00	6.27
time (sec)	N/A	1.397	0.824	0.114	0.000	26.383	0.000	8.214	6.862

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	548	570	0	19333	0	8660	2500
N.S.	1	1.00	0.95	0.99	0.00	33.62	0.00	15.06	4.35
time (sec)	N/A	6.511	1.117	0.134	0.000	84.860	0.000	8.753	7.370

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	58	82	61	63	57
N.S.	1	1.00	0.91	0.82	0.85	1.21	0.90	0.93	0.84
time (sec)	N/A	0.085	0.019	0.040	0.281	0.373	0.059	4.832	0.056

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	51	53	77	56	58	53
N.S.	1	1.00	1.00	0.84	0.87	1.26	0.92	0.95	0.87
time (sec)	N/A	0.078	0.019	0.029	0.280	0.375	0.061	5.196	0.039

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	46	48	72	48	53	47
N.S.	1	1.00	1.00	0.85	0.89	1.33	0.89	0.98	0.87
time (sec)	N/A	0.077	0.017	0.039	0.264	0.362	0.059	3.831	0.897

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	41	43	67	44	45	43
N.S.	1	1.00	1.00	0.84	0.88	1.37	0.90	0.92	0.88
time (sec)	N/A	0.058	0.015	0.027	0.271	0.368	0.061	2.654	0.038

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	38	57	36	40	37
N.S.	1	1.00	1.00	0.86	0.90	1.36	0.86	0.95	0.88
time (sec)	N/A	0.033	0.012	0.025	0.296	0.360	0.057	3.007	0.049

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	44	71	41	47	40
N.S.	1	1.00	1.00	0.86	1.00	1.61	0.93	1.07	0.91
time (sec)	N/A	0.051	0.016	0.030	0.273	0.364	0.065	4.166	0.041

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	45	53	92	51	53	50
N.S.	1	1.00	0.91	0.82	0.96	1.67	0.93	0.96	0.91
time (sec)	N/A	0.071	0.018	0.031	0.270	0.369	0.075	5.429	0.044

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	50	56	97	56	66	55
N.S.	1	1.00	0.88	0.78	0.88	1.52	0.88	1.03	0.86
time (sec)	N/A	0.071	0.018	0.033	0.277	0.365	0.082	6.024	0.919

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	56	58	79	68	58	58
N.S.	1	1.00	1.01	0.80	0.83	1.13	0.97	0.83	0.83
time (sec)	N/A	0.055	0.031	0.033	0.490	0.366	0.082	5.877	0.952

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	58	49	51	74	54	51	50
N.S.	1	1.00	1.02	0.86	0.89	1.30	0.95	0.89	0.88
time (sec)	N/A	0.051	0.032	0.044	0.482	0.371	0.081	5.192	0.054

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	46	48	69	54	48	48
N.S.	1	1.00	1.02	0.82	0.86	1.23	0.96	0.86	0.86
time (sec)	N/A	0.047	0.029	0.030	0.490	0.355	0.080	5.472	0.918

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	41	43	64	48	43	42
N.S.	1	1.00	1.02	0.84	0.88	1.31	0.98	0.88	0.86
time (sec)	N/A	0.043	0.026	0.030	0.485	0.370	0.081	4.621	0.068

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	38	40	59	46	40	40
N.S.	1	1.00	0.96	0.79	0.83	1.23	0.96	0.83	0.83
time (sec)	N/A	0.018	0.028	0.028	0.487	0.369	0.079	3.700	0.072

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	43	45	68	49	45	45
N.S.	1	1.00	0.96	0.81	0.85	1.28	0.92	0.85	0.85
time (sec)	N/A	0.049	0.034	0.034	0.497	0.357	0.087	3.299	0.070

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	48	52	79	56	52	51
N.S.	1	1.00	0.90	0.77	0.84	1.27	0.90	0.84	0.82
time (sec)	N/A	0.054	0.037	0.042	0.486	0.363	0.096	3.808	0.923

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	53	57	84	61	57	57
N.S.	1	1.00	0.88	0.77	0.83	1.22	0.88	0.83	0.83
time (sec)	N/A	0.060	0.041	0.036	0.496	0.379	0.103	5.112	0.916

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	77	58	62	89	66	62	61
N.S.	1	1.00	1.01	0.76	0.82	1.17	0.87	0.82	0.80
time (sec)	N/A	0.065	0.038	0.036	0.497	0.384	0.110	4.764	0.074

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	71	64	71	114	75	61	70
N.S.	1	1.00	0.88	0.79	0.88	1.41	0.93	0.75	0.86
time (sec)	N/A	0.075	0.038	0.034	0.514	0.365	0.103	4.448	0.059

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	62	68	109	76	58	68
N.S.	1	1.00	0.82	0.78	0.85	1.36	0.95	0.72	0.85
time (sec)	N/A	0.081	0.038	0.034	0.503	0.377	0.102	4.482	0.052

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	56	63	104	70	53	63
N.S.	1	1.00	0.80	0.75	0.84	1.39	0.93	0.71	0.84
time (sec)	N/A	0.060	0.043	0.035	0.494	0.376	0.104	2.875	0.928

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	54	60	99	65	50	59
N.S.	1	1.00	0.76	0.75	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.046	0.040	0.032	0.495	0.371	0.101	3.101	0.928

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	60	99	66	50	60
N.S.	1	1.00	0.78	0.74	0.83	1.38	0.92	0.69	0.83
time (sec)	N/A	0.045	0.042	0.029	0.496	0.359	0.102	4.320	0.071

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	60	99	65	50	59
N.S.	1	1.00	0.78	0.74	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.025	0.041	0.031	0.495	0.371	0.099	4.727	0.070

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	58	65	108	71	55	65
N.S.	1	1.00	0.80	0.73	0.82	1.37	0.90	0.70	0.82
time (sec)	N/A	0.069	0.045	0.037	0.488	0.366	0.112	5.462	0.920

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	64	72	119	76	62	71
N.S.	1	1.00	0.91	0.74	0.84	1.38	0.88	0.72	0.83
time (sec)	N/A	0.077	0.039	0.040	0.493	0.364	0.116	5.315	0.923

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	68	77	124	82	67	77
N.S.	1	1.00	0.78	0.73	0.83	1.33	0.88	0.72	0.83
time (sec)	N/A	0.088	0.051	0.038	0.488	0.362	0.123	4.799	0.935

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	74	71	95	87	76	75
N.S.	1	1.00	0.91	0.86	0.83	1.10	1.01	0.88	0.87
time (sec)	N/A	0.088	0.031	0.030	0.529	0.361	0.067	3.371	0.901

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	69	66	90	80	71	69
N.S.	1	1.00	0.90	0.85	0.81	1.11	0.99	0.88	0.85
time (sec)	N/A	0.086	0.021	0.031	0.509	0.374	0.067	3.724	0.051

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	64	59	85	73	66	65
N.S.	1	1.00	0.89	0.86	0.80	1.15	0.99	0.89	0.88
time (sec)	N/A	0.082	0.021	0.039	0.511	0.374	0.066	3.089	0.045

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	59	54	80	68	54	60
N.S.	1	1.00	0.94	0.91	0.83	1.23	1.05	0.83	0.92
time (sec)	N/A	0.069	0.018	0.028	0.507	0.367	0.066	3.311	0.919

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	54	49	70	60	49	69
N.S.	1	1.00	1.00	0.93	0.84	1.21	1.03	0.84	1.19
time (sec)	N/A	0.048	0.015	0.025	0.505	0.364	0.065	4.279	0.048

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	58	55	84	65	62	59
N.S.	1	1.00	1.41	0.88	0.83	1.27	0.98	0.94	0.89
time (sec)	N/A	0.072	0.045	0.030	0.493	0.373	0.072	5.420	0.907

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	63	66	105	76	66	68
N.S.	1	1.00	1.37	0.89	0.93	1.48	1.07	0.93	0.96
time (sec)	N/A	0.093	0.036	0.031	0.506	0.363	0.083	5.380	0.061

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	105	68	71	110	80	79	72
N.S.	1	1.00	1.31	0.85	0.89	1.38	1.00	0.99	0.90
time (sec)	N/A	0.093	0.042	0.032	0.508	0.363	0.087	4.299	0.060

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	110	73	76	115	85	84	78
N.S.	1	1.00	1.26	0.84	0.87	1.32	0.98	0.97	0.90
time (sec)	N/A	0.098	0.046	0.033	0.506	0.368	0.096	4.085	0.065

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	145	296	0	521	71	585	171
N.S.	1	1.00	0.58	1.19	0.00	2.10	0.29	2.36	0.69
time (sec)	N/A	0.231	0.112	0.091	0.000	0.383	0.331	3.745	0.106

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	132	288	0	483	1205	576	164
N.S.	1	1.00	0.56	1.22	0.00	2.04	5.08	2.43	0.69
time (sec)	N/A	0.194	0.098	0.044	0.000	0.374	0.731	3.013	0.943

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	129	285	0	510	60	573	162
N.S.	1	1.00	0.56	1.23	0.00	2.20	0.26	2.47	0.70
time (sec)	N/A	0.199	0.099	0.038	0.000	0.368	0.337	3.687	0.095

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	121	281	0	461	51	566	156
N.S.	1	1.00	0.54	1.25	0.00	2.05	0.23	2.52	0.69
time (sec)	N/A	0.194	0.102	0.037	0.000	0.372	0.330	5.358	0.958

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	115	277	0	461	1185	565	153
N.S.	1	1.00	0.51	1.24	0.00	2.06	5.29	2.52	0.68
time (sec)	N/A	0.142	0.172	0.038	0.000	0.373	0.721	4.033	0.127

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	126	283	0	478	1192	572	159
N.S.	1	1.00	0.55	1.24	0.00	2.09	5.21	2.50	0.69
time (sec)	N/A	0.195	0.116	0.043	0.000	0.407	0.748	3.778	0.136

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	131	288	0	530	60	579	165
N.S.	1	1.00	0.55	1.21	0.00	2.23	0.25	2.43	0.69
time (sec)	N/A	0.201	0.182	0.045	0.000	0.390	0.349	4.470	0.137

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	140	293	0	503	1202	584	171
N.S.	1	1.00	0.57	1.20	0.00	2.05	4.91	2.38	0.70
time (sec)	N/A	0.203	0.196	0.045	0.000	0.380	0.761	4.449	0.143

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	156	298	0	568	1204	588	184
N.S.	1	1.00	0.64	1.23	0.00	2.34	4.95	2.42	0.76
time (sec)	N/A	0.234	0.138	0.043	0.000	0.386	0.718	5.734	0.111

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	155	295	0	564	82	585	182
N.S.	1	1.00	0.64	1.22	0.00	2.33	0.34	2.42	0.75
time (sec)	N/A	0.207	0.128	0.040	0.000	0.380	0.364	4.572	0.942

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	138	291	0	553	71	580	176
N.S.	1	1.00	0.59	1.24	0.00	2.35	0.30	2.47	0.75
time (sec)	N/A	0.204	0.203	0.038	0.000	0.385	0.347	3.210	0.992

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	129	287	0	548	1198	577	173
N.S.	1	1.00	0.54	1.21	0.00	2.30	5.03	2.42	0.73
time (sec)	N/A	0.183	0.194	0.042	0.000	0.371	0.701	4.625	0.146

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	133	287	0	574	1200	577	174
N.S.	1	1.00	0.54	1.17	0.00	2.33	4.88	2.35	0.71
time (sec)	N/A	0.189	0.195	0.045	0.000	0.388	0.716	4.573	1.012

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	129	287	0	580	1195	577	173
N.S.	1	1.00	0.52	1.16	0.00	2.34	4.82	2.33	0.70
time (sec)	N/A	0.174	0.188	0.050	0.000	0.384	0.692	3.986	1.008

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	140	293	0	633	75	582	179
N.S.	1	1.00	0.55	1.16	0.00	2.50	0.30	2.30	0.71
time (sec)	N/A	0.231	0.240	0.046	0.000	0.399	0.361	4.183	0.993

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	139	298	0	652	80	589	185
N.S.	1	1.00	0.53	1.14	0.00	2.49	0.31	2.25	0.71
time (sec)	N/A	0.249	0.215	0.046	0.000	0.396	0.367	7.547	1.022

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	142	151	0	486	0	146	1834
N.S.	1	1.00	0.95	1.01	0.00	3.26	0.00	0.98	12.31
time (sec)	N/A	0.188	0.078	0.112	0.000	0.465	0.000	5.654	1.685

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	721	760	0	0	0	10761	2500
N.S.	1	1.00	1.21	1.28	0.00	0.00	0.00	18.12	4.21
time (sec)	N/A	11.263	1.640	0.097	0.000	0.000	0.000	7.928	4.731

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	575	584	0	23774	0	9170	2500
N.S.	1	1.00	1.22	1.24	0.00	50.48	0.00	19.47	5.31
time (sec)	N/A	4.354	1.192	0.083	0.000	176.559	0.000	7.382	4.218

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	512	543	0	19375	0	8913	2500
N.S.	1	1.00	1.14	1.21	0.00	43.15	0.00	19.85	5.57
time (sec)	N/A	1.957	1.023	0.078	0.000	141.882	0.000	6.922	5.821

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	529	550	0	23991	0	9176	2500
N.S.	1	1.00	1.15	1.20	0.00	52.15	0.00	19.95	5.43
time (sec)	N/A	1.814	1.512	0.145	0.000	128.532	0.000	9.077	7.761

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	612	629	0	0	0	10422	2500
N.S.	1	1.00	1.13	1.16	0.00	0.00	0.00	19.23	4.61
time (sec)	N/A	4.859	1.316	0.152	0.000	0.000	0.000	6.394	8.467

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	31	31	54	58	31
N.S.	1	1.00	1.00	1.05	1.55	1.55	2.70	2.90	1.55
time (sec)	N/A	0.029	0.382	0.034	0.324	0.403	174.572	5.214	1.098

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	278	149	145	271	138	367	225	287
N.S.	1	1.32	0.71	0.69	1.29	0.66	1.75	1.07	1.37
time (sec)	N/A	0.207	0.318	0.170	0.505	0.381	81.994	3.549	1.645

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	213	116	109	199	104	367	163	215
N.S.	1	1.34	0.73	0.69	1.25	0.65	2.31	1.03	1.35
time (sec)	N/A	0.128	0.238	0.168	0.488	0.391	56.862	4.544	1.494

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	149	80	73	127	71	350	103	143
N.S.	1	1.37	0.73	0.67	1.17	0.65	3.21	0.94	1.31
time (sec)	N/A	0.082	0.164	0.143	0.488	0.415	37.089	5.204	1.377

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	151	106	143	98	80	304	0	161
N.S.	1	1.62	1.14	1.54	1.05	0.86	3.27	0.00	1.73
time (sec)	N/A	0.108	0.219	0.158	0.501	0.393	45.402	0.000	2.949

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	155	85	163	117	98	0	0	422
N.S.	1	1.57	0.86	1.65	1.18	0.99	0.00	0.00	4.26
time (sec)	N/A	0.162	0.254	0.157	0.492	0.413	0.000	0.000	5.151

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	182	102	222	184	102	0	0	932
N.S.	1	1.44	0.81	1.76	1.46	0.81	0.00	0.00	7.40
time (sec)	N/A	0.187	0.263	0.157	0.489	0.381	0.000	0.000	10.816

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	248	141	306	256	137	0	0	1621
N.S.	1	1.17	0.67	1.44	1.21	0.65	0.00	0.00	7.65
time (sec)	N/A	0.251	0.398	0.182	0.504	0.391	0.000	0.000	20.051

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	245	134	273	178	134	0	176	1132
N.S.	1	1.13	0.62	1.26	0.82	0.62	0.00	0.81	5.24
time (sec)	N/A	0.136	0.285	0.150	0.481	0.363	0.000	5.198	23.121

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	179	98	191	107	100	0	116	651
N.S.	1	1.40	0.77	1.49	0.84	0.78	0.00	0.91	5.09
time (sec)	N/A	0.061	0.198	0.144	0.487	0.368	0.000	5.718	12.861

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	155	86	148	70	90	0	251	306
N.S.	1	1.52	0.84	1.45	0.69	0.88	0.00	2.46	3.00
time (sec)	N/A	0.079	0.177	0.156	0.493	0.364	0.000	5.663	7.002

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	81	146	81	90	257	555	138
N.S.	1	1.00	0.52	0.93	0.52	0.57	1.64	3.54	0.88
time (sec)	N/A	0.080	0.167	0.152	0.499	0.370	55.950	3.482	2.268

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	87	86	139	76	0	1103	146
N.S.	1	1.00	0.54	0.54	0.87	0.48	0.00	6.89	0.91
time (sec)	N/A	0.097	0.140	0.154	0.492	0.369	0.000	3.511	1.732

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	124	122	211	110	0	1517	218
N.S.	1	1.00	0.55	0.54	0.93	0.49	0.00	6.71	0.96
time (sec)	N/A	0.119	0.189	0.168	0.509	0.396	0.000	6.406	1.817

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	158	158	283	144	0	1931	290
N.S.	1	1.00	0.54	0.54	0.97	0.49	0.00	6.61	0.99
time (sec)	N/A	0.156	0.236	0.161	0.494	0.440	0.000	7.385	1.873

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [131] had the largest ratio of [42]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	26	0.038
2	A	2	1	1.00	24	0.042
3	A	2	1	1.00	23	0.043
4	A	2	1	1.00	26	0.038
5	A	2	1	1.00	26	0.038
6	A	2	1	1.00	26	0.038
7	A	2	1	1.00	26	0.038
8	A	2	1	1.00	26	0.038
9	A	2	1	1.00	26	0.038
10	A	2	1	1.00	26	0.038
11	A	2	1	1.00	28	0.036
12	A	2	1	1.00	26	0.038
13	A	2	1	1.00	25	0.040
14	A	2	1	1.00	28	0.036
15	A	2	1	1.00	28	0.036
16	A	2	1	1.00	28	0.036
17	A	2	1	1.00	28	0.036
18	A	2	1	1.00	28	0.036
19	A	2	1	1.00	28	0.036
20	A	2	1	1.00	28	0.036
21	A	13	11	1.00	28	0.393
22	A	12	11	1.00	28	0.393
23	A	11	10	1.00	28	0.357
24	A	10	9	1.00	26	0.346
25	A	8	7	1.00	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	12	10	1.00	28	0.357
27	A	13	12	1.00	28	0.429
28	A	13	11	1.00	28	0.393
29	A	11	10	1.00	28	0.357
30	A	10	9	1.00	28	0.321
31	A	10	9	1.00	28	0.321
32	A	10	9	1.00	26	0.346
33	A	10	9	1.00	25	0.360
34	A	14	12	1.00	28	0.429
35	A	15	13	1.00	28	0.464
36	A	15	13	1.00	28	0.464
37	A	2	1	1.00	30	0.033
38	A	2	1	1.00	30	0.033
39	A	2	1	1.00	28	0.036
40	A	8	5	1.00	30	0.167
41	A	10	6	0.98	30	0.200
42	A	10	9	1.00	28	0.321
43	A	11	10	1.00	30	0.333
44	A	11	10	1.00	31	0.323
45	A	11	10	1.00	34	0.294
46	A	11	10	1.00	34	0.294
47	A	7	6	1.00	30	0.200
48	A	7	6	1.00	30	0.200
49	A	7	6	1.00	30	0.200
50	A	7	6	1.00	28	0.214
51	A	7	6	1.00	30	0.200
52	A	7	6	1.00	30	0.200
53	A	7	6	1.00	30	0.200
54	A	7	6	1.00	30	0.200
55	A	5	3	1.00	30	0.100
56	A	5	3	1.00	30	0.100
57	A	5	3	1.00	27	0.111
58	A	5	3	1.00	30	0.100
59	A	5	3	1.00	30	0.100
60	A	5	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	7	1.00	30	0.233
62	A	7	7	1.00	30	0.233
63	A	6	6	1.00	30	0.200
64	A	5	5	1.00	28	0.179
65	A	8	7	1.00	30	0.233
66	A	8	7	1.00	30	0.233
67	A	8	7	1.00	30	0.233
68	A	6	4	1.00	30	0.133
69	A	6	4	1.00	30	0.133
70	A	4	3	1.00	30	0.100
71	A	4	3	1.00	27	0.111
72	A	6	4	1.00	30	0.133
73	A	6	4	1.00	30	0.133
74	A	7	5	1.00	31	0.161
75	A	7	5	1.00	31	0.161
76	A	7	5	1.00	31	0.161
77	A	7	5	1.00	31	0.161
78	A	5	4	1.00	29	0.138
79	A	4	3	1.00	31	0.097
80	A	4	3	1.00	31	0.097
81	A	4	3	1.00	31	0.097
82	A	6	4	1.00	31	0.129
83	A	6	4	1.00	31	0.129
84	A	6	4	1.00	31	0.129
85	A	6	4	1.00	31	0.129
86	A	4	3	1.00	28	0.107
87	A	5	3	1.00	31	0.097
88	A	5	3	1.00	31	0.097
89	A	5	3	1.00	31	0.097
90	A	5	3	1.00	31	0.097
91	A	7	5	1.00	31	0.161
92	A	7	5	1.00	31	0.161
93	A	7	5	1.00	31	0.161
94	A	5	4	1.00	31	0.129
95	A	5	4	1.00	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	28	0.143
97	A	6	3	1.00	31	0.097
98	A	6	3	1.00	31	0.097
99	A	6	3	1.00	31	0.097
100	A	8	7	1.00	31	0.226
101	A	8	7	1.00	31	0.226
102	A	8	7	1.00	31	0.226
103	A	8	7	1.00	31	0.226
104	A	6	6	1.00	29	0.207
105	A	8	7	1.00	31	0.226
106	A	8	7	1.00	31	0.226
107	A	8	7	1.00	31	0.226
108	A	8	7	1.00	31	0.226
109	A	12	7	1.00	31	0.226
110	A	12	7	1.00	31	0.226
111	A	12	7	1.00	31	0.226
112	A	12	7	1.00	31	0.226
113	A	10	6	1.00	28	0.214
114	A	12	7	1.00	31	0.226
115	A	12	7	1.00	31	0.226
116	A	12	7	1.00	31	0.226
117	A	13	8	1.00	31	0.258
118	A	13	8	1.00	31	0.258
119	A	13	8	1.00	31	0.258
120	A	11	7	1.00	31	0.226
121	A	11	7	1.00	31	0.226
122	A	11	7	1.00	28	0.250
123	A	13	7	1.00	31	0.226
124	A	13	7	1.00	31	0.226
125	A	7	6	1.00	33	0.182
126	A	6	4	1.00	35	0.114
127	A	6	4	1.00	35	0.114
128	A	4	3	1.00	32	0.094
129	A	6	4	1.00	35	0.114
130	A	6	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	1	1	1.00	42	0.024
132	A	5	4	1.32	35	0.114
133	A	4	3	1.34	35	0.086
134	A	4	3	1.37	33	0.091
135	A	6	5	1.62	35	0.143
136	A	6	6	1.57	35	0.171
137	A	6	6	1.44	35	0.171
138	A	7	7	1.17	35	0.200
139	A	6	6	1.13	35	0.171
140	A	5	5	1.40	32	0.156
141	A	5	5	1.52	35	0.143
142	A	5	5	1.00	35	0.143
143	A	4	4	1.00	35	0.114
144	A	5	5	1.00	35	0.143
145	A	6	5	1.00	35	0.143

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=74

$$\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.038$, Rules used = {1642}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx&=\int\left(aAx^2+aBx^3+(Ab+aC)x^4+bBx^5+(Ac+bC)x^6+Bcx\right.\\&=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Maple [A]
time = 0.07, size = 61, normalized size = 0.82

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{aB\,x^{4}}{4}+\frac{(Ab+aC)x^{5}}{5}+\frac{bB\,x^{6}}{6}+\frac{(Ac+bC)x^{7}}{7}+\frac{Bc\,x^{8}}{8}+\frac{cC\,x^{9}}{9}$	61
norman	$\frac{cC\,x^{9}}{9}+\frac{Bc\,x^{8}}{8}+\left(\frac{Ac}{7}+\frac{bC}{7}\right)x^{7}+\frac{bB\,x^{6}}{6}+\left(\frac{Ab}{5}+\frac{aC}{5}\right)x^{5}+\frac{aB\,x^{4}}{4}+\frac{aA\,x^{3}}{3}$	63
gosper	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}bC+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}aC+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65
risch	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}bC+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}aC+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

Maxima [A]
time = 0.27, size = 60, normalized size = 0.81

$$\frac{1}{9}\,Ccx^{9}+\frac{1}{8}\,Bcx^{8}+\frac{1}{6}\,Bbx^{6}+\frac{1}{7}\,(Cb+Ac)x^{7}+\frac{1}{4}\,Bax^{4}+\frac{1}{5}\,(Ca+Ab)x^{5}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4
 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A]
time = 0.36, size = 60, normalized size = 0.81

$$\frac{1}{9}\,Ccx^{9}+\frac{1}{8}\,Bcx^{8}+\frac{1}{6}\,Bbx^{6}+\frac{1}{7}\,(Cb+Ac)x^{7}+\frac{1}{4}\,Bax^{4}+\frac{1}{5}\,(Ca+Ab)x^{5}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4
 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Sympy [A]
time = 0.01, size = 68, normalized size = 0.92

$$\frac{Aax^{3}}{3}+\frac{Bax^{4}}{4}+\frac{Bbx^{6}}{6}+\frac{Bcx^{8}}{8}+\frac{Ccx^{9}}{9}+x^{7}\left(\frac{Ac}{7}+\frac{Cb}{7}\right)+x^{5}\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c/
7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

Giac [A]
time = 3.79, size = 64, normalized size = 0.86

$$\frac{1}{9}\,Ccx^{9} + \frac{1}{8}\,Bcx^{8} + \frac{1}{7}\,Cbx^{7} + \frac{1}{7}\,Acx^{7} + \frac{1}{6}\,Bbx^{6} + \frac{1}{5}\,Cax^{5} + \frac{1}{5}\,Abx^{5} + \frac{1}{4}\,Bax^{4} + \frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*C
*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

Mupad [B]
time = 0.04, size = 62, normalized size = 0.84

$$\frac{C\,c\,x^9}{9}+\frac{B\,c\,x^8}{8}+\left(\frac{A\,c}{7}+\frac{C\,b}{7}\right)\,x^7+\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{5}+\frac{C\,a}{5}\right)\,x^5+\frac{B\,a\,x^4}{4}+\frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((A*b)/5 + (C*a)/5) + x^7*((A*c)/7 + (C*b)/7) + (A*a*x^3)/3 + (B*a*x^4)
/4 + (B*b*x^6)/6 + (B*c*x^8)/8 + (C*c*x^9)/9
```

3.2 $\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1642}

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(aAx+aBx^2+(Ab+aC)x^3+bBx^4+(Ac+bC)x^5+Bcx\right.\\ &= \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Maple [A]
time = 0.07, size = 61, normalized size = 0.82

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{aB\,x^{3}}{3}+\frac{(Ab+aC)x^{4}}{4}+\frac{bB\,x^{5}}{5}+\frac{(Ac+bC)x^{6}}{6}+\frac{Bc\,x^{7}}{7}+\frac{cC\,x^{8}}{8}$	61
norman	$\frac{cC\,x^{8}}{8}+\frac{Bc\,x^{7}}{7}+\left(\frac{Ac}{6}+\frac{bC}{6}\right)x^{6}+\frac{bB\,x^{5}}{5}+\left(\frac{Ab}{4}+\frac{aC}{4}\right)x^{4}+\frac{aB\,x^{3}}{3}+\frac{aA\,x^{2}}{2}$	63
gosper	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}bC+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}aC+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65
risch	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}x^{6}bC+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}aC+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

Maxima [A]
time = 0.29, size = 60, normalized size = 0.81

$$\frac{1}{8}\,Ccx^{8}+\frac{1}{7}\,Bcx^{7}+\frac{1}{5}\,Bbx^{5}+\frac{1}{6}\,(Cb+Ac)x^{6}+\frac{1}{3}\,Bax^{3}+\frac{1}{4}\,(Ca+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^3
 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A]
time = 0.38, size = 60, normalized size = 0.81

$$\frac{1}{8}\,Ccx^{8}+\frac{1}{7}\,Bcx^{7}+\frac{1}{5}\,Bbx^{5}+\frac{1}{6}\,(Cb+Ac)x^{6}+\frac{1}{3}\,Bax^{3}+\frac{1}{4}\,(Ca+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^3
 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Sympy [A]
time = 0.01, size = 68, normalized size = 0.92

$$\frac{Aax^{2}}{2}+\frac{Bax^{3}}{3}+\frac{Bbx^{5}}{5}+\frac{Bcx^{7}}{7}+\frac{Ccx^{8}}{8}+x^{6}\left(\frac{Ac}{6}+\frac{Cb}{6}\right)+x^{4}\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 + x**6*(A*c/
6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

Giac [A]
time = 4.00, size = 64, normalized size = 0.86

$$\frac{1}{8}Ccx^8+\frac{1}{7}Bcx^7+\frac{1}{6}Cbx^6+\frac{1}{6}Acx^6+\frac{1}{5}Bbx^5+\frac{1}{4}Cax^4+\frac{1}{4}Abx^4+\frac{1}{3}Bax^3+\frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x^5 + 1/4*C
*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

Mupad [B]
time = 0.03, size = 62, normalized size = 0.84

$$\frac{C\,c\,x^8}{8}+\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{6}+\frac{C\,b}{6}\right)\,x^6+\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^4+\frac{B\,a\,x^3}{3}+\frac{A\,a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4*((A*b)/4 + (C*a)/4) + x^6*((A*c)/6 + (C*b)/6) + (A*a*x^2)/2 + (B*a*x^3)
/3 + (B*b*x^5)/5 + (B*c*x^7)/7 + (C*c*x^8)/8
```

3.3 $\int (A+Bx+Cx^2) (a+bx^2+cx^4)\,dx$

Optimal. Leaf size=69

$$aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1671}

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \int \left(aA+aBx+(Ab+aC)x^2+bBx^3+(Ac+bC)x^4+Bcx^5+c\right.$$
$$= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bc$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.00

$$aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.84

method	result	size
default	$aAx+\frac{aB\,x^2}{2}+\frac{(Ab+aC)x^3}{3}+\frac{bB\,x^4}{4}+\frac{(Ac+bC)x^5}{5}+\frac{Bc\,x^6}{6}+\frac{cC\,x^7}{7}$	58
norman	$\frac{cC\,x^7}{7}+\frac{Bc\,x^6}{6}+\left(\frac{Ac}{5}+\frac{bC}{5}\right)x^5+\frac{bB\,x^4}{4}+\left(\frac{Ab}{3}+\frac{aC}{3}\right)x^3+\frac{aB\,x^2}{2}+aAx$	60
gosper	$\frac{1}{7}cC\,x^7+\frac{1}{6}Bc\,x^6+\frac{1}{5}x^5Ac+\frac{1}{5}x^5bC+\frac{1}{4}bB\,x^4+\frac{1}{3}x^3Ab+\frac{1}{3}x^3aC+\frac{1}{2}aB\,x^2+aAx$	62
risch	$\frac{1}{7}cC\,x^7+\frac{1}{6}Bc\,x^6+\frac{1}{5}x^5Ac+\frac{1}{5}x^5bC+\frac{1}{4}bB\,x^4+\frac{1}{3}x^3Ab+\frac{1}{3}x^3aC+\frac{1}{2}aB\,x^2+aAx$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.83

$$\frac{1}{7}\,Ccx^7+\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Bbx^4+\frac{1}{5}\,(Cb+Ac)x^5+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^2
 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Fricas [A]
time = 0.38, size = 57, normalized size = 0.83

$$\frac{1}{7}\,Ccx^7+\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Bbx^4+\frac{1}{5}\,(Cb+Ac)x^5+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^2
 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.94

$$Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 + C
*b/5) + x**3*(A*b/3 + C*a/3)
```

Giac [A]
time = 6.32, size = 61, normalized size = 0.88

$$\frac{1}{7}Ccx^7 + \frac{1}{6}Bcx^6 + \frac{1}{5}Cbx^5 + \frac{1}{5}Acx^5 + \frac{1}{4}Bbx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Abx^3 + \frac{1}{2}Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x^4 + 1/3*C
*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B]
time = 0.03, size = 59, normalized size = 0.86

$$\frac{C\,c\,x^7}{7} + \frac{B\,c\,x^6}{6} + \left(\frac{A\,c}{5} + \frac{C\,b}{5}\right)\,x^5 + \frac{B\,b\,x^4}{4} + \left(\frac{A\,b}{3} + \frac{C\,a}{3}\right)\,x^3 + \frac{B\,a\,x^2}{2} + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((A*b)/3 + (C*a)/3) + x^5*((A*c)/5 + (C*b)/5) + A*a*x + (B*a*x^2)/2 + (
B*b*x^4)/4 + (B*c*x^6)/6 + (C*c*x^7)/7
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} \, dx$

Optimal. Leaf size=65

$$aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6+aA\log(x)$$

```
[Out] a*B*x+1/2*(A*b+C*a)*x^2+1/3*b*B*x^3+1/4*(A*c+C*b)*x^4+1/5*B*c*x^5+1/6*c*C*x
^6+a*A*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx &= \int \left(aB+\frac{aA}{x}+(Ab+aC)x+bBx^2+(Ac+bC)x^3+Bcx^4+c\right.\\ &= aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4+\frac{1}{5}Bcx^5+\frac{1}{6}c\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.00

$$aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6+aA\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Maple [A]
time = 0.01, size = 60, normalized size = 0.92

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{aC}{2}\right)x^{2}+\left(\frac{Ac}{4}+\frac{bC}{4}\right)x^{4}+aBx+\frac{Bb\,x^{3}}{3}+\frac{Bc\,x^{5}}{5}+\frac{cC\,x^{6}}{6}+aA\ln\left(x\right)$	58
default	$\frac{cC\,x^{6}}{6}+\frac{Bc\,x^{5}}{5}+\frac{Ac\,x^{4}}{4}+\frac{bC\,x^{4}}{4}+\frac{Bb\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+aBx+aA\ln\left(x\right)$	60
risch	$\frac{cC\,x^{6}}{6}+\frac{Bc\,x^{5}}{5}+\frac{Ac\,x^{4}}{4}+\frac{bC\,x^{4}}{4}+\frac{Bb\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+aBx+aA\ln\left(x\right)$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*C*x^6+1/5*B*c*x^5+1/4*A*c*x^4+1/4*b*C*x^4+1/3*B*b*x^3+1/2*A*b*x^2+1/2
*C*a*x^2+a*B*x+a*A*ln(x)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.85

$$\frac{1}{6}\,Ccx^{6}+\frac{1}{5}\,Bcx^{5}+\frac{1}{3}\,Bbx^{3}+\frac{1}{4}\,(Cb+Ac)x^{4}+Bax+\frac{1}{2}\,(Ca+Ab)x^{2}+Aa\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

Fricas [A]
time = 0.37, size = 55, normalized size = 0.85

$$\frac{1}{6}\,Ccx^{6}+\frac{1}{5}\,Bcx^{5}+\frac{1}{3}\,Bbx^{3}+\frac{1}{4}\,(Cb+Ac)x^{4}+Bax+\frac{1}{2}\,(Ca+Ab)x^{2}+Aa\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A]
time = 0.05, size = 63, normalized size = 0.97

$$Aa\log\left(x\right)+Bax+\frac{Bbx^{3}}{3}+\frac{Bcx^{5}}{5}+\frac{Ccx^{6}}{6}+x^{4}\left(\frac{Ac}{4}+\frac{Cb}{4}\right)+x^{2}\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 + C
*b/4) + x**2*(A*b/2 + C*a/2)
```

Giac [A]
time = 5.96, size = 60, normalized size = 0.92

$$\frac{1}{6}\,Ccx^{6}+\frac{1}{5}\,Bcx^{5}+\frac{1}{4}\,Cbx^{4}+\frac{1}{4}\,Acx^{4}+\frac{1}{3}\,Bbx^{3}+\frac{1}{2}\,Cax^{2}+\frac{1}{2}\,Abx^{2}+Bax+Aa\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*C
*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))
```

Mupad [B]
time = 0.04, size = 57, normalized size = 0.88

$$x^2\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)+x^4\left(\frac{A\,c}{4}+\frac{C\,b}{4}\right)+B\,a\,x+\frac{B\,b\,x^3}{3}+\frac{B\,c\,x^5}{5}+\frac{C\,c\,x^6}{6}+A\,a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^2*((A*b)/2 + (C*a)/2) + x^4*((A*c)/4 + (C*b)/4) + B*a*x + (B*b*x^3)/3 + (
B*c*x^5)/5 + (C*c*x^6)/6 + A*a*log(x)
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

```
[Out] -a*A/x+(A*b+C*a)*x+1/2*b*B*x^2+1/3*(A*c+C*b)*x^3+1/4*B*c*x^4+1/5*c*C*x^5+a*
B*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+bBx+(Ac+bC)x^2+Bcx^3+c\right.\\ &= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 1.00

$$-\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Maple [A]
time = 0.02, size = 57, normalized size = 0.90

method	result	size
default	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{3}}{3}+\frac{Cb\,x^{3}}{3}+\frac{Bb\,x^{2}}{2}+Abx+aCx-\frac{aA}{x}+aB\ln\left(x\right)$	57
risch	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{3}}{3}+\frac{Cb\,x^{3}}{3}+\frac{Bb\,x^{2}}{2}+Abx+aCx-\frac{aA}{x}+aB\ln\left(x\right)$	57
norman	$\frac{\left(\frac{Ac}{3}+\frac{bC}{3}\right)x^{4}+\left(Ab+aC\right)x^{2}-aA+\frac{Bb\,x^{3}}{2}+\frac{Bc\,x^{5}}{4}+\frac{cC\,x^{6}}{5}}{x}+aB\ln\left(x\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*c*x^3+1/3*C*b*x^3+1/2*B*b*x^2+A*b*x+a*C*x-a*A
/x+a*B*ln(x)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.87

$$\frac{1}{5}\,Ccx^{5}+\frac{1}{4}\,Bcx^{4}+\frac{1}{2}\,Bbx^{2}+\frac{1}{3}\,(Cb+Ac)x^{3}+Ba\log\left(x\right)+(Ca+Ab)x-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x)
+ (C*a + A*b)*x - A*a/x
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.98

$$\frac{12\,Ccx^{6}+15\,Bcx^{5}+30\,Bbx^{3}+20\,(Cb+Ac)x^{4}+60\,Bax\log\left(x\right)+60\,(Ca+Ab)x^{2}-60\,Aa}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x*
log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

Sympy [A]
time = 0.07, size = 58, normalized size = 0.92

$$-\frac{Aa}{x}+Ba\log\left(x\right)+\frac{Bbx^{2}}{2}+\frac{Bcx^{4}}{4}+\frac{Ccx^{5}}{5}+x^{3}\left(\frac{Ac}{3}+\frac{Cb}{3}\right)+x(Ab+Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 +
C*b/3) + x*(A*b + C*a)
```

Giac [A]
time = 6.50, size = 57, normalized size = 0.90

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{3}Cbx^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\log(|x|) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*x
 + A*b*x + B*a*log(abs(x)) - A*a/x
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.89

$$x\,(A\,b+C\,a)+x^3\left(\frac{A\,c}{3}+\frac{C\,b}{3}\right)-\frac{A\,a}{x}+\frac{B\,b\,x^2}{2}+\frac{B\,c\,x^4}{4}+\frac{C\,c\,x^5}{5}+B\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x*(A*b + C*a) + x^3*((A*c)/3 + (C*b)/3) - (A*a)/x + (B*b*x^2)/2 + (B*c*x^4)
/4 + (C*c*x^5)/5 + B*a*log(x)
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=63

$$-\frac{aA}{2x^2}-\frac{aB}{x}+bBx+\frac{1}{2}(Ac+bC)x^2+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4+(Ab+aC)\log(x)$$

```
[Out] -1/2*a*A/x^2-a*B/x+b*B*x+1/2*(A*c+C*b)*x^2+1/3*B*c*x^3+1/4*c*C*x^4+(A*b+C*a
)*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*C
*x^4)/4 + (A*b + a*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx &= \int \left(bB+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+(Ac+bC)x+Bcx^2+cCx^3\right)\\ &= -\frac{aA}{2x^2}-\frac{aB}{x}+bBx+\frac{1}{2}(Ac+bC)x^2+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4+(Ab+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.92

$$-\frac{a(A+2Bx)}{2x^2}+\frac{1}{12}x\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right)+(Ab+aC)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*(A + 2*B*x))/x^2 + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^2
)))/12 + (A*b + a*C)*Log[x]
```

Maple [A]
time = 0.01, size = 58, normalized size = 0.92

method	result	size
default	$\frac{cC\,x^{4}}{4}+\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Cb\,x^{2}}{2}+bBx-\frac{aA}{2x^{2}}-\frac{aB}{x}+\left(Ab+aC\right)\ln\left(x\right)$	58
risch	$\frac{cC\,x^{4}}{4}+\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Cb\,x^{2}}{2}+bBx+\frac{-aBx-\frac{1}{2}aA}{x^{2}}+A\ln\left(x\right)b+C\ln\left(x\right)a$	58
norman	$\frac{\left(\frac{Ac}{2}+\frac{bC}{2}\right)x^{4}+Bb\,x^{3}-\frac{aA}{2}+\frac{Bc\,x^{5}}{3}-aBx+\frac{cC\,x^{6}}{4}}{x^{2}}+\left(Ab+aC\right)\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*c*x^2+1/2*C*b*x^2+b*B*x-1/2*a*A/x^2-a*B/x+(A*
b+C*a)*ln(x)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.87

$$\frac{1}{4}\,Ccx^{4}+\frac{1}{3}\,Bcx^{3}+Bbx+\frac{1}{2}\,(Cb+Ac)x^{2}+(Ca+Ab)\log\left(x\right)-\frac{2\,Bax+Aa}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(x
) - 1/2*(2*B*a*x + A*a)/x^2
```

Fricas [A]
time = 0.37, size = 62, normalized size = 0.98

$$\frac{3\,Ccx^{6}+4\,Bcx^{5}+12\,Bbx^{3}+6\,(Cb+Ac)x^{4}+12\,(Ca+Ab)x^{2}\log\left(x\right)-12\,Bax-6\,Aa}{12\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A*
b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

Sympy [A]
time = 0.15, size = 61, normalized size = 0.97

$$Bbx+\frac{Bcx^{3}}{3}+\frac{Ccx^{4}}{4}+x^{2}\left(\frac{Ac}{2}+\frac{Cb}{2}\right)+(Ab+Ca)\log\left(x\right)+\frac{-Aa-2Bax}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x)
 + (-A*a - 2*B*a*x)/(2*x**2)
```

Giac [A]
time = 4.62, size = 58, normalized size = 0.92

$$\frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+\frac{1}{2}\,Cbx^2+\frac{1}{2}\,Acx^2+Bbx+(Ca+Ab)\log\left(|x|\right)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b)
*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

Mupad [B]
time = 0.03, size = 56, normalized size = 0.89

$$x^2\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)-\frac{\frac{A\,a}{2}+B\,a\,x}{x^2}+\ln\left(x\right)\,\left(A\,b+C\,a\right)+B\,b\,x+\frac{B\,c\,x^3}{3}+\frac{C\,c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((A*c)/2 + (C*b)/2) - ((A*a)/2 + B*a*x)/x^2 + log(x)*(A*b + C*a) + B*b*
x + (B*c*x^3)/3 + (C*c*x^4)/4
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+(Ac+bC)x+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3+bB\log(x)$$

```
[Out] -1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+(A*c+C*b)*x+1/2*B*c*x^2+1/3*c*C*x^3+b
*B*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1642}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)/
2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx=\int \left(Ac\left(1+\frac{bC}{Ac}\right)+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB}{x}+Bcx+cCx^2\right)$$
$$=-\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+(Ac+bC)x+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3+bB\,\mathrm{lo}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.95

$$-\frac{Ab}{x}+Acx+bCx+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3-\frac{a(2A+3x(B+2Cx))}{6x^3}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B +
 2*C*x)))/(6*x^3) + b*B*Log[x]
```

Maple [A]
time = 0.02, size = 55, normalized size = 0.87

method	result	size
default	$\frac{cC\,x^{3}}{3}+\frac{Bc\,x^{2}}{2}+Acx+bCx-\frac{aB}{2x^{2}}-\frac{Ab+aC}{x}-\frac{aA}{3x^{3}}+bB\ln\left(x\right)$	55
risch	$\frac{cC\,x^{3}}{3}+\frac{Bc\,x^{2}}{2}+Acx+bCx+\frac{\left(-Ab-aC\right)x^{2}-\frac{aBx}{2}-\frac{aA}{3}}{x^{3}}+bB\ln\left(x\right)$	56
norman	$\frac{\left(-Ab-aC\right)x^{2}+\left(Ac+bC\right)x^{4}-\frac{aA}{3}+\frac{Bc\,x^{5}}{2}-\frac{aBx}{2}+\frac{cC\,x^{6}}{3}}{x^{3}}+bB\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c*C*x^3+1/2*B*c*x^2+A*c*x+b*C*x-1/2*a*B/x^2-(A*b+C*a)/x-1/3*a*A/x^3+b*B
*ln(x)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.89

$$\frac{1}{3}\,Ccx^{3}+\frac{1}{2}\,Bcx^{2}+Bb\log\left(x\right)+(Cb+Ac)x-\frac{3\,Bax+6\,(Ca+Ab)x^{2}+2\,Aa}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*(
C*a + A*b)*x^2 + 2*A*a)/x^3
```

Fricas [A]
time = 0.40, size = 62, normalized size = 0.98

$$\frac{2\,Ccx^{6}+3\,Bcx^{5}+6\,Bbx^{3}\log\left(x\right)+6\,(Cb+Ac)x^{4}-3\,Bax-6\,(Ca+Ab)x^{2}-2\,Aa}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*x
 - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A]
time = 0.29, size = 63, normalized size = 1.00

$$Bb\log\left(x\right)+\frac{Bcx^{2}}{2}+\frac{Ccx^{3}}{3}+x(Ac+Cb)+\frac{-2Aa-3Bax+x^{2}(-6Ab-6Ca)}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) + (-2*A*a - 3*B*a*x +
x**2*(-6*A*b - 6*C*a))/(6*x**3)
```

Giac [A]
time = 3.90, size = 56, normalized size = 0.89

$$\frac{1}{3}Ccx^3 + \frac{1}{2}Bcx^2 + Cbx + Acx + Bb\log(|x|) - \frac{3\,Bax + 6\,(Ca + Ab)x^2 + 2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x
+ 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

Mupad [B]
time = 0.03, size = 55, normalized size = 0.87

$$x\,(A\,c + C\,b) - \frac{(A\,b + C\,a)\,x^2 + \frac{B\,a\,x}{2} + \frac{A\,a}{3}}{x^3} + \frac{B\,c\,x^2}{2} + \frac{C\,c\,x^3}{3} + B\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x)
```

```
[Out] x*(A*c + C*b) - ((A*a)/3 + x^2*(A*b + C*a) + (B*a*x)/2)/x^3 + (B*c*x^2)/2 +
 (C*c*x^3)/3 + B*b*log(x)
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5} dx$

Optimal. Leaf size=63

$$-\frac{aA}{4x^4}-\frac{aB}{3x^3}-\frac{Ab+aC}{2x^2}-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2+(Ac+bC)\log(x)$$

```
[Out] -1/4*a*A/x^4-1/3*a*B/x^3+1/2*(-A*b-C*a)/x^2-b*B/x+B*c*x+1/2*c*C*x^2+(A*c+C*
b)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (c
*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx &= \int \left(Bc+\frac{aA}{x^5}+\frac{aB}{x^4}+\frac{Ab+aC}{x^3}+\frac{bB}{x^2}+\frac{Ac+bC}{x}+cCx\right)dx\\ &= -\frac{aA}{4x^4}-\frac{aB}{3x^3}-\frac{Ab+aC}{2x^2}-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2+(Ac+bC)\log(\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.98

$$-\frac{a(3A+4Bx+6Cx^2)}{12x^4}+\frac{-Ab-2bBx+cx^3(2B+Cx)}{2x^2}+(Ac+bC)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/12*(a*(3*A + 4*B*x + 6*C*x^2))/x^4 + (-(A*b) - 2*b*B*x + c*x^3*(2*B + C*
x))/(2*x^2) + (A*c + b*C)*Log[x]
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.89

method	result	size
default	$\frac{cC\,x^{2}}{2}+Bcx-\frac{aA}{4x^{4}}-\frac{Ab+aC}{2x^{2}}-\frac{bB}{x}-\frac{aB}{3x^{3}}+\left(Ac+bC\right)\ln\left(x\right)$	56
risch	$\frac{cC\,x^{2}}{2}+Bcx+\frac{-Bb\,x^{3}+\left(-\frac{Ab}{2}-\frac{aC}{2}\right)x^{2}-\frac{aBx}{3}-\frac{aA}{4}}{x^{4}}+A\ln\left(x\right)c+C\ln\left(x\right)b$	57
norman	$\frac{\left(-\frac{Ab}{2}-\frac{aC}{2}\right)x^{2}+Bc\,x^{5}-\frac{aA}{4}-Bb\,x^{3}-\frac{aBx}{3}+\frac{cC\,x^{6}}{2}}{x^{4}}+\left(Ac+bC\right)\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*C*x^2+B*c*x-1/4*a*A/x^4-1/2*(A*b+C*a)/x^2-b*B/x-1/3*a*B/x^3+(A*c+C*b)
*ln(x)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.89

$$\frac{1}{2}\,Ccx^{2}+Bcx+\left(Cb+Ac\right)\log\left(x\right)-\frac{12\,Bbx^{3}+4\,Bax+6\,\left(Ca+Ab\right)x^{2}+3\,Aa}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*(
C*a + A*b)*x^2 + 3*A*a)/x^4
```

Fricas [A]
time = 0.37, size = 62, normalized size = 0.98

$$\frac{6\,Ccx^{6}+12\,Bcx^{5}+12\,\left(Cb+Ac\right)x^{4}\log\left(x\right)-12\,Bbx^{3}-4\,Bax-6\,\left(Ca+Ab\right)x^{2}-3\,Aa}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*B
*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

Sympy [A]
time = 1.08, size = 63, normalized size = 1.00

$$Bcx+\frac{Ccx^{2}}{2}+\left(Ac+Cb\right)\log\left(x\right)+\frac{-3Aa-4Bax-12Bbx^{3}+x^{2}\left(-6Ab-6Ca\right)}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) + (-3*A*a - 4*B*a*x - 12*B*b*x**3 +
 x**2*(-6*A*b - 6*C*a))/(12*x**4)
```

Giac [A]
time = 3.91, size = 57, normalized size = 0.90

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\log(|x|) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x
+ 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.89

$$\ln(x)\,\left(A\,c+C\,b\right)-\frac{B\,b\,x^3+\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)\,x^2+\frac{B\,a\,x}{3}+\frac{A\,a}{4}}{x^4}+B\,c\,x+\frac{C\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x)
```

```
[Out] log(x)*(A*c + C*b) - ((A*a)/4 + x^2*((A*b)/2 + (C*a)/2) + (B*a*x)/3 + B*b*x
^3)/x^4 + B*c*x + (C*c*x^2)/2
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} \, dx$

Optimal. Leaf size=63

$$-\frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ab+aC}{3x^3} - \frac{bB}{2x^2} - \frac{Ac+bC}{x} + cCx + Bc\log(x)$$

```
[Out] -1/5*a*A/x^5-1/4*a*B/x^4+1/3*(-A*b-C*a)/x^3-1/2*b*B/x^2+(-A*c-C*b)/x+c*C*x+
B*c*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1642\}$

$$-\frac{aC+Ab}{3x^3} - \frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ac+bC}{x} - \frac{bB}{2x^2} + Bc\log(x) + cCx$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*c
 + b*C)/x + c*C*x + B*c*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} \, dx &= \int \left(cC + \frac{aA}{x^6} + \frac{aB}{x^5} + \frac{Ab+aC}{x^4} + \frac{bB}{x^3} + \frac{Ac+bC}{x^2} + \frac{Bc}{x}\right) dx \\ &= -\frac{aA}{5x^5} - \frac{aB}{4x^4} - \frac{Ab+aC}{3x^3} - \frac{bB}{2x^2} - \frac{Ac+bC}{x} + cCx + Bc\log(x)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 1.00

$$-\frac{12aA - 60cCx^6 + 30bx^3(B+2Cx) + 5ax(3B+4Cx) + 20Ax^2(b+3cx^2)}{60x^5} + Bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -1/60*(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) + 2
0*A*x^2*(b + 3*c*x^2))/x^5 + B*c*Log[x]
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.89

method	result	size
default	$cCx-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{bB}{2x^2}-\frac{Ac+bC}{x}-\frac{Ab+aC}{3x^3}+Bc\ln\left(x\right)$	56
risch	$cCx+\frac{\left(-Ac-bC\right)x^4-\frac{Bb\,x^3}{2}+\left(-\frac{Ab}{3}-\frac{aC}{3}\right)x^2-\frac{aBx}{4}-\frac{aA}{5}}{x^5}+Bc\ln\left(x\right)$	58
norman	$\frac{\left(-\frac{Ab}{3}-\frac{aC}{3}\right)x^2+\left(-Ac-bC\right)x^4+cC\,x^6-\frac{aA}{5}-\frac{Bb\,x^3}{2}-\frac{aBx}{4}}{x^5}+Bc\ln\left(x\right)$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c*C*x-1/5*a*A/x^5-1/4*a*B/x^4-1/2*b*B/x^2-(A*c+C*b)/x-1/3*(A*b+C*a)/x^3+B*c
*ln(x)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.89

$$Ccx+Bc\log\left(x\right)-\frac{30\,Bbx^3+60\,(Cb+Ac)x^4+15\,Bax+20\,(Ca+Ab)x^2+12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20*
(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Fricas [A]
time = 0.38, size = 62, normalized size = 0.98

$$\frac{60\,Ccx^6+60\,Bcx^5\log\left(x\right)-30\,Bbx^3-60\,(Cb+Ac)x^4-15\,Bax-20\,(Ca+Ab)x^2-12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 15
*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

Sympy [A]
time = 3.15, size = 66, normalized size = 1.05

$$Bc\log\left(x\right)+Ccx+\frac{-12Aa-15Bax-30Bbx^3+x^4(-60Ac-60Cb)+x^2(-20Ab-20Ca)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] B*c*log(x) + C*c*x + (-12*A*a - 15*B*a*x - 30*B*b*x**3 + x**4*(-60*A*c - 60
*C*b) + x**2*(-20*A*b - 20*C*a))/(60*x**5)
```

Giac [A]
time = 4.81, size = 57, normalized size = 0.90

$$Ccx + Bc\log\left(|x|\right) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x
+ 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Mupad [B]
time = 0.78, size = 56, normalized size = 0.89

$$C\,c\,x - \frac{\left(A\,c + C\,b\right)\,x^4 + \frac{B\,b\,x^3}{2} + \left(\frac{A\,b}{3} + \frac{C\,a}{3}\right)\,x^2 + \frac{B\,a\,x}{4} + \frac{A\,a}{5}}{x^5} + B\,c\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x)
```

```
[Out] C*c*x - ((A*a)/5 + x^2*((A*b)/3 + (C*a)/3) + x^4*(A*c + C*b) + (B*a*x)/4 +
(B*b*x^3)/2)/x^5 + B*c*log(x)
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=68

$$-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)$$

```
[Out] -1/6*a*A/x^6-1/5*a*B/x^5+1/4*(-A*b-C*a)/x^4-1/3*b*B/x^3+1/2*(-A*c-C*b)/x^2-
B*c/x+c*C*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1642\}$

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*c
 + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx &= \int \left(\frac{aA}{x^7}+\frac{aB}{x^6}+\frac{Ab+aC}{x^5}+\frac{bB}{x^4}+\frac{Ac+bC}{x^3}+\frac{Bc}{x^2}+\frac{cC}{x}\right)dx\\ &= -\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 1.00

$$-\frac{a(10A+3x(4B+5Cx))+5x^2(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{60x^6}+cC\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/60*(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B
 + 3*b*C*x + 6*B*c*x^2)))/x^6 + c*C*Log[x]
```

Maple [A]
time = 0.01, size = 59, normalized size = 0.87

method	result	size
default	$-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}-\frac{bB}{3x^3}+cC\ln(x)-\frac{aA}{6x^6}$	59
norman	$\frac{\left(-\frac{Ab}{4}-\frac{aC}{4}\right)x^2+\left(-\frac{Ac}{2}-\frac{bC}{2}\right)x^4-\frac{aA}{6}-\frac{Bb\,x^3}{3}-Bc\,x^5-\frac{aBx}{5}}{x^6}+cC\ln(x)$	61
risch	$\frac{\left(-\frac{Ab}{4}-\frac{aC}{4}\right)x^2+\left(-\frac{Ac}{2}-\frac{bC}{2}\right)x^4-\frac{aA}{6}-\frac{Bb\,x^3}{3}-Bc\,x^5-\frac{aBx}{5}}{x^6}+cC\ln(x)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a*B/x^5-1/4*(A*b+C*a)/x^4-1/2*(A*c+C*b)/x^2-B*c/x-1/3*b*B/x^3+c*C*ln(x
)-1/6*a*A/x^6
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.87

$$Cc\log(x)-\frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x
+ 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Fricas [A]
time = 0.38, size = 62, normalized size = 0.91

$$\frac{60\,Ccx^6\log(x)-60\,Bcx^5-20\,Bbx^3-30\,(Cb+Ac)x^4-12\,Bax-15\,(Ca+Ab)x^2-10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 12
*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

Sympy [A]
time = 9.68, size = 70, normalized size = 1.03

$$Cc\log(x)+\frac{-10Aa-12Bax-20Bbx^3-60Bcx^5+x^4(-30Ac-30Cb)+x^2(-15Ab-15Ca)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] C*c*log(x) + (-10*A*a - 12*B*a*x - 20*B*b*x**3 - 60*B*c*x**5 + x**4*(-30*A*
c - 30*C*b) + x**2*(-15*A*b - 15*C*a))/(60*x**6)
```

Giac [A]
time = 4.20, size = 60, normalized size = 0.88

$$Cc\log\left(|x|\right)-\frac{60\,Bcx^{5}+20\,Bbx^{3}+30\,(Cb+Ac)x^{4}+12\,Bax+15\,(Ca+Ab)x^{2}+10\,Aa}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B
*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Mupad [B]
time = 0.79, size = 60, normalized size = 0.88

$$C\,c\,\ln\left(x\right)-\frac{B\,c\,x^5+\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)\,x^4+\frac{B\,b\,x^3}{3}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^2+\frac{B\,a\,x}{5}+\frac{A\,a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x)
```

```
[Out] C*c*log(x) - ((A*a)/6 + x^2*((A*b)/4 + (C*a)/4) + x^4*((A*c)/2 + (C*b)/2) +
 (B*a*x)/5 + (B*b*x^3)/3 + B*c*x^5)/x^6
```

3.11 $\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(A\left(b^2+2ac\right)+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+\left(b^2\right.\right.$$

```
[Out] 1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/3*a*b*B*x^6+1/7*(A*(2*a
*c+b^2)+2*a*b*C)*x^7+1/8*B*(2*a*c+b^2)*x^8+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+
1/5*b*B*c*x^10+1/11*c*(A*c+2*C*b)*x^11+1/12*B*c^2*x^12+1/13*c^2*C*x^13
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1642\}$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+2abBx^5+\left(A\left(b^2+2ac\right.\right.\right.\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(A\left(b^2+\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 159, normalized size = 1.00

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(Ab^2+2aAc+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+b^2C+2acC\right)x^9+\frac{1}{5}bBcx^{10}+\frac{1}{11}c(Ac+2bC)x^{11}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Maple [A]
time = 0.08, size = 142, normalized size = 0.89

method	result
default	$\frac{c^2 C\,x^{13}}{13}+\frac{B\,c^2 x^{12}}{12}+\frac{\left(A\,c^2+2 b c C\right) x^{11}}{11}+\frac{b B c\,x^{10}}{5}+\frac{\left(2 b c A+\left(2 a c+b^2\right) C\right) x^9}{9}+\frac{B\left(2 a c+b^2\right) x^8}{8}+\frac{\left(A\left(2 a c+b^2\right)+2 a b C\right) x^7}{7}+$
norman	$\frac{c^2 C\,x^{13}}{13}+\frac{B\,c^2 x^{12}}{12}+\left(\frac{1}{11} A\,c^2+\frac{2}{11} b c C\right) x^{11}+\frac{b B c\,x^{10}}{5}+\left(\frac{2}{9} b c A+\frac{2}{9} a c C+\frac{1}{9} C\,b^2\right) x^9+\left(\frac{1}{4} a c B+\frac{1}{8} b^2 B\right.$
gosper	$\frac{1}{13} c^2 C\,x^{13}+\frac{1}{12} B\,c^2 x^{12}+\frac{1}{11} x^{11} A\,c^2+\frac{2}{11} x^{11} b c C+\frac{1}{5} b B c\,x^{10}+\frac{2}{9} x^9 b c A+\frac{2}{9} x^9 a c C+\frac{1}{9} x^9 C\,b^2+\frac{1}{4} x$
risch	$\frac{1}{13} c^2 C\,x^{13}+\frac{1}{12} B\,c^2 x^{12}+\frac{1}{11} x^{11} A\,c^2+\frac{2}{11} x^{11} b c C+\frac{1}{5} b B c\,x^{10}+\frac{2}{9} x^9 b c A+\frac{2}{9} x^9 a c C+\frac{1}{9} x^9 C\,b^2+\frac{1}{4} x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1/
9*(2*b*c*A+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*a*
b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x^3
```

Maxima [A]
time = 0.28, size = 143, normalized size = 0.90

$$\frac{1}{13} Cc^2x^{13}+\frac{1}{12} Bc^2x^{12}+\frac{1}{5} Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2\left(Ca+Ab\right)c\right)x^9+\frac{1}{3} Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)x^8+\frac{1}{7}\left(2\,Cab+Ab^2+2\,Aac\right)x^7+\frac{1}{4} Ba^2x^4+\frac{1}{3} Aa^2x^3+\frac{1}{5}\left(Ca^2+2\,Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)
*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*
B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^
2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Fricas [A]
time = 0.40, size = 143, normalized size = 0.90

$$\frac{1}{13} Cc^2x^{13}+\frac{1}{12} Bc^2x^{12}+\frac{1}{5} Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2\left(Ca+Ab\right)c\right)x^9+\frac{1}{3} Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)x^8+\frac{1}{7}\left(2\,Cab+Ab^2+2\,Aac\right)x^7+\frac{1}{4} Ba^2x^4+\frac{1}{3} Aa^2x^3+\frac{1}{5}\left(Ca^2+2\,Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)
*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*
B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^
2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Sympy [A]
time = 0.02, size = 168, normalized size = 1.06

$$\frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Babx^6}{3} + \frac{Bbcx^{10}}{5} + \frac{Bc^2x^{12}}{12} + \frac{Cc^2x^{13}}{13} + x^{11}\left(\frac{Ac^2}{11} + \frac{2Cbc}{11}\right) + x^9\cdot\left(\frac{2Abc}{9} + \frac{2Cac}{9} + \frac{Cb^2}{9}\right) + x^8\left(\frac{Bac}{4} + \frac{Bb^2}{8}\right) + x^7\cdot\left(\frac{2Aac}{7} + \frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^5\cdot\left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**12
/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9 +
2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*b**
2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)
```

Giac [A]
time = 3.40, size = 154, normalized size = 0.97

$$\frac{1}{13}Cc^2x^{13} + \frac{1}{12}Bc^2x^{12} + \frac{2}{11}Cbcx^{11} + \frac{1}{11}Ac^2x^{11} + \frac{1}{5}Bbcx^{10} + \frac{1}{9}Cb^2x^9 + \frac{2}{9}Cacx^9 + \frac{2}{9}Abcx^9 + \frac{1}{8}Bb^2x^8 + \frac{1}{4}Bacx^8 + \frac{2}{7}Cabx^7 + \frac{1}{7}Ab^2x^7 + \frac{2}{7}Aacx^7 + \frac{1}{3}Babx^6 + \frac{1}{5}Ca^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{4}Ba^2x^4 + \frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/5
*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x^8
 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B*a*
b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

Mupad [B]
time = 0.82, size = 141, normalized size = 0.89

$$x^5\left(\frac{C\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^{11}\left(\frac{A\,c^2}{11} + \frac{2\,C\,b\,c}{11}\right) + x^7\left(\frac{A\,b^2}{7} + \frac{2\,C\,a\,b}{7} + \frac{2\,A\,a\,c}{7}\right) + x^9\left(\frac{C\,b^2}{9} + \frac{2\,A\,c\,b}{9} + \frac{2\,C\,a\,c}{9}\right) + \frac{A\,a^2\,x^3}{3} + \frac{B\,a^2\,x^4}{4} + \frac{B\,c^2\,x^{12}}{12} + \frac{C\,c^2\,x^{13}}{13} + \frac{B\,x^8\,\left(b^2+2\,a\,c\right)}{8} + \frac{B\,a\,b\,x^6}{3} + \frac{B\,b\,c\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((C*a^2)/5 + (2*A*a*b)/5) + x^11*((A*c^2)/11 + (2*C*b*c)/11) + x^7*((A*
b^2)/7 + (2*A*a*c)/7 + (2*C*a*b)/7) + x^9*((C*b^2)/9 + (2*A*b*c)/9 + (2*C*a
*c)/9) + (A*a^2*x^3)/3 + (B*a^2*x^4)/4 + (B*c^2*x^12)/12 + (C*c^2*x^13)/13
+ (B*x^8*(2*a*c + b^2))/8 + (B*a*b*x^6)/3 + (B*b*c*x^10)/5
```

3.12 $\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+2ac\right)+2abC\right)x^6+\frac{1}{7}B\left(b^2+2ac\right)x^7+\frac{1}{8}\left(2Abc+(b\right.$$

```
[Out] 1/2*a^2*A*x^2+1/3*a^2*B*x^3+1/4*a*(2*A*b+C*a)*x^4+2/5*a*b*B*x^5+1/6*(A*(2*a
*c+b^2)+2*a*b*C)*x^6+1/7*B*(2*a*c+b^2)*x^7+1/8*(2*A*b*c+(2*a*c+b^2)*C)*x^8+
2/9*b*B*c*x^9+1/10*c*(A*c+2*C*b)*x^10+1/11*B*c^2*x^11+1/12*c^2*C*x^12
```

Rubi [A]
time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1642}

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2Ax+a^2Bx^2+a(2Ab+aC)x^3+2abBx^4+\left(A\left(b^2+2ac\right.\right.\right.\\ &= \frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 159, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(Ab^2+2aAc+2abC\right)x^6+\frac{1}{7}B\left(b^2+2ac\right)x^7+\frac{1}{8}\left(2Abc+b^2C+2acC\right)x^8+\frac{2}{9}bBcx^9+\frac{1}{10}c(Ac+2bC)x^{10}+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Maple [A]
time = 0.09, size = 142, normalized size = 0.89

method	result
default	$\frac{c^2 C x^{12}}{12}+\frac{B c^2 x^{11}}{11}+\frac{(A c^2+2bcC) x^{10}}{10}+\frac{2bBc x^9}{9}+\frac{(2bcA+(2ac+b^2)C) x^8}{8}+\frac{B(2ac+b^2) x^7}{7}+\frac{(A(2ac+b^2)+2abC) x^6}{6}+$
norman	$\frac{c^2 C x^{12}}{12}+\frac{B c^2 x^{11}}{11}+\left(\frac{1}{10}A c^2+\frac{1}{5}bcC\right) x^{10}+\frac{2bBc x^9}{9}+\left(\frac{1}{4}bcA+\frac{1}{4}acC+\frac{1}{8}C b^2\right) x^8+\left(\frac{2}{7}acB+\frac{1}{7}b^2 B\right)$
gosper	$\frac{1}{12}c^2 C x^{12}+\frac{1}{11}B c^2 x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}bcC+\frac{2}{9}bBc x^9+\frac{1}{4}x^8 bcA+\frac{1}{4}x^8 acC+\frac{1}{8}x^8 C b^2+\frac{2}{7}x^7 ac$
risch	$\frac{1}{12}c^2 C x^{12}+\frac{1}{11}B c^2 x^{11}+\frac{1}{10}x^{10}A c^2+\frac{1}{5}x^{10}bcC+\frac{2}{9}bBc x^9+\frac{1}{4}x^8 bcA+\frac{1}{4}x^8 acC+\frac{1}{8}x^8 C b^2+\frac{2}{7}x^7 ac$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B*c*x^9+1/8
*(2*b*c*A+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*(A*(2*a*c+b^2)+2*a*b
*C)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^2
```

Maxima [A]
time = 0.30, size = 143, normalized size = 0.90

$$\frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{2}{9}Bbcx^9+\frac{1}{10}(2Cbc+Ac^2)x^{10}+\frac{1}{8}(Cb^2+2(Ca+Ab)c)x^8+\frac{2}{5}Babx^5+\frac{1}{7}(Bb^2+2Bac)x^7+\frac{1}{6}(2Cab+Ab^2+2Aac)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2+\frac{1}{4}(Ca^2+2Aab)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)*
x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2*B
*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2
*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Fricas [A]
time = 0.35, size = 143, normalized size = 0.90

$$\frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{2}{9}Bbcx^9+\frac{1}{10}(2Cbc+Ac^2)x^{10}+\frac{1}{8}(Cb^2+2(Ca+Ab)c)x^8+\frac{2}{5}Babx^5+\frac{1}{7}(Bb^2+2Bac)x^7+\frac{1}{6}(2Cab+Ab^2+2Aac)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2+\frac{1}{4}(Ca^2+2Aab)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)*
x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2*B
*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2
*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Sympy [A]
time = 0.02, size = 163, normalized size = 1.03

$$\frac{Aa^2x^2}{2} + \frac{Ba^2x^3}{3} + \frac{2Babx^5}{5} + \frac{2Bbcx^9}{9} + \frac{Bc^2x^{11}}{11} + \frac{Cc^2x^{12}}{12} + x^{10}\left(\frac{Ac^2}{10} + \frac{Cbc}{5}\right) + x^8\left(\frac{Abc}{4} + \frac{Cac}{4} + \frac{Cb^2}{8}\right) + x^7 \cdot \left(\frac{2Bac}{7} + \frac{Bb^2}{7}\right) + x^6\left(\frac{Aac}{3} + \frac{Ab^2}{6} + \frac{Cab}{3}\right) + x^4\left(\frac{Aab}{2} + \frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x*
*11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 + C*
a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2/6
+ C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)
```

Giac [A]
time = 4.19, size = 154, normalized size = 0.97

$$\frac{1}{12}Cc^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{1}{5}Cbcx^{10} + \frac{1}{10}Ac^2x^{10} + \frac{2}{9}Bbcx^9 + \frac{1}{8}Cb^2x^8 + \frac{1}{4}Cacx^8 + \frac{1}{4}Abcx^8 + \frac{1}{7}Bb^2x^7 + \frac{2}{7}Bacx^7 + \frac{1}{3}Cabx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{3}Aacx^6 + \frac{2}{5}Babx^5 + \frac{1}{4}Ca^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{3}Ba^2x^3 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x^10 + 2/9*
B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x^8 + 1/7*B*b^2*x^7 +
 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*A*a*c*x^6 + 2/5*B*a*b*
x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

Mupad [B]
time = 0.07, size = 141, normalized size = 0.89

$$x^4\left(\frac{C\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^{10}\left(\frac{A\,c^2}{10} + \frac{C\,b\,c}{5}\right) + x^6\left(\frac{A\,b^2}{6} + \frac{C\,a\,b}{3} + \frac{A\,a\,c}{3}\right) + x^8\left(\frac{C\,b^2}{8} + \frac{A\,c\,b}{4} + \frac{C\,a\,c}{4}\right) + \frac{A\,a^2\,x^2}{2} + \frac{B\,a^2\,x^3}{3} + \frac{B\,c^2\,x^{11}}{11} + \frac{C\,c^2\,x^{12}}{12} + \frac{B\,x^7\,\left(b^2 + 2\,a\,c\right)}{7} + \frac{2\,B\,a\,b\,x^5}{5} + \frac{2\,B\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4*((C*a^2)/4 + (A*a*b)/2) + x^10*((A*c^2)/10 + (C*b*c)/5) + x^6*((A*b^2)/
6 + (A*a*c)/3 + (C*a*b)/3) + x^8*((C*b^2)/8 + (A*b*c)/4 + (C*a*c)/4) + (A*a
^2*x^2)/2 + (B*a^2*x^3)/3 + (B*c^2*x^11)/11 + (C*c^2*x^12)/12 + (B*x^7*(2*a
*c + b^2))/7 + (2*B*a*b*x^5)/5 + (2*B*b*c*x^9)/9
```

3.13 $\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=154

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(A\left(b^2+2ac\right)+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6+\frac{1}{7}\left(2Abc+\left(b^2+\right.\right.$$

```
[Out] a^2*A*x+1/2*a^2*B*x^2+1/3*a*(2*A*b+C*a)*x^3+1/2*a*b*B*x^4+1/5*(A*(2*a*c+b^2
)+2*a*b*C)*x^5+1/6*B*(2*a*c+b^2)*x^6+1/7*(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/4*b*
B*c*x^8+1/9*c*(A*c+2*C*b)*x^9+1/10*B*c^2*x^10+1/11*c^2*C*x^11
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1671}

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC)+\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^
2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2 +
 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2A+a^2Bx+a(2Ab+aC)x^2+2abBx^3+\left(A\left(b^2+2ac\right)+2\right.\right.\\ &= a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(A\left(b^2+2ac\right)\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 154, normalized size = 1.00

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(Ab^2+2aAc+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6+\frac{1}{7}\left(2Abc+b^2C+2acC\right)x^7+\frac{1}{4}bBcx^8+\frac{1}{9}c(Ac+2bC)x^9+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*b^2
 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*C
+ 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Maple [A]
time = 0.08, size = 139, normalized size = 0.90

method	result
default	$\frac{c^{2} C\,x^{11}}{11}+\frac{B\,c^{2} x^{10}}{10}+\frac{\left(A\,c^{2}+2 b c C\right) x^{9}}{9}+\frac{b B c\,x^{8}}{4}+\frac{\left(2 b c A+\left(2 a c+b^{2}\right) C\right) x^{7}}{7}+\frac{B \left(2 a c+b^{2}\right) x^{6}}{6}+\frac{\left(A \left(2 a c+b^{2}\right)+2 a b C\right) x^{5}}{5}+$
norman	$\frac{c^{2} C\,x^{11}}{11}+\frac{B\,c^{2} x^{10}}{10}+\left(\frac{1}{9} A\,c^{2}+\frac{2}{9} b c C\right) x^{9}+\frac{b B c\,x^{8}}{4}+\left(\frac{2}{7} b c A+\frac{2}{7} a c C+\frac{1}{7} C\,b^{2}\right) x^{7}+\left(\frac{1}{3} a c B+\frac{1}{6} b^{2} B\right) x$
gosper	$\frac{1}{11} c^{2} C\,x^{11}+\frac{1}{10} B\,c^{2} x^{10}+\frac{1}{9} x^{9} A\,c^{2}+\frac{2}{9} x^{9} b c C+\frac{1}{4} b B c\,x^{8}+\frac{2}{7} x^{7} b c A+\frac{2}{7} x^{7} a c C+\frac{1}{7} x^{7} C\,b^{2}+\frac{1}{3} x^{6} a c$
risch	$\frac{1}{11} c^{2} C\,x^{11}+\frac{1}{10} B\,c^{2} x^{10}+\frac{1}{9} x^{9} A\,c^{2}+\frac{2}{9} x^{9} b c C+\frac{1}{4} b B c\,x^{8}+\frac{2}{7} x^{7} b c A+\frac{2}{7} x^{7} a c C+\frac{1}{7} x^{7} C\,b^{2}+\frac{1}{3} x^{6} a c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c*x^8+1/7*(
2*b*c*A+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(A*(2*a*c+b^2)+2*a*b*C
)*x^5+1/2*a*b*B*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

Maxima [A]
time = 0.28, size = 140, normalized size = 0.91

$$\frac{1}{11}\,Cc^{2}x^{11}+\frac{1}{10}\,Bc^{2}x^{10}+\frac{1}{4}\,Bbcx^{8}+\frac{1}{9}\,{\left(2\,Cbc+Ac^{2}\right)}x^{9}+\frac{1}{7}\,{\left(Cb^{2}+2\,{\left(Ca+Ab\right)}c\right)}x^{7}+\frac{1}{2}\,Babx^{4}+\frac{1}{6}\,{\left(Bb^{2}+2\,Bac\right)}x^{6}+\frac{1}{5}\,{\left(2\,Cab+Ab^{2}+2\,Aac\right)}x^{5}+\frac{1}{2}\,Ba^{2}x^{2}+Aa^{2}x+\frac{1}{3}\,{\left(Ca^{2}+2\,Aab\right)}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*x
^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B*a
*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x + 1
/3*(C*a^2 + 2*A*a*b)*x^3
```

Fricas [A]
time = 0.39, size = 140, normalized size = 0.91

$$\frac{1}{11}\,Cc^{2}x^{11}+\frac{1}{10}\,Bc^{2}x^{10}+\frac{1}{4}\,Bbcx^{8}+\frac{1}{9}\,{\left(2\,Cbc+Ac^{2}\right)}x^{9}+\frac{1}{7}\,{\left(Cb^{2}+2\,{\left(Ca+Ab\right)}c\right)}x^{7}+\frac{1}{2}\,Babx^{4}+\frac{1}{6}\,{\left(Bb^{2}+2\,Bac\right)}x^{6}+\frac{1}{5}\,{\left(2\,Cab+Ab^{2}+2\,Aac\right)}x^{5}+\frac{1}{2}\,Ba^{2}x^{2}+Aa^{2}x+\frac{1}{3}\,{\left(Ca^{2}+2\,Aab\right)}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*x
^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B*a
*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x + 1
/3*(C*a^2 + 2*A*a*b)*x^3
```

Sympy [A]
time = 0.02, size = 165, normalized size = 1.07

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bbcx^8}{4} + \frac{Bc^2x^{10}}{10} + \frac{Cc^2x^{11}}{11} + x^9\left(\frac{Ac^2}{9} + \frac{2Cbc}{9}\right) + x^7\cdot\left(\frac{2Abc}{7} + \frac{2Cac}{7} + \frac{Cb^2}{7}\right) + x^6\left(\frac{Bac}{3} + \frac{Bb^2}{6}\right) + x^5\cdot\left(\frac{2Aac}{5} + \frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^3\cdot\left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 +
C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c/7
 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 + 2*C
*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

Giac [A]
time = 6.00, size = 151, normalized size = 0.98

$$\frac{1}{11}Cc^2x^{11} + \frac{1}{10}Bc^2x^{10} + \frac{2}{9}Cbcx^9 + \frac{1}{9}Ac^2x^9 + \frac{1}{4}Bbcx^8 + \frac{1}{7}Cb^2x^7 + \frac{2}{7}Cacx^7 + \frac{2}{7}Abcx^7 + \frac{1}{6}Bb^2x^6 + \frac{1}{3}Bacx^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{2}{5}Aacx^5 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9 + 1/4*B*b
*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7 + 1/6*B*b^2*x^6 + 1/
3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/2*B*a*b*x^4
 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B]
time = 0.07, size = 138, normalized size = 0.90

$$x^3\left(\frac{C\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^9\left(\frac{A\,c^2}{9} + \frac{2\,C\,b\,c}{9}\right) + x^5\left(\frac{A\,b^2}{5} + \frac{2\,C\,a\,b}{5} + \frac{2\,A\,a\,c}{5}\right) + x^7\left(\frac{C\,b^2}{7} + \frac{2\,A\,c\,b}{7} + \frac{2\,C\,a\,c}{7}\right) + \frac{B\,a^2\,x^2}{2} + \frac{B\,c^2\,x^{10}}{10} + \frac{C\,c^2\,x^{11}}{11} + \frac{B\,x^6\,(b^2 + 2\,a\,c)}{6} + A\,a^2\,x + \frac{B\,a\,b\,x^4}{2} + \frac{B\,b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*b)/3) + x^9*((A*c^2)/9 + (2*C*b*c)/9) + x^5*((A*b^2
)/5 + (2*A*a*c)/5 + (2*C*a*b)/5) + x^7*((C*b^2)/7 + (2*A*b*c)/7 + (2*C*a*c)
/7) + (B*a^2*x^2)/2 + (B*c^2*x^10)/10 + (C*c^2*x^11)/11 + (B*x^6*(2*a*c + b
^2))/6 + A*a^2*x + (B*a*b*x^4)/2 + (B*b*c*x^8)/4
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} \, dx$

Optimal. Leaf size=150

$$a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3+\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4+\frac{1}{5}B\left(b^2+2ac\right)x^5+\frac{1}{6}\left(2Abc+\left(b^2+2ac\right)C\right)$$

```
[Out] a^2*B*x+1/2*a*(2*A*b+C*a)*x^2+2/3*a*b*B*x^3+1/4*(A*(2*a*c+b^2)+2*a*b*C)*x^4
+1/5*B*(2*a*c+b^2)*x^5+1/6*(2*A*b*c+(2*a*c+b^2)*C)*x^6+2/7*b*B*c*x^7+1/8*c*
(A*c+2*C*b)*x^8+1/9*B*c^2*x^9+1/10*c^2*C*x^10+a^2*A*ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC)+\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx &= \int \left(a^2B+\frac{a^2A}{x}+a(2Ab+aC)x+2abBx^2+\left(A\left(b^2+2ac\right)+2\right.\right.\\ &= a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3+\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 150, normalized size = 1.00

$$a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3+\frac{1}{4}\left(Ab^2+2aAc+2abC\right)x^4+\frac{1}{5}B\left(b^2+2ac\right)x^5+\frac{1}{6}\left(2Abc+b^2C+2acC\right)x^6+\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}+a^2A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Maple [A]
time = 0.02, size = 149, normalized size = 0.99

method	result
norman	$\left(\frac{1}{8}A\,c^2+\frac{1}{4}bcC\right)x^8+\left(abA+\frac{1}{2}a^2C\right)x^2+\left(\frac{2}{5}acB+\frac{1}{5}b^2B\right)x^5+\left(\frac{1}{2}acA+\frac{1}{4}A\,b^2+\frac{1}{2}abC\right)x^4+\left(\frac{1}{3}bc\right.$
default	$\frac{c^2C\,x^{10}}{10}+\frac{B\,c^2x^9}{9}+\frac{A\,c^2x^8}{8}+\frac{Cbc\,x^8}{4}+\frac{2bBc\,x^7}{7}+\frac{Abc\,x^6}{3}+\frac{Cac\,x^6}{3}+\frac{C\,b^2x^6}{6}+\frac{2acB\,x^5}{5}+\frac{B\,b^2x^5}{5}+\frac{Aac\,x^4}{2}+\frac{A\,b^2}{4}$
risch	$\frac{c^2C\,x^{10}}{10}+\frac{B\,c^2x^9}{9}+\frac{A\,c^2x^8}{8}+\frac{Cbc\,x^8}{4}+\frac{2bBc\,x^7}{7}+\frac{Abc\,x^6}{3}+\frac{Cac\,x^6}{3}+\frac{C\,b^2x^6}{6}+\frac{2acB\,x^5}{5}+\frac{B\,b^2x^5}{5}+\frac{Aac\,x^4}{2}+\frac{A\,b^2}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^2*C*x^10+1/9*B*c^2*x^9+1/8*A*c^2*x^8+1/4*C*b*c*x^8+2/7*b*B*c*x^7+1/3
*A*b*c*x^6+1/3*C*a*c*x^6+1/6*C*b^2*x^6+2/5*a*c*B*x^5+1/5*B*b^2*x^5+1/2*A*a*
c*x^4+1/4*A*b^2*x^4+1/2*C*a*b*x^4+2/3*a*b*B*x^3+a*A*b*x^2+1/2*C*a^2*x^2+a^2
*B*x+a^2*A*ln(x)
```

Maxima [A]
time = 0.29, size = 138, normalized size = 0.92

$$\frac{1}{10}Cc^2x^{10}+\frac{1}{9}Bc^2x^9+\frac{2}{7}Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2+2\,Aac\right)x^4+Ba^2x+Aa^2\log\left(x\right)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

Fricas [A]
time = 0.37, size = 138, normalized size = 0.92

$$\frac{1}{10}Cc^2x^{10}+\frac{1}{9}Bc^2x^9+\frac{2}{7}Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2+2\,Aac\right)x^4+Ba^2x+Aa^2\log\left(x\right)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

Sympy [A]
time = 0.12, size = 156, normalized size = 1.04

$$Aa^2\log(x) + Ba^2x + \frac{2Babx^3}{3} + \frac{2Bbcx^7}{7} + \frac{Bc^2x^9}{9} + \frac{Cc^2x^{10}}{10} + x^8\left(\frac{Ac^2}{8} + \frac{Cbc}{4}\right) + x^6\left(\frac{Abc}{3} + \frac{Cac}{3} + \frac{Cb^2}{6}\right) + x^5\cdot\left(\frac{2Bac}{5} + \frac{Bb^2}{5}\right) + x^4\left(\frac{Aac}{2} + \frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9
+ C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 + C
*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*b/2
) + x**2*(A*a*b + C*a**2/2)
```

Giac [A]
time = 4.18, size = 149, normalized size = 0.99

$$\frac{1}{10}Cc^2x^{10} + \frac{1}{9}Bc^2x^9 + \frac{1}{4}Cbcx^8 + \frac{1}{8}Ac^2x^8 + \frac{2}{7}Bbcx^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{3}Cacx^6 + \frac{1}{3}Abcx^6 + \frac{1}{5}Bb^2x^5 + \frac{2}{5}Bacx^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Aacx^4 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*c
*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/5*
B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^3 +
 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

Mupad [B]
time = 0.80, size = 135, normalized size = 0.90

$$x^2\left(\frac{C\,a^2}{2} + A\,b\,a\right) + x^8\left(\frac{A\,c^2}{8} + \frac{C\,b\,c}{4}\right) + x^4\left(\frac{A\,b^2}{4} + \frac{C\,a\,b}{2} + \frac{A\,a\,c}{2}\right) + x^6\left(\frac{C\,b^2}{6} + \frac{A\,c\,b}{3} + \frac{C\,a\,c}{3}\right) + \frac{B\,c^2\,x^9}{9} + \frac{C\,c^2\,x^{10}}{10} + A\,a^2\,\ln(x) + \frac{B\,x^5\,(b^2 + 2\,a\,c)}{5} + B\,a^2\,x + \frac{2\,B\,a\,b\,x^3}{3} + \frac{2\,B\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x)
```

```
[Out] x^2*((C*a^2)/2 + A*a*b) + x^8*((A*c^2)/8 + (C*b*c)/4) + x^4*((A*b^2)/4 + (A
*a*c)/2 + (C*a*b)/2) + x^6*((C*b^2)/6 + (A*b*c)/3 + (C*a*c)/3) + (B*c^2*x^9
)/9 + (C*c^2*x^10)/10 + A*a^2*log(x) + (B*x^5*(2*a*c + b^2))/5 + B*a^2*x +
(2*B*a*b*x^3)/3 + (2*B*b*c*x^7)/7
```

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2} \, dx$

Optimal. Leaf size=145

$$-\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3+\frac{1}{4}B\left(b^2+2ac\right)x^4+\frac{1}{5}\left(2Abc+\left(b^2+2ac\right)C\right)x^5+\frac{1}{3}$$

```
[Out] -a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*(A*(2*a*c+b^2)+2*a*b*C)*x^3+1/4*B*(2
*a*c+b^2)*x^4+1/5*(2*A*b*c+(2*a*c+b^2)*C)*x^5+1/3*b*B*c*x^6+1/7*c*(A*c+2*C*
b)*x^7+1/8*B*c^2*x^8+1/9*c^2*C*x^9+a^2*B*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abBx^2+\frac{1}{7}cx^7(Ac+2bC)+\frac{1}{3}bBcx^6+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2} \, dx &= \int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+\left(A\left(b^2+2ac\right)+2abC\right.\right. \\ &= -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3+\end{aligned}$$

Mathematica [A]
time = 0.06, size = 145, normalized size = 1.00

$$-\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(Ab^2+2aAc+2abC\right)x^3+\frac{1}{4}B\left(b^2+2ac\right)x^4+\frac{1}{5}\left(2Abc+b^2C+2acC\right)x^5+\frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Maple [A]
time = 0.02, size = 147, normalized size = 1.01

method	result
norman	$\frac{\left(\frac{1}{7}A c^2+\frac{2}{7}bcC\right)x^8+\left(\frac{1}{2}acB+\frac{1}{4}b^2 B\right)x^5+\left(\frac{2}{3}acA+\frac{1}{3}A b^2+\frac{2}{3}abC\right)x^4+\left(\frac{2}{5}bcA+\frac{2}{5}acC+\frac{1}{5}C b^2\right)x^6+\left(2abA+a^2 C\right)x^2+abB x^3-a^2 A+\frac{B c^2}{8}}{x}$
default	$\frac{c^2 C x^9}{9}+\frac{B c^2 x^8}{8}+\frac{A c^2 x^7}{7}+\frac{2bcC x^7}{7}+\frac{bBc x^6}{3}+\frac{2Abc x^5}{5}+\frac{2Cac x^5}{5}+\frac{C b^2 x^5}{5}+\frac{acB x^4}{2}+\frac{B b^2 x^4}{4}+\frac{2Aac x^3}{3}+$
risch	$\frac{c^2 C x^9}{9}+\frac{B c^2 x^8}{8}+\frac{A c^2 x^7}{7}+\frac{2bcC x^7}{7}+\frac{bBc x^6}{3}+\frac{2Abc x^5}{5}+\frac{2Cac x^5}{5}+\frac{C b^2 x^5}{5}+\frac{acB x^4}{2}+\frac{B b^2 x^4}{4}+\frac{2Aac x^3}{3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^2*C*x^9+1/8*B*c^2*x^8+1/7*A*c^2*x^7+2/7*b*c*C*x^7+1/3*b*B*c*x^6+2/5*A
*b*c*x^5+2/5*C*a*c*x^5+1/5*C*b^2*x^5+1/2*a*c*B*x^4+1/4*B*b^2*x^4+2/3*A*a*c*
x^3+1/3*A*b^2*x^3+2/3*C*a*b*x^3+a*b*B*x^2+2*a*b*A*x+a^2*C*x-a^2*A/x+a^2*B*l
n(x)
```

Maxima [A]
time = 0.33, size = 137, normalized size = 0.94

$$\frac{1}{9}Cc^2x^9+\frac{1}{8}Bc^2x^8+\frac{1}{3}Bbcx^6+\frac{1}{7}\left(2Cbc+Ac^2\right)x^7+\frac{1}{5}\left(Cb^2+2\left(Ca+Ab\right)c\right)x^5+Babx^2+\frac{1}{4}\left(Bb^2+2Bac\right)x^4+\frac{1}{3}\left(2Cab+Ab^2+2Aac\right)x^3+Ba^2\log\left(x\right)-\frac{Aa^2}{x}+\left(Ca^2+2Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7 +
 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^4
+ 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2 + 2
*A*a*b)*x
```

Fricas [A]
time = 0.37, size = 145, normalized size = 1.00

$$\frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\left(2\,Cbc+Ac^2\right)x^8+504\left(Cb^2+2\left(Ca+Ab\right)c\right)x^6+2520\,Babx^3+630\left(Bb^2+2\,Bac\right)x^5+840\left(2\,Cab+Ab^2+2\,Aac\right)x^4+2520\,Ba^2x\log\left(x\right)-2520\,Aa^2+2520\left(Ca^2+2\,Aab\right)x^2}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*c
^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2 +
 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x) -
 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x
```

Sympy [A]
time = 0.13, size = 156, normalized size = 1.08

$$-\frac{Aa^2}{x}+Ba^2\log{(x)}+Babx^2+\frac{Bbcx^6}{3}+\frac{Bc^2x^8}{8}+\frac{Cc^2x^9}{9}+x^7\left(\frac{Ac^2}{7}+\frac{2Cbc}{7}\right)+x^5\cdot\left(\frac{2Abc}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Bac}{2}+\frac{Bb^2}{4}\right)+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+x\left(2Aab+Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*c
**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 + C*
b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a*b/
3) + x*(2*A*a*b + C*a**2)
```

Giac [A]
time = 3.77, size = 147, normalized size = 1.01

$$\frac{1}{9}Cc^2x^9+\frac{1}{8}Bc^2x^8+\frac{2}{7}Cbcx^7+\frac{1}{7}Ac^2x^7+\frac{1}{3}Bbcx^6+\frac{1}{5}Cb^2x^5+\frac{2}{5}Cacx^5+\frac{2}{5}Abcx^5+\frac{1}{4}Bb^2x^4+\frac{1}{2}Bacx^4+\frac{2}{3}Cabx^3+\frac{1}{3}Ab^2x^3+\frac{2}{3}Aacx^3+Babx^2+Ca^2x+2\,Aabx+Ba^2\log\left(|x|\right)-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*x
^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*B*
a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*a^2
*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x
```

Mupad [B]
time = 0.80, size = 135, normalized size = 0.93

$$x^7\left(\frac{A\,c^2}{7}+\frac{2\,C\,b\,c}{7}\right)+x^3\left(\frac{A\,b^2}{3}+\frac{2\,C\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,C\,a\,c}{5}\right)+x\,\left(C\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{x}+\frac{B\,c^2\,x^8}{8}+\frac{C\,c^2\,x^9}{9}+B\,a^2\,\ln\left(x\right)+\frac{B\,x^4\,\left(b^2+2\,a\,c\right)}{4}+B\,a\,b\,x^2+\frac{B\,b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x)
```

```
[Out] x^7*((A*c^2)/7 + (2*C*b*c)/7) + x^3*((A*b^2)/3 + (2*A*a*c)/3 + (2*C*a*b)/3)
 + x^5*((C*b^2)/5 + (2*A*b*c)/5 + (2*C*a*c)/5) + x*(C*a^2 + 2*A*a*b) - (A*a
^2)/x + (B*c^2*x^8)/8 + (C*c^2*x^9)/9 + B*a^2*log(x) + (B*x^4*(2*a*c + b^2)
)/4 + B*a*b*x^2 + (B*b*c*x^6)/3
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2+\frac{1}{3}B\left(b^2+2ac\right)x^3+\frac{1}{4}\left(2Abc+\left(b^2+2ac\right)C\right)x^4+\frac{2}{5}bBcx^5$$

```
[Out] -1/2*a^2*A/x^2-a^2*B/x+2*a*b*B*x+1/2*(A*(2*a*c+b^2)+2*a*b*C)*x^2+1/3*B*(2*a
*c+b^2)*x^3+1/4*(2*A*b*c+(2*a*c+b^2)*C)*x^4+2/5*b*B*c*x^5+1/6*c*(A*c+2*C*b)
*x^6+1/7*B*c^2*x^7+1/8*c^2*C*x^8+a*(2*A*b+C*a)*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2abBx+\frac{1}{6}cx^6(Ac+2bC)+\frac{2}{5}bBcx^5+\frac{1}{7}Bc^2x^7+\frac{1}{8}c^2Cx^8$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^2
)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*b*
B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a*(2
*A*b + a*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^3}\,dx &= \int \left(2abB+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2ab\right.\right.\\ &= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2+\frac{1}{3}B\left(b^2-\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 139, normalized size = 0.93

$$-\frac{a^2(A+2Bx)}{2x^2}+\frac{1}{6}ax\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right)+\frac{1}{840}x^2\left(70b^2x(4B+3Cx)+56bcx^3(6B+5Cx)+15c^2x^5(8B+7Cx)+140A\left(3b^2+3bcx^2+c^2x^4\right)\right)+a(2Ab+aC)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*(A + 2*B*x))/x^2 + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C
*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*c^
2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A*b
+ a*C)*Log[x]
```

Maple [A]
time = 0.02, size = 146, normalized size = 0.98

method	result
norman	$\frac{\left(\frac{1}{6}A\,c^2+\frac{1}{3}bcC\right)x^8+\left(\frac{2}{3}acB+\frac{1}{3}b^2B\right)x^5+\left(acA+\frac{1}{2}A\,b^2+abC\right)x^4+\left(\frac{1}{2}bcA+\frac{1}{2}acC+\frac{1}{4}C\,b^2\right)x^6-\frac{a^2A}{2}+\frac{B\,c^2x^9}{7}-a^2Bx+\frac{c^2C\,x^{10}}{8}+2abB\,x^3+}{x^2}$
default	$\frac{c^2C\,x^8}{8}+\frac{B\,c^2x^7}{7}+\frac{A\,c^2x^6}{6}+\frac{bcC\,x^6}{3}+\frac{2bBc\,x^5}{5}+\frac{Abc\,x^4}{2}+\frac{Cac\,x^4}{2}+\frac{C\,b^2x^4}{4}+\frac{2Bac\,x^3}{3}+\frac{B\,b^2x^3}{3}+Aac\,x^2+$
risch	$\frac{c^2C\,x^8}{8}+\frac{B\,c^2x^7}{7}+\frac{A\,c^2x^6}{6}+\frac{bcC\,x^6}{3}+\frac{2bBc\,x^5}{5}+\frac{Abc\,x^4}{2}+\frac{Cac\,x^4}{2}+\frac{C\,b^2x^4}{4}+\frac{2Bac\,x^3}{3}+\frac{B\,b^2x^3}{3}+Aac\,x^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^2*C*x^8+1/7*B*c^2*x^7+1/6*A*c^2*x^6+1/3*b*c*C*x^6+2/5*b*B*c*x^5+1/2*A
*b*c*x^4+1/2*C*a*c*x^4+1/4*C*b^2*x^4+2/3*B*a*c*x^3+1/3*B*b^2*x^3+A*a*c*x^2+
1/2*A*b^2*x^2+C*a*b*x^2+2*a*b*B*x-1/2*a^2*A/x^2-a^2*B/x+a*(2*A*b+C*a)*ln(x)
```

Maxima [A]
time = 0.29, size = 139, normalized size = 0.93

$$\frac{1}{8}\,Cc^2x^8+\frac{1}{7}\,Bc^2x^7+\frac{2}{5}\,Bbcx^5+\frac{1}{6}\,(2\,Cbc+Ac^2)x^6+\frac{1}{4}\,(Cb^2+2\,(Ca+Ab)c)x^4+2\,Babx+\frac{1}{3}\,(Bb^2+2\,Bac)x^3+\frac{1}{2}\,(2\,Cab+Ab^2+2\,Aac)x^2+(Ca^2+2\,Aab)\log{(x)}-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6 +
 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^3
+ 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B
*a^2*x + A*a^2)/x^2
```

Fricas [A]
time = 0.36, size = 145, normalized size = 0.97

$$\frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\,(2\,Cbc+Ac^2)x^8+210\,(Cb^2+2\,(Ca+Ab)c)x^6+1680\,Babx^3+280\,(Bb^2+2\,Bac)x^5+420\,(2\,Cab+Ab^2+2\,Aac)x^4-840\,Ba^2x+840\,(Ca^2+2\,Aab)x^2\log{(x)}-420\,Aa^2}{840\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c^
2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2 +
2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(C*a
^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2
```

Sympy [A]
time = 0.21, size = 153, normalized size = 1.03

$$2Babx + \frac{2Bbcx^5}{5} + \frac{Bc^2x^7}{7} + \frac{Cc^2x^8}{8} + a(2Ab + Ca)\log(x) + x^6\left(\frac{Ac^2}{6} + \frac{Cbc}{3}\right) + x^4\left(\frac{Abc}{2} + \frac{Cac}{2} + \frac{Cb^2}{4}\right) + x^3\cdot\left(\frac{2Bac}{3} + \frac{Bb^2}{3}\right) + x^2\left(Aac + \frac{Ab^2}{2} + Cab\right) + \frac{-Aa^2 - 2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*a
)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4)
+ x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) + (-A*a**2
- 2*B*a**2*x)/(2*x**2)
```

Giac [A]
time = 3.33, size = 148, normalized size = 0.99

$$\frac{1}{8}Cc^2x^8 + \frac{1}{7}Bc^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{6}Ac^2x^6 + \frac{2}{5}Bbcx^5 + \frac{1}{4}Cb^2x^4 + \frac{1}{2}Cacx^4 + \frac{1}{2}Abcx^4 + \frac{1}{3}Bb^2x^3 + \frac{2}{3}Bacx^3 + Cabx^2 + \frac{1}{2}Ab^2x^2 + Aacx^2 + 2\,Babx + (Ca^2 + 2\,Aab)\log(|x|) - \frac{2\,Ba^2x + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*x
^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*B*
a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2*A*
a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

Mupad [B]
time = 0.79, size = 135, normalized size = 0.91

$$x^6\left(\frac{A\,c^2}{6} + \frac{C\,b\,c}{3}\right) + \ln(x)\,\left(C\,a^2 + 2\,A\,b\,a\right) + x^2\left(\frac{A\,b^2}{2} + C\,a\,b + A\,a\,c\right) + x^4\left(\frac{C\,b^2}{4} + \frac{A\,c\,b}{2} + \frac{C\,a\,c}{2}\right) - \frac{\frac{A\,a^2}{2} + B\,a^2\,x}{x^2} + \frac{B\,c^2\,x^7}{7} + \frac{C\,c^2\,x^8}{8} + \frac{B\,x^3\,\left(b^2 + 2\,a\,c\right)}{3} + \frac{2\,B\,b\,c\,x^5}{5} + 2\,B\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x)
```

```
[Out] x^6*((A*c^2)/6 + (C*b*c)/3) + log(x)*(C*a^2 + 2*A*a*b) + x^2*((A*b^2)/2 + A
*a*c + C*a*b) + x^4*((C*b^2)/4 + (A*b*c)/2 + (C*a*c)/2) - ((A*a^2)/2 + B*a^
2*x)/x^2 + (B*c^2*x^7)/7 + (C*c^2*x^8)/8 + (B*x^3*(2*a*c + b^2))/3 + (2*B*b
*c*x^5)/5 + 2*B*a*b*x
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2+\frac{1}{3}\left(2Abc+\left(b^2+2ac\right)C\right)x^3+\frac{1}{2}bBc$$

```
[Out] -1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+(A*(2*a*c+b^2)+2*a*b*C)*x+1/2*
B*(2*a*c+b^2)*x^2+1/3*(2*A*b*c+(2*a*c+b^2)*C)*x^3+1/2*b*B*c*x^4+1/5*c*(A*c+
2*C*b)*x^5+1/6*B*c^2*x^6+1/7*c^2*C*x^7+2*a*b*B*ln(x)
```

Rubi [A]
time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)+\frac{1}{5}cx^5(Ac+2bC)+\frac{1}{2}bBcx^4+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c)
 + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^3)
/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/
7 + 2*a*b*B*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^4}\,dx &= \int \left(Ab^2\left(1+\frac{2a(Ac+bC)}{Ab^2}\right)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\right.\\ &= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\end{aligned}$$

Mathematica [A]
time = 0.05, size = 151, normalized size = 1.01

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{-2aAb-a^2C}{x}+\left(Ab^2+2aAc+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2+\frac{1}{3}\left(2Abc+b^2C+2acC\right)x^3+\frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A*
c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^3
)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)
/7 + 2*a*b*B*Log[x]
```

Maple [A]
time = 0.02, size = 143, normalized size = 0.96

method	result
norman	$\frac{\left(\frac{1}{5}A\,c^2+\frac{2}{5}bcC\right)x^8+\left(acB+\frac{1}{2}b^2B\right)x^5+\left(\frac{2}{3}bcA+\frac{2}{3}acC+\frac{1}{3}C\,b^2\right)x^6+\left(-2abA-a^2C\right)x^2+\left(2acA+A\,b^2+2abC\right)x^4-\frac{a^2A}{3}+\frac{B\,c^2x^9}{6}-\frac{a^2Bx}{2}+}{x^3}$
default	$\frac{c^2C\,x^7}{7}+\frac{c^2B\,x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{bBc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2acAx+$
risch	$\frac{c^2C\,x^7}{7}+\frac{c^2B\,x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{bBc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2acAx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c^2*C*x^7+1/6*c^2*B*x^6+1/5*A*c^2*x^5+2/5*C*b*c*x^5+1/2*b*B*c*x^4+2/3*A
*b*c*x^3+2/3*C*a*c*x^3+1/3*C*b^2*x^3+B*a*c*x^2+1/2*B*b^2*x^2+2*a*c*A*x+A*b^
2*x+2*a*b*C*x-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x-1/3*a^2*A/x^3+2*a*b*B*ln(x)
```

Maxima [A]
time = 0.27, size = 140, normalized size = 0.94

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bbcx^4+\frac{1}{5}\,(2\,Cbc+Ac^2)x^5+\frac{1}{3}\,(Cb^2+2\,(Ca+Ab)c)x^3+2\,Bab\log\left(x\right)+\frac{1}{2}\,(Bb^2+2\,Bac)x^2+(2\,Cab+Ab^2+2\,Aac)x-\frac{3\,Ba^2x+2\,Aa^2+6\,(Ca^2+2\,Aab)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5 +
 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*c)
*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

Fricas [A]
time = 0.37, size = 145, normalized size = 0.97

$$\frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\,(2\,Cbc+Ac^2)x^8+70\,(Cb^2+2\,(Ca+Ab)c)x^6+420\,Babx^3\log\left(x\right)+105\,(Bb^2+2\,Bac)x^5+210\,(2\,Cab+Ab^2+2\,Aac)x^4-105\,Ba^2x-70\,Aa^2-210\,(Ca^2+2\,Aab)x^2}{210\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)*
x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^2
+ 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*A*a
^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A]
time = 0.40, size = 160, normalized size = 1.07

$$2Bab\log(x) + \frac{Bbcx^4}{2} + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Cbc}{5}\right) + x^3 \cdot \left(\frac{2Abc}{3} + \frac{2Cac}{3} + \frac{Cb^2}{3}\right) + x^2\left(Bac + \frac{Bb^2}{2}\right) + x(2Aac + Ab^2 + 2Cab) + \frac{-2Aa^2 - 3Ba^2x + x^2(-12Aab - 6Ca^2)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c**
2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c +
B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2
*(-12*A*a*b - 6*C*a**2))/(6*x**3)
```

Giac [A]
time = 3.88, size = 146, normalized size = 0.98

$$\frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bbcx^4 + \frac{1}{3}Cb^2x^3 + \frac{2}{3}Cacx^3 + \frac{2}{3}Abcx^3 + \frac{1}{2}Bb^2x^2 + Bacx^2 + 2Cabx + Ab^2x + 2Aacx + 2Bab\log(|x|) - \frac{3Ba^2x + 2Aa^2 + 6(Ca^2 + 2Aab)x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*x
^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*c*
x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a^2*
x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Mupad [B]
time = 0.06, size = 137, normalized size = 0.92

$$x^5\left(\frac{Ac^2}{5} + \frac{2Cbc}{5}\right) - \frac{x^2(Ca^2 + 2Aba) + \frac{Aa^2}{3} + \frac{Ba^2x}{2}}{x^3} + x(Ab^2 + 2Cab + 2Aac) + x^3\left(\frac{Cb^2}{3} + \frac{2Acb}{3} + \frac{2Cac}{3}\right) + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + \frac{Bx^2(b^2 + 2ac)}{2} + \frac{Bbcx^4}{2} + 2Bab\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x)
```

```
[Out] x^5*((A*c^2)/5 + (2*C*b*c)/5) - (x^2*(C*a^2 + 2*A*a*b) + (A*a^2)/3 + (B*a^2
*x)/2)/x^3 + x*(A*b^2 + 2*A*a*c + 2*C*a*b) + x^3*((C*b^2)/3 + (2*A*b*c)/3 +
 (2*C*a*c)/3) + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + (B*x^2*(2*a*c + b^2))/2 + (
B*b*c*x^4)/2 + 2*B*a*b*log(x)
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} \, dx$

Optimal. Leaf size=148

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x}+B\left(b^2+2ac\right)x+\frac{1}{2}\left(2Abc+\left(b^2+2ac\right)C\right)x^2+\frac{2}{3}bBcx^3+\frac{1}{4}c(Ac+2bC)x^4$$

```
[Out] -1/4*a^2*A/x^4-1/3*a^2*B/x^3-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x+B*(2*a*c+b^2)*
x+1/2*(2*A*b*c+(2*a*c+b^2)*C)*x^2+2/3*b*B*c*x^3+1/4*c*(A*c+2*C*b)*x^4+1/5*B
*c^2*x^5+1/6*c^2*C*x^6+(A*(2*a*c+b^2)+2*a*b*C)*ln(x)
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}cx^4(Ac+2bC)+\frac{2}{3}bBcx^3+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -1/4*(a^2*A)/x^4 - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)/
x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^3)
/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 + 2*
a*c) + 2*a*b*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} \, dx = \int \left(B\left(b^2+2ac\right)+\frac{a^2A}{x^5}+\frac{a^2B}{x^4}+\frac{a(2Ab+aC)}{x^3}+\frac{2abB}{x^2}+\frac{A(b^2\right.$$

$$= -\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x}+B\left(b^2+2ac\right)x+\frac{1}{2}(2A$$

Mathematica [A]
time = 0.05, size = 130, normalized size = 0.88

$$-\frac{a^2(3A+4Bx+6Cx^2)}{12x^4}+\frac{a(-Ab-2bBx+cx^3(2B+Cx))}{x^2}+\frac{1}{60}x\left(30b^2(2B+Cx)+10bcx(6A+x(4B+3Cx))+c^2x^3(15A+2x(6B+5Cx))\right)+\left(A\left(b^2+2ac\right)+2abC\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -1/12*(a^2*(3*A + 4*B*x + 6*C*x^2))/x^4 + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*B
 + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x)) +
 c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*Log[
x]
```

Maple [A]
time = 0.02, size = 139, normalized size = 0.94

method	result
default	$\frac{c^2 C x^6}{6}+\frac{B c^2 x^5}{5}+\frac{A c^2 x^4}{4}+\frac{C b c x^4}{2}+\frac{2 b B c x^3}{3}+A b c x^2+C a c x^2+\frac{C b^2 x^2}{2}+2 a c B x+b^2 B x-\frac{a^2 A}{4 x^4}-\frac{a(2\ldots}{}$
norman	$\frac{\left(\frac{1}{4} A c^2+\frac{1}{2} b c C\right) x^8+\left(-a b A-\frac{1}{2} a^2 C\right) x^2+\left(b c A+a c C+\frac{1}{2} C b^2\right) x^6+\left(2 a c B+b^2 B\right) x^5-\frac{a^2 A}{4}+\frac{B c^2 x^9}{5}-\frac{a^2 B x}{3}+\frac{c^2 C x^{10}}{6}-2 a b B x^3+\frac{2 b B c x^7}{3}}{x^4}$
risch	$\frac{c^2 C x^6}{6}+\frac{B c^2 x^5}{5}+\frac{A c^2 x^4}{4}+\frac{C b c x^4}{2}+\frac{2 b B c x^3}{3}+A b c x^2+C a c x^2+\frac{C b^2 x^2}{2}+2 a c B x+b^2 B x+\frac{-2 a b B x^3+(\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*c^2*x^4+1/2*C*b*c*x^4+2/3*b*B*c*x^3+A*b*c
*x^2+C*a*c*x^2+1/2*C*b^2*x^2+2*a*c*B*x+b^2*B*x-1/4*a^2*A/x^4-1/2*a*(2*A*b+C
*a)/x^2-2*a*b*B/x-1/3*a^2*B/x^3+(2*A*a*c+A*b^2+2*C*a*b)*ln(x)
```

Maxima [A]
time = 0.27, size = 139, normalized size = 0.94

$$\frac{1}{6} Cc^2x^6 + \frac{1}{5} Bc^2x^5 + \frac{2}{3} Bbcx^3 + \frac{1}{4} \left(2 Cbc + Ac^2\right)x^4 + \frac{1}{2} \left(Cb^2 + 2 \left(Ca + Ab\right)c\right)x^2 + \left(Bb^2 + 2 Bac\right)x + \left(2 Cab + Ab^2 + 2 Aac\right) \log\left(x\right) - \frac{24 Babx^3 + 4 Ba^2x + 3 Aa^2 + 6 \left(Ca^2 + 2 Aab\right)x^2}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4 +
 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b^2
 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 +
2*A*a*b)*x^2)/x^4
```

Fricas [A]
time = 0.38, size = 145, normalized size = 0.98

$$\frac{10 Cc^2x^{10} + 12 Bc^2x^9 + 40 Bbcx^7 + 15 \left(2 Cbc + Ac^2\right)x^8 + 30 \left(Cb^2 + 2 \left(Ca + Ab\right)c\right)x^6 - 120 Babx^3 + 60 \left(Bb^2 + 2 Bac\right)x^5 + 60 \left(2 Cab + Ab^2 + 2 Aac\right)x^4 \log\left(x\right) - 20 Ba^2x - 15 Aa^2 - 30 \left(Ca^2 + 2 Aab\right)x^2}{60 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x^
8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)
*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2 -
30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Sympy [A]
time = 1.44, size = 153, normalized size = 1.03

$$\frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)+x\left(2Bac+Bb^2\right)+\left(2Aac+Ab^2+2Cab\right)\log\left(x\right)+\frac{-3Aa^2-4Ba^2x-24Babx^3+x^2\left(-12Aab-6Ca^2\right)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] 2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2)
+ x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b**
2 + 2*C*a*b)*log(x) + (-3*A*a**2 - 4*B*a**2*x - 24*B*a*b*x**3 + x**2*(-12*A
*a*b - 6*C*a**2))/(12*x**4)
```

Giac [A]
time = 6.23, size = 142, normalized size = 0.96

$$\frac{1}{6}Cc^2x^6+\frac{1}{5}Bc^2x^5+\frac{1}{2}Cbcx^4+\frac{1}{4}Ac^2x^4+\frac{2}{3}Bbcx^3+\frac{1}{2}Cb^2x^2+Cacx^2+Abcx^2+Bb^2x+2\,Bacx+\left(2\,Cab+Ab^2+2\,Aac\right)\log\left(|x|\right)-\frac{24\,Babx^3+4\,Ba^2x+3\,Aa^2+6\,\left(Ca^2+2\,Aab\right)x^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*x
^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a*b
 + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2
+ 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Mupad [B]
time = 0.06, size = 134, normalized size = 0.91

$$x^4\,\left(\frac{A\,c^2}{4}+\frac{C\,b\,c}{2}\right)-\frac{x^2\,\left(\frac{C\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{4}+\frac{B\,a^2\,x}{3}+2\,B\,a\,b\,x^3}{x^4}+x^2\,\left(\frac{C\,b^2}{2}+A\,c\,b+C\,a\,c\right)+\ln\left(x\right)\,\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right)+\frac{B\,c^2\,x^5}{5}+\frac{C\,c^2\,x^6}{6}+B\,x\,\left(b^2+2\,a\,c\right)+\frac{2\,B\,b\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x)
```

```
[Out] x^4*((A*c^2)/4 + (C*b*c)/2) - (x^2*((C*a^2)/2 + A*a*b) + (A*a^2)/4 + (B*a^2
*x)/3 + 2*B*a*b*x^3)/x^4 + x^2*((C*b^2)/2 + A*b*c + C*a*c) + log(x)*(A*b^2
+ 2*A*a*c + 2*C*a*b) + (B*c^2*x^5)/5 + (C*c^2*x^6)/6 + B*x*(2*a*c + b^2) +
(2*B*b*c*x^3)/3
```

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=143

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{A(b^2+2ac)+2abC}{x}+\left(2Abc+\left(b^2+2ac\right)C\right)x+bBcx^2+\frac{1}{3}c(Ac+2bC)x^3$$

```
[Out] -1/5*a^2*A/x^5-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3-a*b*B/x^2+(-A*(2*a*c+b^2
)-2*a*b*C)/x+(2*A*b*c+(2*a*c+b^2)*C)*x+b*B*c*x^2+1/3*c*(A*c+2*C*b)*x^3+1/4*
B*c^2*x^4+1/5*c^2*C*x^5+B*(2*a*c+b^2)*ln(x)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A(2ac+b^2)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6} dx = \int \left(2Abc\left(1+\frac{b\left(1+\frac{2ac}{b^2}\right)C}{2Ac}\right)+\frac{a^2A}{x^6}+\frac{a^2B}{x^5}+\frac{a(2Ab+aC)}{x^4}+\right.$$

$$= -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{A(b^2+2ac)+2abC}{x}+($$

Mathematica [A]
time = 0.05, size = 142, normalized size = 0.99

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{Ab^2+2aAc+2abC}{x}+2Abcx+\left(b^2+2ac\right)Cx+bBcx^2+\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 + 2*a*c)*C*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Maple [A]
time = 0.02, size = 135, normalized size = 0.94

method	result
default	$\frac{c^2 C x^5}{5}+\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2 C b c x^3}{3}+b B c x^2+2 b c A x+2 a c C x+C b^2 x-\frac{a^2 A}{5 x^5}-\frac{a^2 B}{4 x^4}-\frac{a b B}{x^2}-\frac{2 a c A+}{}$
risch	$\frac{c^2 C x^5}{5}+\frac{B c^2 x^4}{4}+\frac{A c^2 x^3}{3}+\frac{2 C b c x^3}{3}+b B c x^2+2 b c A x+2 a c C x+C b^2 x+\frac{\left(-2 a c A-A b^2-2 a b C\right) x^4-a b B x^3}{}$
norman	$\frac{\left(\frac{1}{3} A c^2+\frac{2}{3} b c C\right) x^8+\left(-\frac{2}{3} a b A-\frac{1}{3} a^2 C\right) x^2+\left(2 b c A+2 a c C+C b^2\right) x^6+\left(-2 a c A-A b^2-2 a b C\right) x^4+b B c x^7-\frac{a^2 A}{5}+\frac{B c^2 x^9}{4}-\frac{a^2 B x}{4}+\frac{c^2 C x^{10}}{5}}{x^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*c^2*x^3+2/3*C*b*c*x^3+b*B*c*x^2+2*b*c*A*x
+2*a*c*C*x+C*b^2*x-1/5*a^2*A/x^5-1/4*a^2*B/x^4-a*b*B/x^2-(2*A*a*c+A*b^2+2*C
*a*b)/x-1/3*a*(2*A*b+C*a)/x^3+B*(2*a*c+b^2)*ln(x)
```

Maxima [A]
time = 0.28, size = 138, normalized size = 0.97

$$\frac{1}{5} C c^2 x^5+\frac{1}{4} B c^2 x^4+B b c x^2+\frac{1}{3} \left(2 C b c+A c^2\right) x^3+\left(C b^2+2 \left(C a+A b\right) c\right) x+\left(B b^2+2 B a c\right) \log\left(x\right)-\frac{60 B a b x^3+60 \left(2 C a b+A b^2+2 A a c\right) x^4+15 B a^2 x+12 A a^2+20 \left(C a^2+2 A a b\right) x^2}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C*
b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3 +
60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*
A*a*b)*x^2)/x^5
```

Fricas [A]
time = 0.36, size = 145, normalized size = 1.01

$$\frac{12 C c^2 x^{10}+15 B c^2 x^9+60 B b c x^7+20 \left(2 C b c+A c^2\right) x^8+60 \left(C b^2+2 \left(C a+A b\right) c\right) x^6+60 \left(B b^2+2 B a c\right) x^5 \log\left(x\right)-60 B a b x^3-60 \left(2 C a b+A b^2+2 A a c\right) x^4-15 B a^2 x-12 A a^2-20 \left(C a^2+2 A a b\right) x^2}{60 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x^
8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) - 60
*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2 - 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Sympy [A]
time = 4.47, size = 155, normalized size = 1.08

$$Bbcx^2 + \frac{Bc^2x^4}{4} + B\left(2ac + b^2\right)\log\left(x\right) + \frac{Cc^2x^5}{5} + x^3\left(\frac{Ac^2}{3} + \frac{2Cbc}{3}\right) + x\left(2Abc + 2Cac + Cb^2\right) + \frac{-12Aa^2 - 15Ba^2x - 60Babx^3 + x^4(-120Aac - 60Ab^2 - 120Cab) + x^2(-40Aab - 20Ca^2)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**3
*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) + (-12*A*a**2 - 15
*B*a**2*x - 60*B*a*b*x**3 + x**4*(-120*A*a*c - 60*A*b**2 - 120*C*a*b) + x**
2*(-40*A*a*b - 20*C*a**2))/(60*x**5)
```

Giac [A]
time = 5.55, size = 140, normalized size = 0.98

$$\frac{1}{5}Cc^2x^5 + \frac{1}{4}Bc^2x^4 + \frac{2}{3}Cbcx^3 + \frac{1}{3}Ac^2x^3 + Bbcx^2 + Cb^2x + 2\,Cacx + 2\,Abcx + \left(Bb^2 + 2\,Bac\right)\log\left(|x|\right) - \frac{60\,Babx^3 + 60\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 15\,Ba^2x + 12\,Aa^2 + 20\,(Ca^2 + 2\,Aab)x^2}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2 +
 C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(60
*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Mupad [B]
time = 0.05, size = 136, normalized size = 0.95

$$x^3\left(\frac{A\,c^2}{3} + \frac{2\,C\,b\,c}{3}\right) - \frac{x^2\left(\frac{C\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + \frac{A\,a^2}{5} + x^4\,\left(A\,b^2 + 2\,C\,a\,b + 2\,A\,a\,c\right) + \frac{B\,a^2\,x}{4} + B\,a\,b\,x^3}{x^5} + x\,\left(C\,b^2 + 2\,A\,c\,b + 2\,C\,a\,c\right) + \ln\left(x\right)\,\left(B\,b^2 + 2\,B\,a\,c\right) + \frac{B\,c^2\,x^4}{4} + \frac{C\,c^2\,x^5}{5} + B\,b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x)
```

```
[Out] x^3*((A*c^2)/3 + (2*C*b*c)/3) - (x^2*((C*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/5
+ x^4*(A*b^2 + 2*A*a*c + 2*C*a*b) + (B*a^2*x)/4 + B*a*b*x^3)/x^5 + x*(C*b^2
 + 2*A*b*c + 2*C*a*c) + log(x)*(B*b^2 + 2*B*a*c) + (B*c^2*x^4)/4 + (C*c^2*x
^5)/5 + B*b*c*x^2
```

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{A(b^2+2ac)+2abC}{2x^2}-\frac{B(b^2+2ac)}{x}+2bBcx+\frac{1}{2}c(Ac+2bC)x^2+\frac{1}{3}Bc^2x^3$$

```
[Out] -1/6*a^2*A/x^6-1/5*a^2*B/x^5-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3+1/2*(-A*(2
*a*c+b^2)-2*a*b*C)/x^2-B*(2*a*c+b^2)/x+2*b*B*c*x+1/2*c*(A*c+2*C*b)*x^2+1/3*
B*c^2*x^3+1/4*c^2*C*x^4+(2*A*b*c+(2*a*c+b^2)*C)*ln(x)
```

Rubi [A]
time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A(2ac+b^2)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B(2ac+b^2)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac+2bC)+2bBcx+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -1/6*(a^2*A)/x^6 - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)/
(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b*B
*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*c +
 (b^2 + 2*a*c)*C)*Log[x]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = \int \left(2bBc+\frac{a^2A}{x^7}+\frac{a^2B}{x^6}+\frac{a(2Ab+aC)}{x^5}+\frac{2abB}{x^4}+\frac{A(b^2+2ac)}{x^3}\right.$$
$$= -\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{A(b^2+2ac)+2abC}{2x^2}-$$

Mathematica [A]
time = 0.06, size = 144, normalized size = 0.97

$$-\frac{b^2B}{x}+bcx(2B+Cx)+\frac{1}{12}c^2x^3(4B+3Cx)+\frac{A(-b^2+c^2x^4)}{2x^2}-\frac{a^2(10A+3x(4B+5Cx))}{60x^6}-\frac{a(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{6x^4}+\left(2Abc+\left(b^2+2ac\right)C\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 +
c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(b
+ 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b^2
+ 2*a*c)*C)*Log[x]
```

Maple [A]
time = 0.02, size = 136, normalized size = 0.91

method	result
default	$\frac{c^2 C x^4}{4}+\frac{B c^2 x^3}{3}+\frac{A c^2 x^2}{2}+Cbc x^2+2bBcx-\frac{a^2 B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2acA+A b^2+2abC}{2x^2}-\frac{B(2ac+b^2)}{x}-\frac{2abB}{3x^3}+$
norman	$\frac{(\frac{1}{2}A c^2+bcC)x^8+(-\frac{1}{2}abA-\frac{1}{4}a^2C)x^2+(-acA-\frac{1}{2}A b^2-abC)x^4+(-2acB-b^2B)x^5-\frac{a^2A}{6}+\frac{B c^2 x^9}{3}-\frac{a^2 Bx}{5}+\frac{c^2 C x^{10}}{4}-\frac{2abB x^3}{3}+2bBc}{x^6}$
risch	$\frac{c^2 C x^4}{4}+\frac{B c^2 x^3}{3}+\frac{A c^2 x^2}{2}+Cbc x^2+2bBcx+\frac{(-2acB-b^2B)x^5+(-acA-\frac{1}{2}A b^2-abC)x^4-\frac{2abB x^3}{3}+(-\frac{1}{2}abA-\frac{1}{4}a^2C)}{x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*c^2*x^2+C*b*c*x^2+2*b*B*c*x-1/5*a^2*B/x^5
-1/4*a*(2*A*b+C*a)/x^4-1/2*(2*A*a*c+A*b^2+2*C*a*b)/x^2-B*(2*a*c+b^2)/x-2/3*
a*b*B/x^3+(2*A*b*c+2*C*a*c+C*b^2)*ln(x)-1/6*a^2*A/x^6
```

Maxima [A]
time = 0.27, size = 140, normalized size = 0.94

$$\frac{1}{4}Cc^2x^4+\frac{1}{3}Bc^2x^3+2Bbcx+\frac{1}{2}(2Cbc+Ac^2)x^2+(Cb^2+2(Ca+Ab)c)\log(x)-\frac{40Babx^3+60(Bb^2+2Bac)x^5+30(2Cab+Ab^2+2Aac)x^4+12Ba^2x+10Aa^2+15(Ca^2+2Aab)x^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C*
b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x
^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2
 + 2*A*a*b)*x^2)/x^6
```

Fricas [A]
time = 0.37, size = 145, normalized size = 0.97

$$\frac{15Cc^2x^{10}+20Bc^2x^9+120Bbcx^7+30(2Cbc+Ac^2)x^8+60(Cb^2+2(Ca+Ab)c)x^6\log(x)-40Babx^3-60(Bb^2+2Bac)x^5-30(2Cab+Ab^2+2Aac)x^4-12Ba^2x-10Aa^2-15(Ca^2+2Aab)x^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*x
^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 + 2
*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2 -
15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Sympy [A]
time = 29.62, size = 158, normalized size = 1.06

$$2Bbcx + \frac{Bc^2x^3}{3} + \frac{Cc^2x^4}{4} + x^2\left(\frac{Ac^2}{2} + Cbc\right) + \left(2Abc + 2Cac + Cb^2\right)\log\left(x\right) + \frac{-10Aa^2 - 12Ba^2x - 40Babx^3 + x^5(-120Bac - 60Bb^2) + x^4(-60Aac - 30Ab^2 - 60Cab) + x^2(-30Aab - 15Ca^2)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A*
b*c + 2*C*a*c + C*b**2)*log(x) + (-10*A*a**2 - 12*B*a**2*x - 40*B*a*b*x**3
+ x**5*(-120*B*a*c - 60*B*b**2) + x**4*(-60*A*a*c - 30*A*b**2 - 60*C*a*b) +
 x**2*(-30*A*a*b - 15*C*a**2))/(60*x**6)
```

Giac [A]
time = 5.53, size = 141, normalized size = 0.95

$$\frac{1}{4}Cc^2x^4 + \frac{1}{3}Bc^2x^3 + Cbcx^2 + \frac{1}{2}Ac^2x^2 + 2\,Bbcx + \left(Cb^2 + 2\,Cac + 2\,Abc\right)\log\left(|x|\right) - \frac{40\,Babx^3 + 60\,(Bb^2 + 2\,Bac)x^5 + 30\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 12\,Ba^2x + 10\,Aa^2 + 15\,(Ca^2 + 2\,Aab)x^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C*
b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B
*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15
*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Mupad [B]
time = 0.06, size = 136, normalized size = 0.91

$$x^2\left(\frac{A\,c^2}{2} + C\,b\,c\right) - \frac{x^2\left(\frac{C\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^5\left(B\,b^2 + 2\,B\,a\,c\right) + \frac{A\,a^2}{6} + x^4\left(\frac{A\,b^2}{2} + C\,a\,b + A\,a\,c\right) + \frac{B\,a^2\,x}{5} + \frac{2\,B\,a\,b\,x^3}{3}}{x^6} + \ln\left(x\right)\,\left(C\,b^2 + 2\,A\,c\,b + 2\,C\,a\,c\right) + \frac{B\,c^2\,x^3}{3} + \frac{C\,c^2\,x^4}{4} + 2\,B\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x)
```

```
[Out] x^2*((A*c^2)/2 + C*b*c) - (x^2*((C*a^2)/4 + (A*a*b)/2) + x^5*(B*b^2 + 2*B*a
*c) + (A*a^2)/6 + x^4*((A*b^2)/2 + A*a*c + C*a*b) + (B*a^2*x)/5 + (2*B*a*b*
x^3)/3)/x^6 + log(x)*(C*b^2 + 2*A*b*c + 2*C*a*c) + (B*c^2*x^3)/3 + (C*c^2*x
^4)/4 + 2*B*b*c*x
```

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$\frac{(Ac-bC)x}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c}-\frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(Abc\right.}{}$$

```
[Out] (A*c-C*b)*x/c^2+1/2*B*x^2/c+1/3*C*x^3/c-1/4*b*B*ln(c*x^4+b*x^2+a)/c^2-1/2*B
*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2
)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c-b^2*C+a
*c*C+(-A*c*(-2*a*c+b^2)+b*(-3*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*(A*b*c-b^2*C+a*c*C+(A*c*(-2*a*c+b^2)-b*(-3*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.21, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1676, 1293, 1180, 211, 12, 1128, 717, 648, 632, 212, 642}

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x(Ac-bC)}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b^2*C + a*c
*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 -
3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*B
*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^5}{a+bx^2+cx^4}\,dx + \int \frac{x^4(A+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx^3}{3c} + B\int \frac{x^5}{a+bx^2+cx^4}\,dx - \frac{\int \frac{x^2(3aC-3(Ac-bC)x^2)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Cx^3}{3c} + \frac{1}{2}B\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right) + \frac{\int \frac{-3a(Ac-bC)-3(\ldots}{a+bx\ldots}}{3\ldots}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} + \frac{B\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} - \frac{\left(Abc-b^2C+a\ldots\right.}{\ldots}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{\ldots}}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\ldots}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{\ldots}}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\ldots}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\tan^{\ldots}}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\ldots}$$

Mathematica [A]
time = 0.39, size = 460, normalized size = 1.36

$$\frac{12\sqrt{c}(Ac-bC)x+6Bc^{3/2}x^2+4c^{3/2}Cx^3+\frac{6\sqrt{2}\left(Ac\left(b^2-2ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{6\sqrt{2}\left(-Ac\left(b^2-2ac+b\sqrt{b^2-4ac}\right)+\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{3B\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{3B\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{12c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]*
(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 -
 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*Sqrt
[2]*(-(A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2*Sqrt
[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
```

```
3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(12*c^(5/2)
)
```

Maple [A]
time = 0.07, size = 331, normalized size = 0.98

method	result
risch	$\frac{C x^3}{3c}+\frac{B x^2}{2c}+\frac{Ax}{c}-\frac{bCx}{c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(-bBc\_R^3+\left(-bcA-acC+C b^2\right)\_R^2-acB\_R-acA+abC\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}}{2c^2}$
default	$\frac{\frac{1}{3}cC x^3+\frac{1}{2}Bc x^2+Acx-bCx}{c^2}+\frac{\left(2ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}-4abc+b^3\right)\left(\frac{B\ln\left(-b-2c x^2+\sqrt{-4ac+b^2}\right)}{2}+\frac{\left(-2A\right.}{}\right.}{2c\left(4ac-b^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/3*c*C*x^3+1/2*B*c*x^2+A*c*x-b*C*x)+4/c*(1/8*(2*a*c*(-4*a*c+b^2)^(1
/2)-b^2*(-4*a*c+b^2)^(1/2)-4*a*b*c+b^3)/c/(4*a*c-b^2)*(1/2*B*ln(-b-2*c*x^2+
(-4*a*c+b^2)^(1/2))+1/2*(-2*A*c-C*(-4*a*c+b^2)^(1/2)+b*C)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)))-1/8*(2*a*c*(-4*a*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+4*a*b*c-b^3)/c/(
4*a*c-b^2)*(1/2*B*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(2*A*c-C*(-4*a*c+b^2
)^(1/2)-b*C)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*b*c*x^3 +
B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a),
 x)/c^2
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5305 vs. $2(294) = 588$.
time = 6.58, size = 5305, normalized size = 15.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*B*b*log(abs(c*x^4 + b*x^2 + a))/c^2 - 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 +
 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^
3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sq
```

rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2
 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*C*c^2 + 2*(sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a
*c)*a^2*c^5)*A*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^
2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^
3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5
- 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(c) - (2*b^5*
c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6
 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a
*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4
 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 -
2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*C)*arctan(2*sqrt(1/2)*
x/sqrt((b*c^7 + sqrt(b^2*c^14 - 4*a*c^15))/c^8))/((a*b^4*c^4 - 8*a^2*b^2*c^
5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) -
1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2
*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq

```
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)...
```

Mupad [B]
time = 0.96, size = 2588, normalized size = 7.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (C*b)/c^2) + symsum(log((C^3*a^4*c - C^3*a^3*b^2 - A*B^2*a^3*c^2 +
 A*C^2*a^2*b^3 + A^2*C*a^3*c^2 + A^3*a^2*b*c^2 + A*B^2*a^2*b^2*c - 2*A^2*C*
a^2*b^2*c - B^2*C*a^3*b*c)/c^3 - root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 -
256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*
z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80
*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a
*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^
2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A
^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z
 - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^
3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z
 + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^
2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c -
 C^4*a^5, z, k)*(root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4
- 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b
^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z
^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 3
6*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a
^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 -
 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*
b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z +
16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b
*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c -
B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*
((x*(16*B*a^2*c^5 + 8*B*b^4*c^3 - 36*B*a*b^2*c^4))/c^3 - (16*A*a^2*c^5 - 4*
A*a*b^2*c^4 + 4*C*a*b^3*c^3 - 16*C*a^2*b*c^4)/c^3 + (root(128*a*b^2*c^6*z^4
 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4
*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 +
8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*
b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2
```

```
- 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B
^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z -
 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^
3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z
- 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c
+ 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a
^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*x*(8*b^3*c^5 - 32*a*b*c^6))/c^3) + (8*B
*C*a^3*c^3 - 4*A*B*a^2*b*c^3)/c^3 + (x*(2*C^2*b^6 + 2*B^2*b^5*c + 4*A^2*a^2
*c^4 + 2*A^2*b^4*c^2 - 4*C^2*a^3*c^3 - 4*A*C*b^5*c + 18*C^2*a^2*b^2*c^2 - 1
2*C^2*a*b^4*c - 8*A^2*a*b^2*c^3 - 10*B^2*a*b^3*c^2 + 6*B^2*a^2*b*c^3 + 20*A
*C*a*b^3*c^2 - 20*A*C*a^2*b*c^3))/c^3) + (x*(B*C^2*a^2*b^3 - B^3*a^3*c^2 +
B^3*a^2*b^2*c + A^2*B*a^2*b*c^2 + 2*A*B*C*a^3*c^2 - 2*B*C^2*a^3*b*c - 2*A*B
*C*a^2*b^2*c))/c^3)*root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z
^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*
a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^
3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2
+ 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^
2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^
2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a
^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z
 + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^
3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c
 - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z,
k), k, 1, 4) + (B*x^2)/(2*c) + (C*x^3)/(3*c)
```

3.22 $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=278

$$\frac{Bx}{c}+\frac{Cx^2}{2c}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2-}}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] B*x/c+1/2*C*x^2/c+1/4*(A*c-C*b)*ln(c*x^4+b*x^2+a)/c^2+1/2*(A*b*c+2*C*a*c-C*
b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)-1/2*B*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*
c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1676, 1265, 787, 648, 632, 212, 642, 12, 1136, 1180, 211}

$$\frac{(2acC+Abc+b^2(-C))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}+\frac{Cx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4]
)/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx &= \int \frac{Bx^4}{a+bx^2+cx^4}\,dx + \int \frac{x^3(A+Cx^2)}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Cx)}{a+bx+cx^2}\,dx, x, x^2\right) + B\int \frac{x^4}{a+bx^2+cx^4}\,dx \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} + \frac{\text{Subst}\left(\int \frac{-aC+(Ac-bC)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} - \frac{B\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{\left(B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(B\left(b+\frac{}{\sqrt{}}\right.\right.}{} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b+\frac{}{\sqrt{}}\right.}{} \\
&= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b+\frac{}{\sqrt{}}\right.}{}
\end{aligned}$$

Mathematica [A]

time = 0.27, size = 377, normalized size = 1.36

$$\frac{4Bcx+2cCx^2-\frac{2\sqrt{2}B\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}B\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{\left(Ac\left(-b+\sqrt{b^2-4ac}\right)+\left(b^2-2ac-b\sqrt{b^2-4ac}\right)C\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(-Ac\left(b+\sqrt{b^2-4ac}\right)+\left(b^2-2ac+b\sqrt{b^2-4ac}\right)C\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b^2 -
4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 -
2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[
b^2 - 4*a*c])/(4*c^2)
```

Maple [A]
time = 0.06, size = 419, normalized size = 1.51

method	result
risch	$\frac{C\,x^2}{2c}+\frac{Bx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left((Ac-bC)\_R^3-Bb\_R^2-aC\_R-aB\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{\frac{1}{2}C\,x^2+Bx}{c}+\frac{-\frac{\left(-bcA\sqrt{-4ac+b^2}-4c^2aA+A\,b^2c-2C\sqrt{-4ac+b^2}\,ac+C\sqrt{-4ac+b^2}\,b^2+4Cabc-C\,b^3\right)\ln\left(-b-2c\,x^2+\sqrt{\ldots}\right)}{4c}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*C*x^2+B*x)+1/c/(4*a*c-b^2)*(-1/4*(-b*c*A*(-4*a*c+b^2)^(1/2)-4*c^2*
a*A+A*b^2*c-2*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C
*b^3)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-2*a*c*B*(-4*a*c+b^2)^(1/2)+
b^2*B*(-4*a*c+b^2)^(1/2)+4*a*b*B*c-b^3*B)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/c/(4*a*c
-b^2)*(1/4*(-b*c*A*(-4*a*c+b^2)^(1/2)+4*c^2*a*A-A*b^2*c-2*C*(-4*a*c+b^2)^(1
/2)*a*c+C*(-4*a*c+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)/c*ln(b+2*c*x^2+(-4*a*c+b^
2)^(1/2))+1/2*(-2*a*c*B*(-4*a*c+b^2)^(1/2)+b^2*B*(-4*a*c+b^2)^(1/2)-4*a*b*B
*c+b^3*B)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C*a*x + B*a
)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [**C**] Result contains complex when optimal does not.
time = 61.88, size = 1329593, normalized size = 4782.71

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/48*(24*C*c*x^2 - 12*c^2*(sqrt(2*sqrt(b^2 - 4*a*c)*c^2*(2*(C^3*a^2*b + B^2
*C*a^2*c + 2*A^2*C*a*b*c - A^3*a*c^2 - (B^2*a*b*c + (a*b^2 + a^2*c)*C^2)*A)
/(b^2*c^4 - 4*a*c^5) + ((b^2*c^2 - 6*a*c^3)*A^2 + (b^3*c - 3*a*b*c^2)*B^2 -
 2*(b^3*c - 5*a*b*c^2)*A*C + (b^4 - 4*a*b^2*c - 2*a^2*c^2)*C^2)*(C*b - A*c)
/((b^2*c^4 - 4*a*c^5)*c^2) - (C*b - A*c)^3/c^6)/(C*b^2 - (2*C*a + A*b)*c) -
 ((b^2*c^2 - 2*a*c^3)*A^2 + (b^3*c - 3*a*b*c^2)*B^2 - 2*(b^3*c - 3*a*b*c^2)
*A*C + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*C^2)/(b^2*c^4 - 4*a*c^5) - ((b^2*c^2 -
 6*a*c^3)*A^2 + (b^3*c - 3*a*b*c^2)*B^2 - 2*(b^3*c - 5*a*b*c^2)*A*C + (b^4
- 4*a*b^2*c - 2*a^2*c^2)*C^2)/(b^2*c^4 - 4*a*c^5) + 2*(C*b - A*c)^2/c^4) +
(C*b - A*c)/c^2 - (C*b^2 - (2*C*a + A*b)*c)/(sqrt(b^2 - 4*a*c)*c^2))*log(C^
5*a^2*b^7 - A*C^4*a*b^8 + 2*(4*A^4*C*a^3*b + A^5*a^2*b^2 + 2*(A*B^4 + A^2*B
^2*C + 2*A^3*C^2)*a^4)*c^5 - (A^5*a*b^4 + 4*(5*B^2*C^3 - 2*A*C^4)*a^5 - 4*(
2*B^4*C - 2*A*B^2*C^2 - 3*A^2*C^3)*a^4*b + (7*A*B^4 - 15*A^2*B^2*C + 30*A^3
*C^2)*a^3*b^2 - (8*A^3*B^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3520 vs. 2(232) = 464.
time = 7.76, size = 3520, normalized size = 12.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(C*b - A*c)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(C*c*x^2 + 2*B*c*x)/
c^2 - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*
(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c
^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b
^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 + sqrt(b^2*c^10 - 4*a
*c^11))/c^6))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^
2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32
*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
+ 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8
*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^
2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5
+ 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^
4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
```

```
(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 -
 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x
/sqrt((b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^3 - 8*a^2*b^2*c^4
 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1
/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*
c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b
^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^7 - 10*a*b^5
*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*
a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 + (b^6 - 10*a*b^4*c - 2*b^5*c + 32*
a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*
c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b
^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 -
4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A - (b^7*c - 10*a*b^5
*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 -
 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c - 6*a*b^4*c^2 - 2*b^5*
c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4)*sqrt(b^2 - 4*a*c
))*C)*log(x^2 + 1/2*(b*c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6)/((a*b^4*c - 8*
a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^
4)*c^2*abs(c)) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 +
8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16
*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)...
```

Mupad [B]
time = 1.53, size = 2696, normalized size = 9.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log((B^3*a^2*b*c - B*C^2*a^3*c + A^2*B*a^2*c^2 + B*C^2*a^2*b^2 - 2*A
*B*C*a^2*b*c)/c^2 - root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z
^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C
*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2
 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a
*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c
^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*
a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z +
12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a
*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*
```

C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A
^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*
a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a
^4, z, k)*(root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*
C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*
z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C
*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*
z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 -
4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z
^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*
a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z
- 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c
^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*
b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c -
B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k)
((x(16*C*a^2*c^4 - 8*A*b^3*c^3 + 8*C*b^4*c^2 + 32*A*a*b*c^4 - 36*C*a*b^2*
c^3))/c^2 - (16*B*a^2*c^4 - 4*B*a*b^2*c^3)/c^2 + (root(128*a*b^2*c^5*z^4 -
16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^
3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c
^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 4
8*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*
a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2
- 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2
*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z +
8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*
z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*
b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^
3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A
^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k)*x*(8*b^3*c^4 - 32*a*b*c^5))/c^2) +
(8*A*B*a^2*c^3 - 4*B*C*a^2*b*c^2)/c^2 + (x*(2*C^2*b^5 + 2*B^2*b^4*c + 2*A^2
*b^3*c^2 + 4*B^2*a^2*c^3 - 4*A*C*b^4*c - 8*A*C*a^2*c^3 - 10*A^2*a*b*c^3 - 1
0*C^2*a*b^3*c - 8*B^2*a*b^2*c^2 + 6*C^2*a^2*b*c^2 + 20*A*C*a*b^2*c^2))/c^2)
- (x*(C^3*a^3*c - C^3*a^2*b^2 + A*C^2*a*b^3 + A^3*a*b*c^2 - A*B^2*a^2*c^2
+ A^2*C*a^2*c^2 + A*B^2*a*b^2*c - 2*A^2*C*a*b^2*c - B^2*C*a^2*b*c))/c^2)*ro
ot(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z
^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^
4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^
2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2*
a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z
^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^
6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z +
16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b
^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3
*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2
*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*

```
b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k), k, 1, 4) +
(C*x^2)/(2*c) + (B*x)/c
```

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=270

$$\frac{Cx}{c}+\frac{\left(Ac-bC-\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(Ac-bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b+}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] C*x/c+1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1
/2))/c/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(A*c-b*C+(-A*b*c+(-2*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*(A*c-b*C+(A*b*c+2*C*a*c-C*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)
*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1676, 1293, 1180, 211, 12, 1128, 648, 632, 212, 642}

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^3}{a+bx^2+cx^4}\,dx + \int \frac{x^2(A+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx}{c} + B\int \frac{x^3}{a+bx^2+cx^4}\,dx - \frac{\int \frac{aC+(-Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{Cx}{c} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right) - \frac{\left(-Ac+bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\int}{2c}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\right.}{}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\right.}{}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\right.}{}$$

Mathematica [A]
time = 0.24, size = 360, normalized size = 1.33

$$\frac{4\sqrt{c}\,Cx - \frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(-Ac\left(b+\sqrt{b^2-4ac}\right)+\left(b^2-2ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{B\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{B\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(-(A*c*(
b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c]
)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

Maple [A]
time = 0.07, size = 267, normalized size = 0.99

method	result
risch	$\frac{Cx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(Bc\_R^3+(Ac-bC)\_R^2-aC\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}}{2c}$
default	$\frac{Cx}{c}+\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)\left(\frac{B\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{2}+\frac{\left(-2Ac-C\sqrt{-4ac+b^2}+bC\right)\sqrt{2}\,\mathrm{arctanh}\left(\ldots\right)}{2\sqrt{\left(-b+\sqrt{-4ac+\ldots}\right.}}\ldots\right.}{2c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*x/c+1/2*(b*(-4*a*c+b^2)^(1/2)+4*a*c-b^2)/c/(4*a*c-b^2)*(1/2*B*ln(-b-2*c*x
^2+(-4*a*c+b^2)^(1/2))+1/2*(-2*A*c-C*(-4*a*c+b^2)^(1/2)+b*C)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)))-1/2*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/c/(4*a*c-b^2)*(1/2*B*ln(b+2*c
*x^2+(-4*a*c+b^2)^(1/2))+1/2*(2*A*c-C*(-4*a*c+b^2)^(1/2)-b*C)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^2 + a), x)
/c
```

Fricas [C] Result contains complex when optimal does not.
time = 57.71, size = 861800, normalized size = 3191.85

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(3*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4 + (b^2 - 4*a*c)^(3/2)*b*c^2)*A^2 + (2*b^5*c + 32*a^2*b*c^3 + sqrt(b^
2 - 4*a*c)*b^4*c + (b^2 - 4*a*c)^(3/2)*b^2*c - 4*(4*b^3*c^2 + sqrt(b^2 - 4*
a*c)*b^2*c^2 + 3*(b^2 - 4*a*c)^(3/2)*c^2)*a)*B^2 - 4*(b^5*c + 16*a^2*b*c^3
+ (b^2 - 4*a*c)^(3/2)*b^2*c - 2*(4*b^3*c^2 + (b^2 - 4*a*c)^(3/2)*c^2)*a)*A*
C + 2*(b^6 + 24*a^2*b^2*c^2 - 16*a^3*c^3 + (b^2 - 4*a*c)^(3/2)*b^3 - 3*(3*b
^4*c + (b^2 - 4*a*c)^(3/2)*b*c)*a)*C^2 + 2*(sqrt(2)*b^5*c^2*sqrt(-(C^2*b^3
+ (4*A*C*a + A^2*b)*c^2 - (3*C^2*a*b + 2*A*C*b^2)*c - (C^2*b^2 + A^2*c^2 -
(C^2*a + 2*A*C*b)*c)*sqrt(b^2 - 4*a*c))/(b^2*c^3 - 4*a*c^4)) + 16*sqrt(2)*a
^2*b*c^4*sqrt(-(C^2*b^3 + (4*A*C*a + A^2*b)*c^2 - (3*C^2*a*b + 2*A*C*b^2)*c
 - (C^2*b^2 + A^2*c^2 - (C^2*a + 2*A*C*b)*c)*sqrt(b^2 - 4*a*c))/(b^2*c^3 -
4*a*c^4)) + sqrt(2)*(b^2 - 4*a*c)^(3/2)*b^2*c^2*sqrt(-(C^2*b^3 + (4*A*C*a +
 A^2*b)*c^2 - (3*C^2*a*b + 2*A*C*b^2)*c - (C^2*b^2 + A^2*c^2 - (C^2*a + 2*A
*C*b)*c)*sqrt(b^2 - 4*a*c ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3844 vs. 2(227) = 454.
time = 6.79, size = 3844, normalized size = 14.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/4*B*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^
4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
```

```
b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3
 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a
*c)*a*b*c^3)*C*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 -
 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 -
4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2
*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)
*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*
C)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^
4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5
- 4*a^2*c^6)*c^2) + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*
A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 - 2*(sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4
+ 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2
- 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 -
 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c...
```

Mupad [B]
time = 2.00, size = 1890, normalized size = 7.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 12
8*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2
*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C
^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2
- 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*
z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2
*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c -
 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*
c - A^4*a*c^2 - C^4*a^3, z, k)*((8*B*C*a^2*c^2 - 4*A*B*a*b*c^2)/c - root(12
8*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 +
16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*A*C*b^4*c*z
^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3*c*z^2 + 16*
A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^2*a^2*c^3*z^2
 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16*B*C^2*a^2*b*c
*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z + 16*B^3*a^2*c^
2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^2*C^2*a^2*c + 2
```

```
*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A^4*a*c^2 - C^4*
a^3, z, k)*((16*C*a^2*c^3 - 4*C*a*b^2*c^2)/c + (x*(8*B*b^3*c^2 - 32*B*a*b*c
^3))/c - (root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B
*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^
2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*
a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 9
6*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z +
 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*
z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*
A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c -
 A^4*a*c^2 - C^4*a^3, z, k)*x*(8*b^3*c^3 - 32*a*b*c^4))/c) + (x*(2*C^2*b^4
- 4*A^2*a*c^3 + 2*B^2*b^3*c + 2*A^2*b^2*c^2 + 4*C^2*a^2*c^2 - 4*A*C*b^3*c -
 10*B^2*a*b*c^2 - 8*C^2*a*b^2*c + 12*A*C*a*b*c^2))/c) - (A^3*a*c^2 - C^3*a^
2*b + A*C^2*a*b^2 + A*C^2*a^2*c - B^2*C*a^2*c + A*B^2*a*b*c - 2*A^2*C*a*b*c
)/c - (x*(B^3*a*b*c + A^2*B*a*c^2 + B*C^2*a*b^2 - B*C^2*a^2*c - 2*A*B*C*a*b
*c))/c)*root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a
*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2
+ 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*
b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*
B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 1
6*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z
+ 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^
2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A
^4*a*c^2 - C^4*a^3, z, k), k, 1, 4) + (C*x)/c
```

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=223

$$-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{(2}{}$$

```
[Out] 1/4*C*ln(c*x^4+b*x^2+a)/c-1/2*(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^
(1/2))/c/(-4*a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/
2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1676, 1261, 648, 632, 212, 642, 12, 1144, 211}

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{C\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (B*Sqrt[b + Sqrt[b^2
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^2}{a+bx^2+cx^4}\,dx + \int \frac{x(A+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{a+bx+cx^2}\,dx, x, x^2\right) + B\int \frac{x^2}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\left(B\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(B\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.$$

$$= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-}}{\sqrt{2}\sqrt{c}\sqrt{}}$$

$$= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-}}{\sqrt{2}\sqrt{c}\sqrt{}}$$

Mathematica [A]
time = 0.23, size = 240, normalized size = 1.08

$$\frac{-2\sqrt{2}B\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+2\sqrt{2}B\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)+\left(2Ac+\left(-b+\sqrt{b^2-4ac}\right)C\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)-\left(2Ac-\left(b+\sqrt{b^2-4ac}\right)C\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]] + (2*A*c + (-
b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*c -
(b + Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[
b^2 - 4*a*c])
```

Maple [A]
time = 0.05, size = 313, normalized size = 1.40

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(C\_R^3+B\_R^2+A\_R\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$

default	$4c$	$\left(\frac{-\frac{\left(2A\sqrt{-4ac+b^2}\,c-C\sqrt{-4ac+b^2}\,b-4acC+C\,b^2\right)\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)}{4c}+\frac{\left(-B\sqrt{-4ac+b^2}\,b-4ac\right.}{2\sqrt{}}}{4c(4ac-b^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(1/4/c/(4*a*c-b^2)*(-1/4*(2*A*(-4*a*c+b^2)^(1/2)*c-C*(-4*a*c+b^2)^(1/2)
*b-4*a*c*C+C*b^2)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-B*(-4*a*c+b^2)^
(1/2)*b-4*a*c*B+b^2*B)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^2)*(1/4*(2*A*(
-4*a*c+b^2)^(1/2)*c-C*(-4*a*c+b^2)^(1/2)*b+4*a*c*C-C*b^2)/c*ln(b+2*c*x^2+(-
4*a*c+b^2)^(1/2))+1/2*(-B*(-4*a*c+b^2)^(1/2)*b+4*a*c*B-b^2*B)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 14.29, size = 845032, normalized size = 3789.38

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/48*(12*c*(2*sqrt(-1/2*sqrt(b^2 - 4*a*c)*c*(2*(C^3*a + A^2*C*c - (C^2*b +
 B^2*c)*A)/(b^2*c^2 - 4*a*c^3) - C^3/c^3 + (B^2*b*c + 2*A*C*b*c - 2*A^2*c^2
 + (b^2 - 6*a*c)*C^2)*C/((b^2*c^2 - 4*a*c^3)*c))/(C*b - 2*A*c) - 1/4*(B^2*b
*c + 2*A*C*b*c - 2*A^2*c^2 + (b^2 - 6*a*c)*C^2)/(b^2*c^2 - 4*a*c^3) - 1/4*(
```

```
B^2*b*c - 2*A*C*b*c + 2*A^2*c^2 + (b^2 - 2*a*c)*C^2)/(b^2*c^2 - 4*a*c^3) +
1/2*C^2/c^2) - C/c - (C*b - 2*A*c)/(sqrt(b^2 - 4*a*c)*c))*log(-C^5*a*b^3 +
A*C^4*b^4 + 8*A^5*c^4 - 4*(5*A^4*C*b + (3*A*B^4 - 3*A^2*B^2*C - 2*A^3*C^2)*
a)*c^3 + 1/4*(B^2*b^4*c^3 - 8*B^2*a*b^2*c^4 + 16*B^2*a^2*c^5)*(2*sqrt(-1/2*
sqrt(b^2 - 4*a*c)*c*(2*(C^3*a + A^2*C*c - (C^2*b + B^2*c)*A)/(b^2*c^2 - 4*a
*c^3) - C^3/c^3 + (B^2*b*c + 2*A*C*b*c - 2*A^2*c^2 + (b^2 - 6*a*c)*C^2)*C/(
(b^2*c^2 - 4*a*c^3)*c))/(C*b - 2*A*c) - 1/4*(B^2*b*c + 2*A*C*b*c - 2*A^2*c^
2 + (b^2 - 6*a*c)*C^2)/(b^2*c^2 - 4*a*c^3) - 1/4*(B^2*b*c - 2*A*C*b*c + 2*A
^2*c^2 + (b^2 - 2*a*c)*C^2)/(b^2*c^2 - 4*a*c^3) + 1/2*C^2/c^2) - C/c - (C*b
 - 2*A*c)/(sqrt(b^2 - 4*a ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2368 vs. 2(179) = 358.
time = 5.19, size = 2368, normalized size = 10.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*C*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a
^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*
a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/s
qrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a
*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*c^2) + 1/8*((2
*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
```

```
 sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2
- 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*
c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c
^4 - 4*a^2*c^5)*c^2) + 1/16*(2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*
c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 - (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2
+ 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) -
 (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*
b^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2
 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^
4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b
^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2 -
 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c
- 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b
*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a
^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) + 1/16*(2*(b^5*c
- 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^
4 + (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2 + 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4
*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b
^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c +
 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs
(c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b
^3*c^4 - 4*a*b*c^5 - (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2
 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*
c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*s
qrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*c^2*abs(c))
```

Mupad [B]
time = 1.89, size = 2500, normalized size = 11.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(A^3*c^2*x - B^3*a*c - B*C^2*a*b - 8*root(128*a*b^2*c^3*z^4 - 16*
b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 1
6*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 +
 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z
^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z
 + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z -
 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*
c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a
^2 - A^4*c^2, z, k)^3*b^3*c^2*x - C^3*a*b*x + A*C^2*b^2*x - 2*C^2*root(128*
a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 25
6*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 4
0*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2
 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*
z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z
 + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C
^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c -
A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^3*x + 32*root(128*a*b^2*c^3*z^4 -
16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3
+ 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^
2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^
2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*
c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*
z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C
*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^
4*a^2 - A^4*c^2, z, k)^3*a*b*c^3*x - 4*A*root(128*a*b^2*c^3*z^4 - 16*b^4*c^
2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^
4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^
2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8
*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*
A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3
*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*
A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A
^4*c^2, z, k)^2*b^2*c^2*x - 8*A*B*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 -
 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3
 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c
^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^
2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a
*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*
z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a
*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2,
 z, k)*a*c^2 + A*B^2*b*c*x + A*C^2*a*c*x - 2*A^2*C*b*c*x - B^2*C*a*c*x + 2*
A*B*C*a*c + 16*A*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4
- 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2
*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^
3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2
```

```
*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*
b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z
 + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b
- A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x
+ 2*A^2*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a
*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*
A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 -
 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2
- 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z -
 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^
2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2
*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b*c^2*x + 4*B^2*roo
t(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^
3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z
^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c
^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*
a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a
*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2
*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*
a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x - 2*B^2*root(128*a*b^2
*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a
^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^...
```

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4} dx$

Optimal. Leaf size=211

$$\frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{B\,\mathrm{ta}}{}$$

```
[Out] -B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(
1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(C+(-2*A*c+C*b)/(-4*a*c+b^2)^(1/2))*2^(1
/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C -
(2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \int \frac{Bx}{a+bx^2+cx^4}\,dx + \int \frac{A+Cx^2}{a+bx^2+cx^4}\,dx$$

$$= B\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\Big(C$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(-}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2}}}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(-}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2}}}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(-}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2}}}$$

Mathematica [A]
time = 0.14, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(2Ac+\left(-b+\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac+\left(b+\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + B\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right) - B\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 242, normalized size = 1.15

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{2}+\frac{\left(-2Ac-C\sqrt{-4ac+b^2}+bC\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/2*B*ln(-b-2*c*x^2+(-4*a*c+b^2
)^(1/2))+1/2*(-2*A*c-C*(-4*a*c+b^2)^(1/2)+b*C)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4*(
-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/2*B*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2
*(2*A*c-C*(-4*a*c+b^2)^(1/2)-b*C)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 27.60, size = 578003, normalized size = 2739.35

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(1/4)^(2/3)*(-I*sqrt(3) + 1)*(3*(4*(b^2 - 4*a*c)^(3/2)*A*C*a*c - (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3 + (b^2 - 4*a*c)^(3/2)*b*c)*A^2 + 2*(4*sqrt
(b^2 - 4*a*c)*a^2*c^2 - (sqrt(b^2 - 4*a*c)*b^2*c - 2*(b^2 - 4*a*c)^(3/2)*c)
*a)*B^2 - (8*a^2*b^2*c - 16*a^3*c^2 - (b^4 - (b^2 - 4*a*c)^(3/2)*b)*a)*C^2
- 4*(sqrt(1/2)*a*b^4*c*sqrt(-(C^2*a*b - (4*A*C*a - A^2*b)*c + (C^2*a - A^2*
c)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - 8*sqrt(1/2)*a^2*b^2*c^2*sqrt
(-(C^2*a*b - (4*A*C*a - A^2*b)*c + (C^2*a - A^2*c)*sqrt(b^2 - 4*a*c))/(a*b^
2*c - 4*a^2*c^2)) + 16*sqrt(1/2)*a^3*c^3*sqrt(-(C^2*a*b - (4*A*C*a - A^2*b)
*c + (C^2*a - A^2*c)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)))*B)/((b^2 -
4*a*c)^(3/2)*a*b^2*c - 4*(b^2 - 4*a*c)^(3/2)*a^2*c^2) + 2*(sqrt(1/2)*(b^2 -
 4*a*c)^(3/2)*sqrt(-(C^2*a*b - (4*A*C*a - A^2*b)*c + (C^2*a - A^2*c)*sqrt(b
^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) + (b^2 - 4*a*c)*B)^2/(b^2 - 4*a*c)^3)/(
27*(sqrt(1/2)*(b^2 - 4*a*c)^(3/2)*a*b^2*c*(-(C^2*a*b - (4*A*C*a - A^2*b)*c
+ (C^2*a - A^2*c)*sqrt(b^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1712 vs. 2(171) = 342.
time = 6.86, size = 1712, normalized size = 8.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*B*log(x^2 + 1/2*(
b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*
c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3
 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2
```

```
- 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*C)*arctan(2
*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b
^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^
4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^
2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*
A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 -
 2*(b^2 - 4*a*c)*a*c^2)*C)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*
c^2 - 4*a^2*c^3)*abs(c)) + 1/4*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*
c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b^3*c^2
+ 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*B*log(x^2 +
 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*
c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2)
```

Mupad [B]
time = 2.31, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(A*B^2*c^2 - A^2*C*c^2 + B^3*c^2*x - C^3*a*c + A*C^2*b*c - 8*root
(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^
2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^
2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C
^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2
*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*
b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*b^3*c^2*x - 16*A*r
oot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c
```

```
*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*
B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*
B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B
^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*a*c^3 - 4*A^2*r
oot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c
*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*
B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*
B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B
^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*c^3*x + 4*A*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2
 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2
*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^
2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*
C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b
*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*b^2*c^2 + 32*root(1
6*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2
- 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*
c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2
*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C
*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b*
c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*a*b*c^3*x - 4*B*root
(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^
2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^
2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C
^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2
*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*
b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*b^2*c^2*x + 4*A*B*
root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*
c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*
C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16
*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A
*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*
B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*b*c^2 - 8*B*C*ro
ot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*
z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*
a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B
*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B
^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^
2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*a*c^2 - 2*A*B*C*c^
2*x + B*C^2*b*c*x + 16*B*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^
3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 1
6*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2
+ 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z
```

```
 - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3
*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^
2, z, k)^2*a*c^3*x + 2*B^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b*c^2*x + 4*C^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + ...
```

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}+\frac{(Ab-2aC)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{2a\sqrt{b^2-4ac}}$$

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*C*a)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c
^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1676, 1265, 814, 648, 632, 212, 642, 12, 1107, 211}

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}\,B\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4]
)/(4*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx &= \int \frac{B}{a+bx^2+cx^4}\,dx+\int \frac{A+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)+B\int \frac{1}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aC-Acx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)+\frac{(Bc)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d
\end{aligned}$$

$$= \frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= \frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.29, size = 285, normalized size = 1.24

$$\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,B\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}+\frac{A\log(x)}{a}-\frac{\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{4a\sqrt{b^2-4ac}}-\frac{\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) - 2*a
*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - ((A*(-
b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*
Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.06, size = 251, normalized size = 1.10

method	result
default	$\frac{4c\left(\frac{\sqrt{-4ac+b^2}\left(-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2aC\right)\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{4c}+\frac{aB\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{\cdots}}{\sqrt{(-b+\sqrt{-\cdots}}}\right)}{\sqrt{(-b+\sqrt{-4ac+\cdots}}}\right)}{16ac-4b^2}\cdots\right.}{\cdots}$
risch	$\frac{A\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^4c^2-8a^3b^2c+a^2b^4\right)\_Z^4+\left(32Aa^3c^2-16Aa^2b^2c+2Aab^4\right)\_Z^3+\left(24a^2c^2A^2-10ab^2cA^2+b^4A^2-8ACa^2bc+2A\cdots\right.\right.}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*((-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(-1/4*(-A*(-4*a*c+b^2)^(1/2)-A*b+2
*a*C)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+a*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+(-4*a*
c+b^2)^(1/2)/(16*a*c-4*b^2)*(1/4*(A*(-4*a*c+b^2)^(1/2)-A*b+2*a*C)/c*ln(b+2*
c*x^2+(-4*a*c+b^2)^(1/2))+a*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))+A*ln(x)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b*x^2 + a),
 x)/a
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2337 vs. 2(186) = 372.
time = 6.43, size = 2337, normalized size = 10.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(abs(c*x^4 + b*x^2 + a))/a + A*log(abs(x))/a + 1/4*((sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 -
 16*a*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^
3 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) + (2*b^3*c^3 -
8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*
```

```
b*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b^2*c^2 - 4*a^5*c^3
))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 2*(b^2 - 4*a*c)*b^2*
c - 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*B)*arctan(2*sqrt(
1/2)*x/sqrt((a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a*b^4 -
 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3
)*c^2) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*
c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c
^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - 2*(a*
b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*
c^3 - 4*a^2*b*c^4 + (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8
*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2
- 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^
2*c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A
- 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4
 + a*b^3*c^4 - 4*a^2*b*c^5 - (a*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b
^2*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^2*b*c + sqrt(a^4*b^2*c^2 - 4
*a^5*c^3))/(a^2*c^2))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 +
8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*abs(c)) - 1/16*((b^6*c - 8*a*b^4
*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (
b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*
a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4
*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 + (a*b^4*c -
8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*
c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5*c^3 + 16*
a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 - (b^5*c^2 - 4*a*b^3*c^3
- 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A - 2*(a*b^5*c^2 - 8*a^2*b^3*c^3
- 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 - (a
*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*C)*l
og(x^2 + 1/2*(a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4
 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a
^3*c^3)*c^2*abs(c))
```

Mupad [B]

time = 1.49, size = 2258, normalized size = 9.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(x*(B^4*c^3 + C^4*a*c^2 + A^2*C^2*c^3 - 3*A*B^2*C*c^3 - A*C^3*b*c
^2 + B^2*C^2*b*c^2) - root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4
*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^
2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*
C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 -
4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*
B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*
b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2
*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(A
*B^2*b*c^3 - 5*A^3*c^4 - 13*A*C^2*a*c^3 + 6*A^2*C*b*c^3 + 17*B^2*C*a*c^3 +
C^3*a*b*c^2 + A*C^2*b^2*c^2 - 4*B^2*C*b^2*c^2) - root(128*a^3*b^2*c*z^4 - 2
56*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 -
 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2
 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*
z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*
c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z
- 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*
b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4
*a^2 - A^4*c^2, z, k)*(x*(60*A^2*a*c^4 - 16*A^2*b^2*c^3 + 4*B^2*b^3*c^2 + 3
6*C^2*a^2*c^3 + 8*A*C*b^3*c^2 - 14*B^2*a*b*c^3 - 10*C^2*a*b^2*c^2 - 28*A*C*
a*b*c^3) + root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*
A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 -
 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z
^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z
^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2
*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A
*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*
B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(240*A*a^2*c^
4 + 12*A*b^4*c^2 - 108*A*a*b^2*c^3 + 4*C*a*b^3*c^2 - 16*C*a^2*b*c^3) + root
(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3
 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^
2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^
3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a
*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*
b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*
A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a
*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*x*(320*a^3*c^4 + 24*a*b^4*c^2 -
 176*a^2*b^2*c^3) - 4*B*a*b^3*c^2 + 16*B*a^2*b*c^3) + 4*A*B*b^3*c^2 + 8*B*C
*a^2*c^3 - 12*A*B*a*b*c^3 - 4*B*C*a*b^2*c^2) + B^3*a*c^3 + 4*A^2*B*b*c^3 +
6*A*B*C*a*c^3 - 4*A*B*C*b^2*c^2 + B*C^2*a*b*c^2) + A*B^3*c^3 - 2*A^2*B*C*c^
3 + A*B*C^2*b*c^2)*root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^
```

```
4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b
*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2
*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A
^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2
*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3
*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*
b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4
) + (A*log(x))/a
```

3.27 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=260

$$-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{b+\sqrt{b^2-\ldots}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -A/a/x+B*ln(x)/a-1/4*B*ln(c*x^4+b*x^2+a)/a+1/2*b*B*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(A*b-2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(A+(-A*b+2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1676, 1295, 1180, 211, 12, 1128, 719, 29, 648, 632, 212, 642}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c])
 + (B*Log[x])/a - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \frac{B}{x\left(a+bx^2+cx^4\right)}\,dx + \int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$= -\frac{A}{ax} - \frac{\int \frac{Ab-aC+Acx^2}{a+bx^2+cx^4}\,dx}{a} + B\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$= -\frac{A}{ax} + \frac{1}{2}B\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right) - \frac{\left(c\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\right)\int \cdots}{2a}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{A\cdots}{\sqrt{b^2\cdots}}\right.\cdots}{\cdots}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{A\cdots}{\sqrt{b^2\cdots}}\right.\cdots}{\cdots}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{A\cdots}{\sqrt{b^2\cdots}}\right.\cdots}{\cdots}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{A\cdots}{\sqrt{b^2\cdots}}\right.\cdots}{\cdots}$$

<u>Mathematica</u> [**A**]
time = 0.69, size = 315, normalized size = 1.21

$$-\frac{\frac{4A}{x}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}-4B\log(x)+\frac{B\left(b+\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{B\left(-b+\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A*(-b + Sqrt[b^2 - 4*a*c])
```

```
+ 2*a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2 - 4*
a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(-b + S
qrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/
a
```

Maple [A]

time = 0.07, size = 325, normalized size = 1.25

method	result
default	$\frac{4c\left(\frac{-\frac{\left(-B\sqrt{-4ac+b^2}\,b+4acB-b^2B\right)\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{4c}+\frac{\left(-Ab\sqrt{-4ac+b^2}+4acA-Ab^2+2C\sqrt{-4ac+b}\right.}{2\sqrt{\left(-b+\sqrt{\right.}}}}{16ac-4b^2}\right.}{}$
risch	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(16a^5c^2-8a^4b^2c+a^3b^4\right)\_Z^4+\left(32Ba^4c^2-16Ba^3b^2c+2Ba^2b^4\right)\_Z^3+\left(12A^2a^2bc^2-7A^2ab^3c+A^2b^5-16A\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(1/(16*a*c-4*b^2)*(-1/4*(-B*(-4*a*c+b^2)^(1/2)*b+4*a*c*B-b^2*B)/c*ln(
-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-A*b*(-4*a*c+b^2)^(1/2)+4*a*c*A-A*b^2+2
*C*(-4*a*c+b^2)^(1/2)*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(1/4*(-B*(
-4*a*c+b^2)^(1/2)*b-4*a*c*B+b^2*B)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(
-A*b*(-4*a*c+b^2)^(1/2)-4*a*c*A+A*b^2+2*C*(-4*a*c+b^2)^(1/2)*a)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))))-A/a/x+B*ln(x)/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*x
^2 + a), x)/a - A/(a*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3508 vs. 2(218) = 436.
time = 7.46, size = 3508, normalized size = 13.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a + B*log(abs(x))/a - A/(a*x) - 1/8*((2*
b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 -
 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3
- 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2
*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(c) - 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(
```

b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 -
4*a*c)*a^2*c^3)*C*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^
4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 -
2*(b^2 - 4*a*c)*a*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b
^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^
2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) + 1/8*((2*b^4*
c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a
*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16
*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c
^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(c) + 2*(sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a
*c)*a^2*c^3)*C*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^4 -
8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3

```
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b
^2 - 4*a*c)*a*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c - sqrt(a^4*b^2*c
^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^2 +
16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) - 1/16*((b^6*c - 8
*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c
^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*...
```

Mupad [B]
time = 1.02, size = 2588, normalized size = 9.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*
B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z
^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2
*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 -
96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z
- 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c
*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2
*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c
- B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16
*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 -
 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2
*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 -
 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 -
 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*
c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c
- B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2
*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 2
56*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 -
 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2
*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 6
4*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^
2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c
^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c
^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*
C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*((16*A*a
^3*c^4 + 4*A*a*b^4*c^2 + 16*C*a^3*b*c^3 - 20*A*a^2*b^2*c^3 - 4*C*a^2*b^3*c^
2)/a + (x*(240*B*a^4*c^4 + 12*B*a^2*b^4*c^2 - 108*B*a^3*b^2*c^3))/a^2 + (ro
ot(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z
^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*
b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2
```

```
+ 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^
2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*
b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a
^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^
2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 -
 A^4*c^3, z, k)*x*(320*a^5*c^4 + 24*a^3*b^4*c^2 - 176*a^4*b^2*c^3))/a^2) -
(8*A*B*a^2*c^4 + 4*A*B*b^4*c^2 - 16*A*B*a*b^2*c^3 - 4*B*C*a*b^3*c^2 + 12*B*
C*a^2*b*c^3)/a + (x*(4*A^2*b^5*c^2 + 60*B^2*a^3*c^4 - 16*B^2*a^2*b^2*c^3 +
4*C^2*a^2*b^3*c^2 - 72*A*C*a^3*c^4 - 28*A^2*a*b^3*c^3 + 50*A^2*a^2*b*c^4 -
14*C^2*a^3*b*c^3 + 48*A*C*a^2*b^2*c^3 - 8*A*C*a*b^4*c^2))/a^2) - (C^3*a^2*c
^3 + 7*A*B^2*a*c^4 + A^2*C*a*c^4 - 4*A*B^2*b^2*c^3 - A*C^2*a*b*c^3 + 4*B^2*
C*a*b*c^3)/a + (x*(5*B^3*a^2*c^4 - 4*A^2*B*b^3*c^3 - B*C^2*a^2*b*c^3 - 26*A
*B*C*a^2*c^4 + 14*A^2*B*a*b*c^4 + 8*A*B*C*a*b^2*c^3))/a^2) - (A*B^3*c^4 - A
^2*B*C*c^4 - B*C^3*a*c^3 + A*B*C^2*b*c^3)/a + (x*(A^4*c^5 + C^4*a^2*c^3 + A
^2*C^2*b^2*c^3 - 2*A^3*C*b*c^4 + A^2*B^2*b*c^4 + 2*A^2*C^2*a*c^4 - 2*A*B^2*
C*a*c^4 - 2*A*C^3*a*b*c^3))/a^2)*root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 -
 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^
3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*
a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^
2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^
2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b
^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b
*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*
B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k), k, 1, 4) - A/(a*x) + (B
*log(x))/a
```

3.28 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=288

$$-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/4*(A*b-C*a)*ln(c*x^4+b*x^2+a)/a^2-
1/2*(A*(-2*a*c+b^2)-a*b*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/
2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4
*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1676, 1265, 814, 648, 632, 212, 642, 12, 1137, 1180, 211}

$$-\frac{(A(b^2-2ac)-abC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log(a+bx^2+cx^4)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2
- 4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
(A*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*
Sqrt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2
+ c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \int \frac{B}{x^2\left(a+bx^2+cx^4\right)}\,dx + \int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right) + B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$= -\frac{B}{ax} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2} + \frac{-Ab+aC}{a^2x} + \frac{A(b^2-ac)-abC+c(Ab-aC)x}{a^2\left(a+bx+cx^2\right)}\right) dx$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{(Ab-aC)\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{A(b^2-ac)-abC+c(Ab-aC)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2} -$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}}{}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}}{}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}}{}$$

Mathematica [A]
time = 0.62, size = 377, normalized size = 1.31

$$\frac{-\frac{2aA}{x^2} - \frac{4aB}{x} - \frac{2\sqrt{2}\,aB\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\,aB\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + 4(-Ab+aC)\log(x) + \frac{\left(A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)-a\left(b+\sqrt{b^2-4ac}\right)C\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(A\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)+a\left(b-\sqrt{b^2-4ac}\right)C\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^2 - 2
*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqrt[b^2 -
```

```
 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A]
time = 0.08, size = 300, normalized size = 1.04

method	result
default	$\frac{4c\left(-\frac{\left(b^2-4ac+b\sqrt{-4ac+b^2}\right)\left(-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2aC\right)\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{4c}+\frac{aB\sqrt{2}\ \operatorname{arctanh}\left(\frac{\cdots}{\sqrt{(\cdots}}\right.}{\sqrt{(-b+\cdots}}\right.}{32ac-8b^2}\right.\cdots}{\cdots}$
risch	$\frac{-\frac{Bx}{a}-\frac{A}{2a}}{x^2}-\frac{\ln(x)Ab}{a^2}+\frac{\ln(x)C}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16c^2a^6-8b^2ca^5+b^4a^4\right)\_Z^4+\left(-32A\,a^4bc^2+16A\,a^3b^3c-2A\,a^2b^5+32C\,a^5c^2-16C\,a^4\cdots\right.\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*c*(-(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/(32*a*c-8*b^2)*(-1/4*(-A*(-4*a*c
+b^2)^(1/2)-A*b+2*a*C)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+a*B*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)))-(b*(-4*a*c+b^2)^(1/2)+4*a*c-b^2)/(32*a*c-8*b^2)*(1/4*(A*(-4*a*c+
b^2)^(1/2)-A*b+2*a*C)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+a*B*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))))-1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b
+ (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A)/
(a*x^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3354 vs. $2(240) = 480$.
time = 6.26, size = 3354, normalized size = 11.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(C*a - A*b)*log(abs(c*x^4 + b*x^2 + a))/a^2 + (C*a - A*b)*log(abs(x))/
a^2 - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 + 2*(sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5
*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B
*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
```

qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b
*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2
- 2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2)
+ 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3
- 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - 2*(sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^
2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*B*ab
s(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b*c
- sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 -
2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) +
1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b
^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c
- 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 32*
a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 - 4*a*c))*A*abs(
c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3 + 8*a^2*b^3*c^
3 + a*b^4*c^3 - 4*a^2*b^2*c^4 + (a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16
*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*
abs(c) + (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^4 + 12*a*b^4*c^
4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 + 8*a^2*b*c^6 + (
b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*c^4 + b^4*c^4 -
2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A - (a*b^6*c^2 - 8*a^2*b^4*c^3 - 2*a*b^5*c
^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*c^5 - (a*b^5*c^
2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2
+ 1/2*(a^4*b*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/((a^3*b^4 - 8*a^
4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)
*c^2*abs(c)) + 1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 1
2*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2
*b*c^5 + (b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3

```
+ b^4*c^3 - 32*a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 -
 4*a*c))*A*abs(c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3
 + 8*a^2*b^3*c^3 + a*b^4*c^3 - 4*a^2*b^2*c^4 + (a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b
^2 - 4*a*c))*C*abs(c) - (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^
4 + 12*a*b^4*c^4 + b^5*c^4 - 32*a^3*b*c^5 - 16*...
```

Mupad [B]
time = 1.17, size = 2500, normalized size = 8.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*
C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2
*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C
*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*
z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 -
4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z
^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C
*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z
+ 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3
*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*
c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 -
 A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)
*(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2
*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16
*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*
c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*
B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b
^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^
2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^
2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*
a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A
*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A
*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*
b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*(root(12
8*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 +
256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^
4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8
*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b
^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 -
96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2
```

```
 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A
*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z
 + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*
c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*
c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A
^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*((16*B*a^5*c^4 +
4*B*a^3*b^4*c^2 - 20*B*a^4*b^2*c^3)/a^3 + (x*(240*C*a^5*c^4 - 224*A*a^4*b*c
^4 - 12*A*a^2*b^5*c^2 + 104*A*a^3*b^3*c^3 + 12*C*a^3*b^4*c^2 - 108*C*a^4*b^
2*c^3))/a^3 + (root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 +
128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5
*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72
*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*
c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^
2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c
^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A
^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*
c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2
*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*
a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c
^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z
, k)*x*(320*a^6*c^4 + 24*a^4*b^4*c^2 - 176*a^5*b^2*c^3))/a^3) - (8*B*C*a^4*
c^4 + 20*A*B*a^2*b^3*c^3 + 4*B*C*a^2*b^4*c^2 - 16*B*C*a^3*b^2*c^3 - 4*A*B*a
*b^5*c^2 - 20*A*B*a^3*b*c^4)/a^3 + (x*(36*A^2*a^3*c^5 + 60*C^2*a^4*c^4 + 22
*A^2*a^2*b^2*c^4 - 28*B^2*a^2*b^3*c^3 - 16*C^2*a^3*b^2*c^3 - 8*A^2*a*b^4*c^
3 + 4*B^2*a*b^5*c^2 + 50*B^2*a^3*b*c^4 + 24*A*C*a^2*b^3*c^3 - 92*A*C*a^3*b*
c^4))/a^3) - (A^2*B*a^2*c^5 + 7*B*C^2*a^3*c^4 - 4*A^2*B*a*b^2*c^4 - 4*B*C^2
*a^2*b^2*c^3 + 4*A*B*C*a*b^3*c^3 - 4*A*B*C*a^2*b*c^4)/a^3 + (x*(2*A^3*b^3*c
^4 + 5*C^3*a^3*c^4 - 12*A^3*a*b*c^5 - 17*A*B^2*a^2*c^5 + 13*A^2*C*a^2*c^5 +
 6*A*B^2*a*b^2*c^4 - 9*A*C^2*a^2*b*c^4 + 2*A^2*C*a*b^2*c^4 - 4*B^2*C*a*b^3*
c^3 + 14*B^2*C*a^2*b*c^4))/a^3) - (A^3*B*b*c^5 + B*C^3*a^2*c^4 - A^2*B*C*a*
c^5 - A*B*C^2*a*b*c^4)/a^3 + (x*(A^4*c^6 + B^4*a*c^5 - A^3*C*b*c^5 + A^2*C^
2*a*c^5 + B^2*C^2*a*b*c^4 - 3*A*B^2*C*a*c^5))/a^3)*root(128*a^5*b^2*c*z^4 -
 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z
^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*
b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 +
40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2
*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a...
```

3.29 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=412

$$\frac{(2Ac-bC)x}{2c(b^2-4ac)}+\frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(Abc+(b^2-6ac)C-\frac{Ac(b^2+\cdots}{\sqrt{\cdots}}\right.\cdots}{2\sqrt{2}\,c^{3/2}(b\cdots}$$

```
[Out] 1/2*(2*A*c-C*b)*x/c/(-4*a*c+b^2)+1/2*B*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)-1/2*x^3*(A*b-2*a*C+(2*A*c-C*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2
*a*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(
x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(-A*c
*(4*a*c+b^2)-b*(-8*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(A*c*(4*a*c+b^2)+b*(-8*a*c+b^2)*C)/
(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A]
time = 0.89, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1676, 1289, 1293, 1180, 211, 12, 1128, 736, 632, 212}

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C(b^2-6ac)+Abc\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C(b^2-6ac)+Abc\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^3(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{x(2Ac-bC)}{2c(b^2-4ac)}+\frac{2aB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 + 4*a
*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*
a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2
*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{Bx^5}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^4(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{x^2(3(Ab-2aC)+}{a+bx^2}}{2(b^2-}$$

$$= \frac{(2Ac-bC)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)}\right.$$

$$= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

$$= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

$$= \frac{(2Ac-bC)x}{2c(b^2-4ac)} + \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x^3(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Mathematica [**A**]

time = 0.88, size = 444, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2(bx^2(-Acx+b(B+Cx))+a(b(B+Cx)-2cx(A+x(B+Cx))))}{c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-Ac\left(b^2+4ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^{3/2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(Ac\left(b^2+4ac+b\sqrt{b^2-4ac}\right)+\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{c^{3/2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{4aB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{4aB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C*
x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*a*
c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c
*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(
A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4
*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*a
*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [**A**]

time = 0.08, size = 550, normalized size = 1.33

method	result
risch	$\frac{-\frac{\left(bcA+2acC-C\,b^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)B\,x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{a(2Ac-bC)x}{2\left(4ac-b^{2}\right)c}+\frac{abB}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{\left(bcA-6acC+C\,b^{2}\right)\_R^{2}}{c\left(4ac-b^{2}\right)}+\frac{4\_}{4c}\right.}{2c\_R}\right.}{4}$
default	$\frac{-\frac{\left(bcA+2acC-C\,b^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)B\,x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{a(2Ac-bC)x}{2\left(4ac-b^{2}\right)c}+\frac{abB}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{-2B\sqrt{-4ac+b^{2}}\,ac\ln\left(-b-2c\,x^{2}+\sqrt{-4ac+b^{2}}\right)+\frac{(4c}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)*B/c/(4*a*c-b^
2)*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*a/(4*a*c-b^2)/c*b*B)/(c*x^4+b*
x^2+a)+2/(4*a*c-b^2)*(1/4/c/(4*a*c-b^2)*(-2*B*(-4*a*c+b^2)^(1/2)*a*c*ln(-b-
2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*c^2*a*A*(-4*a*c+b^2)^(1/2)+A*b^2*c*(-4*a
*c+b^2)^(1/2)+4*A*a*b*c^2-A*b^3*c-8*C*(-4*a*c+b^2)^(1/2)*a*b*c+C*(-4*a*c+b^
2)^(1/2)*b^3-24*C*a^2*c^2+10*C*a*b^2*c-C*b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/
(4*a*c-b^2)*(2*B*(-4*a*c+b^2)^(1/2)*a*c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/
2*(4*c^2*a*A*(-4*a*c+b^2)^(1/2)+A*b^2*c*(-4*a*c+b^2)^(1/2)-4*A*a*b*c^2+A*b^
3*c-8*C*(-4*a*c+b^2)^(1/2)*a*b*c+C*(-4*a*c+b^2)^(1/2)*b^3+24*C*a^2*c^2-10*C
*a*b^2*c+C*b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a*
b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4
*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (6*
C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5220 vs. 2(361) = 722.
time = 8.91, size = 5220, normalized size = 12.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*b^2*x^3 - 2*C*a*c*x^3 - A*b*c*x^3 + B*b^2*x^2 - 2*B*a*c*x^2 + C*a*b
*x - 2*A*a*c*x + B*a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*
b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2
 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4
*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 +
16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3
*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4
```

*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a
*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(b^2*c - 4*a*c^2) - (2*b^7*
c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b
*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 +
 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*C)*arctan(2*sqr
t(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c
- 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a
^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64
*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*
abs(c)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20
*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^
2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c

```
 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*s...
```

Mupad [B]
time = 1.77, size = 2500, normalized size = 6.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(- root(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680
*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c
^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^
5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c
*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*
z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5
*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*
a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536
*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*
b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*
a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c
^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^
3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*
c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4
*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c -
960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144
*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C
^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*
c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c
^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*
a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k)*(root(1572864*a^5*b^2*c^8*
z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z
^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C
*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 204
8*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288
*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^
2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 +
24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*
z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c
^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z +
768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672
*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 499
2*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z
- 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B
^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*
```

```
B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^
3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b
*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 7
68*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5
 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*
a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*
c^4, z, k)*((x*(1024*B*a^4*c^6 - 16*B*a*b^6*c^3 + 192*B*a^2*b^4*c^4 - 768*B
*a^3*b^2*c^5))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (
2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*C*a*b^7*c^2 - 1024*C*a^4*b*c^5 + 384*A
*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*C*a^2*b^5*c^3 + 768*C*a^3*b^3*c^4)/
(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (root(1572864*a^
5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2
*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^4
 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3
*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*z
^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2
- 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*
c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^
2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768
*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^
3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7
*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c
^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*
b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z
 + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2
 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*
B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2
*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3
*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C
^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 1
6*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 1
6*A^4*a^3*c^4, z, k)*x*(16*b^9*c^3 - 256*a*b^7*...
```

3.30 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=347

$$\frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{a(2Ac-bC)+(Abc-b^2C+2acC)\,x^2}{2c(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b}}}$$

```
[Out] 1/2*B*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(a*(2*A*c-C*b)+(A*b*c+
2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(A*b-2*C*a)*arctanh((2*c
*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*B*arctan(x*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1676, 1265, 791, 632, 212, 12, 1134, 1180, 211}

$$-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2acC+Abc+b^2(-C))+a(2Ac-bC)}{2c(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{B\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2*A*c - b*C
) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx &= \int \frac{Bx^4}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^3(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Cx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right) + B\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{B\int \cdots}{2(\cdots} \\
&= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{\Big(B\Big(\cdots}{\cdots} \\
&= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{B\Big(b^2\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 358, normalized size = 1.03

$$\frac{1}{4}\left(-\frac{2(bx^2(Ac-bC+Bcx)+a(2Ac-bC+2cx(B+Cx)))}{c(-b^2+4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}B\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2(Ab-2aC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2(Ab-2aC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x))))/(c*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*
```

```
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(A*b -
2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(A*b
 - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.06, size = 458, normalized size = 1.32

method	result
risch	$\frac{-\frac{bB\,x^3}{2(4ac-b^2)}-\frac{(bcA+2acC-C\,b^2)x^2}{2c(4ac-b^2)}-\frac{xaB}{4ac-b^2}-\frac{a(2Ac-bC)}{2(4ac-b^2)c}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\_R^2bB}{4ac-b^2}-\frac{2(Ab-2aC)\_R}{4ac-b^2}+\frac{2aB}{4ac-b^2}\right)}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{-\frac{bB\,x^3}{2(4ac-b^2)}-\frac{(bcA+2acC-C\,b^2)x^2}{2c(4ac-b^2)}-\frac{xaB}{4ac-b^2}-\frac{a(2Ac-bC)}{2(4ac-b^2)c}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-\frac{\left(-4bcA\sqrt{-4ac+b^2}+8C\sqrt{-4ac+b^2}\,ac\right)\ln\left(-b-2c\,x^2+\right.}{4c}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/(4*a*c-b^2)*b*B*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^2-1/(4*
a*c-b^2)*x*a*B-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)+2/(4*a*c-b^
2)*c*(1/4/c/(4*a*c-b^2)*(-1/4*(-4*b*c*A*(-4*a*c+b^2)^(1/2)+8*C*(-4*a*c+b^2)
^(1/2)*a*c)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*a*c*B*(-4*a*c+b^2)^(
1/2)+b^2*B*(-4*a*c+b^2)^(1/2)+4*a*b*B*c-b^3*B)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c
/(4*a*c-b^2)*(1/4*(-4*b*c*A*(-4*a*c+b^2)^(1/2)+8*C*(-4*a*c+b^2)^(1/2)*a*c)/
c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*a*c*B*(-4*a*c+b^2)^(1/2)+b^2*B*(-
4*a*c+b^2)^(1/2)-4*a*b*B*c+b^3*B)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a + A*b)*c)*x^
2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2
) + 1/2*integrate((B*b*x^2 - 2*B*a - 2*(2*C*a - A*b)*x)/(c*x^4 + b*x^2 + a)
, x)/(b^2 - 4*a*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3229 vs. 2(296) = 592.
time = 5.33, size = 3229, normalized size = 9.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c*x^3 - C*b^2*x^2 + 2*C*a*c*x^2 + A*b*c*x^2 + 2*B*a*c*x - C*a*b +
2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*((2*b^3*c^2 - 8*a*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2
 - 4*a*c)^2*B + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt
```

```
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*
a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2*b^7*c^2 - 8*a*
b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*B)*arctan(2*
sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*
a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a
^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) - 1/16*((2*
b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a
*c)*b*c^2)*(b^2 - 4*a*c)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b
^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2
*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2
*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^
4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4
*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a
^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64
*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(
c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c - 4*a*b*c^2
- 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - 2*(a*b^3*c - 4
*a^2*b*c^2 - 2*a*b^2*c^2 + a*b*c^3 + (a*b^2*c - 4*a^2*c^2 - 2*a*b*c^2 + a*c
^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^
2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3
*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*A + 2*(a*b^5*c - 8*a^2*b^3*c
```

```
^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 -
 (a*b^4*c - 4*a^2*b^2*c^2 - 2*a*b^3*c^2 + a*b^2*c^3)*sqrt(b^2 - 4*a*c))*C)*
log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)
*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c +
 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c)) +
1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c - 4*a*b*c^2 - 2*b^
2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - 2*(a*b^3*c - 4*a^2*b
*c^2 - 2*a*b^2*c^2 + a*b*c^3 - (a*b^2*c - 4*a^2*c^2 - 2*a*b*c^2 + a*c^3)*sq
rt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16
*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 -
 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*A + 2*...
```

Mupad [B]
time = 1.61, size = 2500, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a
^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*
c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 -
24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^
2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^
3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*
b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2
*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z
 - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z
+ 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*
B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c +
 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^
2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*
B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*((256*A*B*a^2*b
^2*c^3 + 128*B*C*a^2*b^3*c^2 - 64*A*B*a*b^4*c^2 - 512*B*C*a^3*b*c^3)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(1572864*a^5*b^2*c^6*z
^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^
4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*
a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*
a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^
2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 81
92*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^
2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 -
16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2
*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*
```

```
a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 5
12*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b
^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2
*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*
c - 9*B^4*a*b^4, z, k)*((x*(16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3
 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^
3 - 1536*C*a^3*b^2*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)) - (2048*B*a^4*c^5 - 32*B*a*b^6*c^2 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*
c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a
^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^
2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4
+ 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2
 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z
^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*
c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^
3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^
5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z
 - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z
 - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a
^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A
^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^
2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c -
 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4
*b*c^6 + 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c))) + (x*(8*A^2*b^5*c^2 - 2*B^2*b^6*c + 64*B^2*a^3*c^
4 + 32*C^2*a^2*b^3*c^2 - 32*A^2*a*b^3*c^3 + 4*B^2*a*b^4*c^2 - 128*C^2*a^3*b
*c^3 + 128*A*C*a^2*b^2*c^3 - 32*A*C*a*b^4*c^2))/(4*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))) - (3*B^3*a*b^3*c + 32*B*C^2*a^3*c^2 + 4*B^3*a^2*
b*c^2 + 8*A^2*B*a*b^2*c^2 - 32*A*B*C*a^2*b*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)) + (x*(4*A^3*b^3*c^2 - 32*C^3*a^3*c^2 + A*B^2*b^4*
c + 4*A*B^2*a*b^2*c^2 + 48*A*C^2*a^2*b*c^2 - 24*A^2*C*a*b^2*c^2 - 8*B^2*C*a
^2*b*c^2 - 2*B^2*C*a*b^3*c))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)))*root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*
b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7
*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 245
76*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 +
 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z
^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5
*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^
8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z -
1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 7
68*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 204...
```

3.31 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4}}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac-}{a+bx^2+cx^4}}{2(b^2-4a}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) -$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\Big(2Ac-bC-}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\Big(2Ac-bC-}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\Big(2Ac-bC-}{2\sqrt{2}}$$

Mathematica [A]
time = 0.67, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.07, size = 456, normalized size = 1.28

method	result
risch	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-bC)\_R^2}{4ac-b^2}-\frac{2\_RbB}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln(x-\_R}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{B\sqrt{-4ac+b^2}\,b\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-4bcA\sqrt{-4ac+\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*b*B+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-1/(4*a*c-b^2)*a*B)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/4/c/(4*a*
c-b^2)*(B*(-4*a*c+b^2)^(1/2)*b*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*
c*A*(-4*a*c+b^2)^(1/2)-8*c^2*a*A+2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4
*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C*b^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^
2)*(-B*(-4*a*c+b^2)^(1/2)*b*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*
(-4*a*c+b^2)^(1/2)+8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c
+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4440 vs. 2(306) = 612.
time = 6.10, size = 4440, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
```

```
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
```

```
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4...
```

Mupad [B]
time = 1.67, size = 2500, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
```

4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2

```
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 32768...
```

3.32 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=317

$$-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*B*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(-A*b+2*a*C-(2*A*c-C*
b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1676, 1261, 652, 632, 212, 12, 1133, 1180, 211}

$$\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(B*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b - 2*a*C
 + (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(2
*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c
]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A
*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx &= \int \frac{Bx^2}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{A+Cx}{(a+bx+cx^2)^2}\,dx, x, x^2\right) + B\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx \\
&= -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} \\
&= -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\Big(Bc\Big(2b-\sqrt{b^2-}}{} \\
&= -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}\,\Big(2b-\sqrt{b^2}}{\sqrt{2}\,(b^2}
\end{aligned}$$

Mathematica [A]
time = 0.80, size = 335, normalized size = 1.06

$$\frac{1}{2}\left(\frac{2aC-A(b+2cx^2)+x(-bB+bCx-2Bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}B\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(-2Ac+bC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{(2Ac-bC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

Maple [A]
time = 0.16, size = 510, normalized size = 1.61

method	result
risch	$\frac{\frac{cB\,x^3}{4ac-b^2}+\frac{(2Ac-bC)x^2}{8ac-2b^2}+\frac{bBx}{8ac-2b^2}+\frac{Ab-2aC}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{2c\_R^2B}{4ac-b^2}+\frac{2(2Ac-bC)\_R}{4ac-b^2}-\frac{bB}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(\frac{\frac{B\left(4ac-b^2\right)x}{8c}+\frac{8c^2aA-2A\,b^2c-4C\sqrt{-4ac+b^2}\,ac+C\sqrt{-4ac+b^2}\,b^2-4Cabc+C\,b^3}{16c^2}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\left(4A\sqrt{-4ac+b^2}\,c-2C\sqrt{-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)^2*((1/8*B*(4*a*c-b^2)/c*x+1/16*(8*c^2*a*A-2*A*b^2
*c-4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)/c^2
)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/16*(4*A*(-4*a*c+b^2)^(1/2)*c-2*C
*(-4*a*c+b^2)^(1/2)*b)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/8*(-2*B*(-4*a*c
+b^2)^(1/2)*b+4*a*c*B-b^2*B)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4/c/(4*a*c-b^2)^2*((-1/8
*B*(4*a*c-b^2)/c*x-1/16/c^2*(8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c
-C*(-4*a*c+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3))/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)
^(1/2))-1/16*(-4*A*(-4*a*c+b^2)^(1/2)*c+2*C*(-4*a*c+b^2)^(1/2)*b)/c*ln(-b-2
*c*x^2+(-4*a*c+b^2)^(1/2))+1/8*(2*B*(-4*a*c+b^2)^(1/2)*b+4*a*c*B-b^2*B)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*c*x^3 + B*b*x - (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/((b^2*c - 4*a*c^
2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*B*c*x^2
- B*b - 2*(C*b - 2*A*c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3014 vs. 2(270) = 540.
time = 8.34, size = 3014, normalized size = 9.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*c*x^3 - C*b*x^2 + 2*A*c*x^2 + B*b*x - 2*C*a + A*b)/((c*x^4 + b*x^
2 + a)*(b^2 - 4*a*c)) - 1/8*((2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2*B - (sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a
*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3
 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*B*abs(b^2 - 4*a*c) - 2*
(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
```

```
a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^2 - 4*a*c)*a*b^2*c^3)*B)*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5
*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^
3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((2*b^2*c^2
- 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 -
4*a*c)^2*B + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
+ 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*
c)*a*b*c^2)*B*abs(b^2 - 4*a*c) - 2*(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c
^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^
2 - 4*a*c)*a*b^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b
^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)
))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a
*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2
- 4*a*c)*abs(c)) - 1/8*(2*(b^3*c^2 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 + (b^2*c
^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - (b^4*
c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*
c^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b
^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*
b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2
- 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c
 - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(
b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c
^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8
*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c)) - 1/8*(2*(b^3*c^2
 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 - (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt
(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - (b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 - (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2
- 4*a*c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^
```

```
4 + b^3*c^4 - 4*a*b*c^5 - (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqr
t(b^2 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a
*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c
^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*
c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 -
 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b...
```

Mupad [B]
time = 1.60, size = 2500, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((4*B^3*a*c^4 + 3*B^3*b^2*c^3 + 8*A^2*B*b*c^4 + 2*B*C^2*b^3*c^2 -
 8*A*B*C*b^2*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ro
ot(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^
4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256
*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3
*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^
2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C
^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 +
24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 -
32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2
*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a
^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 1
92*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b
^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 2
4*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B
^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*(root(1572864*a^6*b^2*c^5*z^
4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4
 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a
^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a
^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2
*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192
*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2
 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 1
6*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*
a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b
^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 51
2*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*
c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*
B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 -
 256*A^4*a*c^4, z, k)*((x*(512*A*a^3*c^6 - 8*A*b^6*c^3 + 4*C*b^7*c^2 + 96*A
```

```
*a*b^4*c^4 - 48*C*a*b^5*c^3 - 256*C*a^3*b*c^5 - 384*A*a^2*b^2*c^5 + 192*C*a
^2*b^3*c^4))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (8*B*b^7*c^
2 - 96*B*a*b^5*c^3 - 512*B*a^3*b*c^5 + 384*B*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a^6*b^2*c^5*z^4 - 98304
0*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a
^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*
z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^
2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^
2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*
b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A
^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9
*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z
 - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z -
2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a
*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*
B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c
^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*
a*c^4, z, k)*x*(8*b^9*c^2 - 128*a*b^7*c^3 + 2048*a^4*b*c^6 + 768*a^2*b^5*c^
4 - 2048*a^3*b^3*c^5))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) -
(256*A*B*a^2*c^5 - 16*A*B*b^4*c^3 + 8*B*C*b^5*c^2 - 128*B*C*a^2*b*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*B^2*a^2*c^5 - 8*A
^2*b^3*c^4 + 5*B^2*b^4*c^3 - 2*C^2*b^5*c^2 + 8*A*C*b^4*c^3 + 32*A^2*a*b*c^5
 - 24*B^2*a*b^2*c^4 + 8*C^2*a*b^3*c^3 - 32*A*C*a*b^2*c^4))/(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(8*A^3*c^5 - C^3*b^3*c^2 + 4*A*B^2*b
*c^4 - 12*A^2*C*b*c^4 + 6*A*C^2*b^2*c^3 - 2*B^2*C*b^2*c^3))/(b^6 - 64*a^3*c
^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))*root(1572864*a^6*b^2*c^5*z^4 - 983040*a^
5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b
^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2
- 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^
2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^
2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*
c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a
^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2
 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 7
68*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048
*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c
^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384...
```

3.33 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=368

$$-\frac{B(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(Ab-2aC+\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{}}$$

```
[Out] -1/2*B*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(A*b^2-2*a*A*c-a*b*C+
c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*B*c*arctanh((2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*a*C+(A*(-12*a*c+b^2)+4*a*b*C)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*a*C+(12*A*
a*c-A*b^2-4*C*a*b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1687, 1192, 1180, 211, 12, 1121, 628, 632, 212}

$$\frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2Bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{B(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(B*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*b^2 - 2*
a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 -
12*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx - \cdots$$

$$= \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\mathrm{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) \cdots$$

$$= -\frac{B(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \cdots$$

$$= -\frac{B(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \cdots$$

$$= -\frac{B(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(Ab^2-2aAc-abC+c(Ab-2aC)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \cdots$$

Mathematica [A]
time = 0.80, size = 393, normalized size = 1.07

$$\frac{1}{4}\left(\frac{2ab(B+Cx)-2Abx(b+cx^2)+4acx(A+x(B+Cx))}{a\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-2a\left(-2b+\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac-b\sqrt{b^2-4ac}\right)+2a\left(2b+\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{4Bc\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4Bc\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c + b*Sq
rt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c]) + 2*
a*(2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*B*c*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*B*c*Log[b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.14, size = 579, normalized size = 1.57

method	result
risch	$\frac{-\frac{c(Ab-2aC)x^3}{2a(4ac-b^2)}+\frac{cx^2B}{4ac-b^2}+\frac{(2acA-Ab^2+abC)x}{2a(4ac-b^2)}+\frac{bB}{8ac-2b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{c(Ab-2aC)\_R^2}{a(4ac-b^2)}+\frac{4\_RcB}{4ac-b^2}+\frac{6acA-}{a(4a}\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(\frac{\frac{\left(4A\sqrt{-4ac+b^2}\,ac-A\sqrt{-4ac+b^2}\,b^2-4Aabc+Ab^3+8a^2cC-2Cab^2\right)x}{16ac}+\frac{B(4ac-b^2)}{8c}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{2aB\sqrt{-4ac+b^2}\,\ln\left(b+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)^2*((1/16*(4*A*(-4*a*c+b^2)^(1/2)*a*c-A*(-4*a*c+b^
2)^(1/2)*b^2-4*A*a*b*c+A*b^3+8*a^2*c*C-2*C*a*b^2)/a/c*x+1/8*B*(4*a*c-b^2)/c
)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/8/a*(2*a*B*(-4*a*c+b^2)^(1/2)*ln
(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(12*A*(-4*a*c+b^2)^(1/2)*a*c-A*(-4*a*c+b
^2)^(1/2)*b^2-4*A*a*b*c+A*b^3-4*C*(-4*a*c+b^2)^(1/2)*a*b+8*a^2*c*C-2*C*a*b^
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))))-1/4/c/(4*a*c-b^2)^2*((-1/16*(-4*A*(-4*a*c+b^2)^(1/2
)*a*c+A*(-4*a*c+b^2)^(1/2)*b^2-4*A*a*b*c+A*b^3+8*a^2*c*C-2*C*a*b^2)/a/c*x-1
/8*B*(4*a*c-b^2)/c)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+1/8/a*(2*a*B*(-4
*a*c+b^2)^(1/2)*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-12*A*(-4*a*c+b^2)^(
1/2)*a*c+A*(-4*a*c+b^2)^(1/2)*b^2-4*A*a*b*c+A*b^3+4*C*(-4*a*c+b^2)^(1/2)*a*
```

```
b+8*a^2*c*C-2*C*a*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c)
*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^
2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6*A*
a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5158 vs. $2(323) = 646$.
time = 7.62, size = 5158, normalized size = 14.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a*c*x^3 - A*b*c*x^3 + 2*B*a*c*x^2 + C*a*b*x - A*b^2*x + 2*A*a*c*x
 + B*a*b)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*
```

```
b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*
b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C + 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*
a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2
- 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)
*A*abs(a*b^2 - 4*a^2*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3
+ 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*C*abs(a*b^2 - 4*a^
2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a
^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A +
 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 -
```

```
4*a*c)*a^4*b^2*c^3)*C)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt(
(a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^
2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2
+ 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3
+ 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4
*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c
 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - ...
```

Mupad [B]
time = 1.67, size = 2500, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((B*b)/(2*(4*a*c - b^2)) + (x*(2*A*a*c - A*b^2 + C*a*b))/(2*a*(4*a*c - b^2)
) + (B*c*x^2)/(4*a*c - b^2) - (c*x^3*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + symsum(log((5*A^3*b^3*c^4 + 8*C^3*a^3*c^4 + 6*C^3*a^2*b
^2*c^3 - 36*A^3*a*b*c^5 - 96*A*B^2*a^2*c^5 + 72*A^2*C*a^2*c^5 - 3*A^2*C*b^4
*c^3 + 16*A*B^2*a*b^2*c^4 + 3*A*C^2*a*b^3*c^3 - 60*A*C^2*a^2*b*c^4 + 18*A^2
*C*a*b^2*c^4 + 16*B^2*C*a^2*b*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
+ 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 10
48576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^
5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*
C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*
a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a
^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512
*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^
2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 -
 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 409
```

6*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B
*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A
^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C
^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C
*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*
b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c
^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 -
496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^
2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a
^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 25
6*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*(root(1572864*a^8*
b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b
^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 +
576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z
^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2
+ 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 -
8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4
*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b
^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B
^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 - 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*
c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c
*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*
z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^
4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z +
192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 -
960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^
2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B
^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 - 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b
*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C
*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*
A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 256*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*
A^4*a^2*c^5, z, k)*((x*(1024*B*a^5*c^6 - 16*B*a^2*b^6*c^3 + 192*B*a^3*b^4*c
^4 - 768*B*a^4*b^2*c^5))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*C*a^5*b*c^5 - 288*A*a^2*
b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*C*a^2*b^7*c^2 - 192*
C*a^3*b^5*c^3 + 768*C*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
+ 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1
048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a
^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A
*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2
*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*
a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 51
2*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z
^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2

```
- 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 40
96*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*
B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*
A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*
C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*
C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*...
```

3.34 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=403

$$\frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b}}$$

```
[Out] 1/2*B*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b
^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-6*a*b*
c+b^3)+4*a^2*c*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^
(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2+1/4*B*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a
/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*B*arctan(x*2^(
1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$,
Rules used = {1676, 1265, 836, 814, 648, 632, 212, 642, 12, 1106, 1180, 211}

$$\frac{\left(4a^2cC+A(b^3-6abc)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A(b^2-2ac)+cx^2(Ab-2aC)-abC}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{B\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx(-2ac+b^2+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c) + 4*a
^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx &= \int \frac{B}{\left(a+bx^2+cx^4\right)^2}\,dx+\int \frac{A+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)+B\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\Big(\int \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\Big(\int \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\,\Big(b \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\,\Big(b \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\,\Big(b \\
&= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\sqrt{c}\,\Big(b
\end{aligned}
$$

Mathematica [A]
time = 0.93, size = 458, normalized size = 1.14

$$
\frac{-\frac{2a\left(abC+2acx(B+Cx)-bBx(b+cx^2)-A(b^2-2ac+bcx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\,aB\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\,aB\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+4A\log(x)-\frac{\left(A\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)+4a^2cC\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{\left(A\left(-b^3+6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)-4a^2cC\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b*
c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2 -
```

```
12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*
B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt
[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4
*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2
- 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^
(3/2))/(4*a^2)
```

Maple [A]
time = 0.10, size = 566, normalized size = 1.40

method	result
default	$-\frac{\frac{\frac{abBc\,x^3}{8ac-2b^2}+\frac{ac(Ab-2aC)x^2}{8ac-2b^2}-\frac{aB\left(2ac-b^2\right)x}{2\left(4ac-b^2\right)}-\frac{a\left(2acA-A\,b^2+abC\right)}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(12Aabc\sqrt{-4ac+b^2}-2A\,b^3\sqrt{-4ac+b^2}-32A\,a^2c^2\right.}{}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*a*b*B*c/(4*a*c-b^2)*x^3+1/2*a*c*(A*b-2*C*a)/(4*a*c-b^2)*x^2-1/
2*a*B*(2*a*c-b^2)/(4*a*c-b^2)*x-1/2*a*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2))/(c
*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(-1/4*(12*A*a*b*c*(-4*a*c+b
^2)^(1/2)-2*A*b^3*(-4*a*c+b^2)^(1/2)-32*A*a^2*c^2+16*A*a*b^2*c-2*A*b^4-8*C*
(-4*a*c+b^2)^(1/2)*a^2*c)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-12*a^2*
c*B*(-4*a*c+b^2)^(1/2)+B*a*b^2*(-4*a*c+b^2)^(1/2)-4*a^2*b*B*c+B*a*b^3)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(1/4*(12*A*a*b*c*(-4*a*c+b^2)^(1/2)-2*A
*b^3*(-4*a*c+b^2)^(1/2)+32*A*a^2*c^2-16*A*a*b^2*c+2*A*b^4-8*C*(-4*a*c+b^2)^
(1/2)*a^2*c)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-12*a^2*c*B*(-4*a*c+b^
2)^(1/2)+B*a*b^2*(-4*a*c+b^2)^(1/2)+4*a^2*b*B*c-B*a*b^3)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
)))+A*ln(x)/a^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 - (2*C*a - A*b)*c*x^2 - C*a*b + A*b^2 - 2*A*a*c + (B*b^2 - 2
*B*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*
b*c)*x^2) + 1/2*integrate((B*a*b*c*x^2 + B*a*b^2 - 6*B*a^2*c - 2*(A*b^2*c -
 4*A*a*c^2)*x^3 - 2*(A*b^3 + (2*C*a^2 - 5*A*a*b)*c)*x)/(c*x^4 + b*x^2 + a),
 x)/(a^2*b^2 - 4*a^3*c) + A*log(x)/a^2
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6024 vs. 2(348) = 696.
time = 7.38, size = 6024, normalized size = 14.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*A*log(abs(c*x^4 + b*x^2 + a))/a^2 + A*log(abs(x))/a^2 + 1/16*((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
```

b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*B + 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 2*a^
4*b^8*c^2 + 120*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 28*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^3*c^4 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 240*
a^6*b^4*c^4 + 384*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 192*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 704*a^7*b^2*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*c^6 - 768*a^8*c^6 + 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 28*(
b^2 - 4*a*c)*a^5*b^4*c^3 + 128*(b^2 - 4*a*c)*a^6*b^2*c^4 - 192*(b^2 - 4*a*c
)*a^7*c^5)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4
- 56*a^9*b^9*c^5 + 576*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8
- 6144*a^13*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^9*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^10*c^3 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^10*b^7*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^9*b^8*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^9*c^4 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^11*b^5*c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^10*b^6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^9*b^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^12*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^13*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^12*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^12*b*c^8 - 2*(b^2 - 4*a*c)*a^8*b^9*c^4 + 48*(b^2
- 4*a*c)*a^9*b^7*c^5 - 384*(b^2 - 4*a*c)*a^10*b^5*c^6 + 1280*(b^2 - 4*a*c)*
a^11*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^12*b*c^8)*B)*arctan(2*sqrt(1/2)*x/sqrt(
(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^2
+ 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^
2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4
)))/((a^6*b^8*c - 16*a^7*b^6*c^2 - 2*a^6*b^7*c^2 + 96*a^8*b^4*c^3 + 24*a^7*
b^5*c^3 + a^6*b^6*c^3 - 256*a^9*b^2*c^4 - 96*a^8*b^3*c^4 - 12*a^7*b^4*c^4 +
256*a^10*c^5 + 128*a^9*b*c^5 + 48*a^8*b^2*c^5 - 64*a^9*c^6)*abs(a^4*b^4*c
- 8*a^5*b^2*c^2 + 16*a^6*c^3)*abs(c)) - 1/16*((a^4*b^4*c - 8*a^5*b^2*c^2 +
16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*

```
c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c
^2 - 2*(b^2 - 4*a*c)*b*c^2)*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^8*c - 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 - 2*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 + 2*a^4*b^8*c^2 + 120*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 28*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^6*c^3 - 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
b^2*c^4 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 14*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 + 240*a^6*b^4*c^4 + 384*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 192*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*
b^2*c^5 - 704*a^7*b^2*c^5 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
c^6 + 768*a^8*c^6 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 + 28*(b^2 - 4*a*c)*a^5*b^4*
c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + 192*(b^2 - 4*a*c)*a^7*c^5)*B*abs(a^4*
b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4 - 56*a^9*b^9*c^5 + 57
6*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8 - 6144*a^13*b*c^9 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^11*c^2 + 28
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 ...
```

Mupad [B]
time = 1.84, size = 2500, normalized size = 6.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*A*a*c - A*b^2 + C*a*b)/(2*a*(4*a*c - b^2)) + (B*x*(2*a*c - b^2))/(2*a*(
4*a*c - b^2)) - (c*x^2*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)) - (B*b*c*x^3)/(2*
a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(root(1572864*a^9*b^2*c^5
*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*
z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 - 256*a^4*b^12*z^4 + 15728
64*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3
- 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 -
 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 901
12*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z
^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^
2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*
c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*
a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 +
 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2
- 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*
A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*
B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2
*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15
```

360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^
3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^
5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c
^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z
 - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*
A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 96
0*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*
C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^
3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*
a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B
^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^
5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2
c^6, z, k)(root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680
*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^
10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^
4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3
*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*
c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4
*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2
*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C
^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 +
24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c
^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2
*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a
^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c
^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z -
 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c
^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*
a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A
*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^
3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^
2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c
^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^
3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5
*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c
^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 +
 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*
a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 -
2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3
*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*((1032*A*B*a^3*b^5*c^4 - 1
52*A*B*a^2*b^7*c^3 - 768*B*C*a^6*c^6 - 2944*A*B*a^4*b^3*c^5 + 16*B*C*a^3*b^
6*c^3 - 208*B*C*a^4*b^4*c^4 + 768*B*C*a^5*b^2*c^5 + 8*A*B*a*b^9*c^2 + 2944*
A*B*a^5*b*c^6)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) +
 root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3
*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 -

```
 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 +
327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3
- 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 25
6*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2...
```

3.35 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=514

$$-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(A\left(3b\right.\right.}{}$$

```
[Out] 1/2*(10*A*a*c-3*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a
/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/
a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/2*b*B*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+B*ln(x)/a^2-1/4*B*ln(c*x^4+b*x^2+
a)/a^2-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
-a*C*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))+A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)
^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*A*b^2-10*a*A*c-a*b*C+(-A*(-16*a*b*c+3*b^3)+a*(-12*a*c+b^2)*C
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.07, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1676, 1291, 1295, 1180, 211, 12, 1128, 754, 814, 648, 632, 212, 642}

$$-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\frac{A\left(3b^3-16abc\right)-aC\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-10aAc-abC+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{bB\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{B\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(3*A*b^2 - 10*a*A*c - a*b*C)/(a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c +
 b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b
*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt
[c]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*
c]) - a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 - 16*
a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/
(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
```

```
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{B}{x\left(a+bx^2+cx^4\right)^2}\,dx+\int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+B\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx-\frac{\int \frac{-3Ab^2+10}{}}{}$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\text{Subst}\left(\int\right.$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+}{2a\left(b^2-4ac\right)x\left(a\right.}$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+}{2a\left(b^2-4ac\right)x\left(a\right.}$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+}{2a\left(b^2-4ac\right)x\left(a\right.}$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+}{2a\left(b^2-4ac\right)x\left(a\right.}$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+}{2a\left(b^2-4ac\right)x\left(a\right.}$$

$$=-\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+}{2a\left(b^2-4ac\right)x\left(a\right.}$$

Mathematica [A]

time = 1.26, size = 559, normalized size = 1.09

$$\frac{-\frac{4A}{x}+\frac{-4a^2c(B+Cx)-2Ab^2x(b+cx^2)+2a(2Ac^2x^3+b^2(B+Cx)+bcx(3A+x(B+Cx)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)+a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)+a\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+4B\log(x)-\frac{B\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{B\left(-b^3+6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3
+ b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] +
 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2
*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c + b*Sqrt[
b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B*(b^3 - 6
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A]
time = 0.14, size = 667, normalized size = 1.30

method	result
default	$-\frac{\frac{c\left(2acA-A\,b^2+abC\right)x^3}{8ac-2b^2}+\frac{x^2abBc}{8ac-2b^2}+\frac{\left(3Aabc-A\,b^3-2a^2cC+Ca\,b^2\right)x}{8ac-2b^2}-\frac{aB\left(2ac-b^2\right)}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(12abBc\sqrt{-4ac+b^2}-2b^3B\sqrt{-4ac+\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^3+1/2/(4*a*c-b^2)*x^2*a*
b*B*c+1/2*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a*c-b^2)*x-1/2*a*B*(2*a*c-
b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(-1/4*(
12*a*b*B*c*(-4*a*c+b^2)^(1/2)-2*b^3*B*(-4*a*c+b^2)^(1/2)-32*a^2*B*c^2+16*a*
```

```
b^2*B*c-2*b^4*B)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(16*A*a*b*c*(-4*a*
c+b^2)^(1/2)-3*A*b^3*(-4*a*c+b^2)^(1/2)-40*A*a^2*c^2+22*A*a*b^2*c-3*A*b^4-1
2*C*(-4*a*c+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*a*b^2-4*C*a^2*b*c+C*a*b^3
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(1/4*(12*a*b*B*c*(-4*a*c+b^2)^(1/
2)-2*b^3*B*(-4*a*c+b^2)^(1/2)+32*a^2*B*c^2-16*a*b^2*B*c+2*b^4*B)/c*ln(b+2*c
*x^2+(-4*a*c+b^2)^(1/2))+1/2*(16*A*a*b*c*(-4*a*c+b^2)^(1/2)-3*A*b^3*(-4*a*c
+b^2)^(1/2)+40*A*a^2*c^2-22*A*a*b^2*c+3*A*b^4-12*C*(-4*a*c+b^2)^(1/2)*a^2*c
+C*(-4*a*c+b^2)^(1/2)*a*b^2+4*C*a^2*b*c-C*a*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))-1/a^
2*A/x+B*ln(x)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b*c*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A
*a^2*c + (C*a*b^2 - 3*A*b^3 - (2*C*a^2 - 11*A*a*b)*c)*x^2 + (B*a*b^2 - 2*B*
a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b
^2 - 4*a^4*c)*x) + 1/2*integrate((C*a*b^2 - 3*A*b^3 - 2*(B*b^2*c - 4*B*a*c^
2)*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^2 - (6*C*a^2 - 13*A*a*b)*c -
2*(B*b^3 - 5*B*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + B*lo
g(x)/a^2
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 9016 vs. $2(453) = 906$.
time = 9.02, size = 9016, normalized size = 17.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a*b*c*x
^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b*c*x^3 + C*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*C*a^2*c*x^2 + 11*A*a*b*c*x^2 + B*a*b^2*x - 2*B*a^2*c*x - 2*A*a*b^2 +
 8*A*a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((a^4*b^4*c
- 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c
)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^
2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*
a*b*c^2)*C + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 49*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 6*a^4*b^9*c^2 + 300*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^6*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 98*a^
5*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 304*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 37*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 600*a^6*b^5*c^4 + 832*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 + 416*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^7*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 +
1632*a^7*b^3*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 -
1664*a^8*b*c^6 + 6*(b^2 - 4*a*c)*a^4*b^7*c^2 - 74*(b^2 - 4*a*c)*a^5*b^5*c^3
 + 304*(b^2 - 4*a*c)*a^6*b^3*c^4 - 416*(b^2 - 4*a*c)*a^7*b*c^5)*A*abs(a^4*b
^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^2 - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 2*a^5*b^8*c^2 + 120*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^3 + 28*sqrt(2)*sqrt(b*c +
```

```
sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^3 + 36*a^6*b^6*c^3 - 352*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 14
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 240*a^7*b^4*c^4 + 38
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*c^5 + 192*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^7*b^2*c^5 + 704*a^8*b^2*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*c^6 - 768*a^9*c^6 + 2*(b^2 - 4*a*c)*a^5*b^6*c^2 - 28*(b^2 - 4*a*c)*a^6
*b^4*c^3 + 128*(b^2 - 4*a*c)*a^7*b^2*c^4 - 192*(b^2 - 4*a*c)*a^8*c^5)*C*abs
(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (6*a^8*b^12*c^4 - 128*a^9*b^10*c
^5 + 1088*a^10*b^8*c^6 - 4608*a^11*b^6*c^7 + 9728*a^12*b^4*c^8 - 8192*a^13*
b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b
^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*
b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*
b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
10*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^9*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^8*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^11*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^10*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^12*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^11*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^10*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^13*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^12*b^3*c^7 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^11*b^4*c^7 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^8 - 6*(b^2 - 4*a*c)*a^8*b^10*c^4 + 104*(b
^2 - 4*a*c)*a^9*b^8*c^5 - 672*(b^2 - 4*a*c)*a^10*b^6*c^6 + 1920*(b^2 - 4*a*
c)*a^11*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^12*b^2*c^8)*A - (2*a^9*b^11*c^4 - 56
*a^10*b^9*c^5 + 576*a^11*b^7*c^6 - 2816*a^12*b^5*c^7 + 6656*a^13*b^3*c^8 -
6144*a^14*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^9*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq...
```

Mupad [B]
time = 2.47, size = 2500, normalized size = 4.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*
a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a^1
1*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4
```

*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*
b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10
*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A
*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*
C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 43008
0*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 6144
0*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^
2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4
*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483
840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^
2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 -
144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z -
5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z
- 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*
z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c
^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a
^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a
*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b
^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^
2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a
*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a
*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*
c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 420
0*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*
c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 3
24*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5
- 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*
c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*
z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 -
983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*
z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 -
2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c
^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*
b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b
^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a
^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C
^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^
2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*
b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 332
32*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^
2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*
C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B
*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 1228
80*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 4
8*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^

```
4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*
c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5
568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4
- 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3
 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6
 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 +
5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^
4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4
- 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^
2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 -
 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4
- 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 +
 6144*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*
a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 614
40*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*
B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358
400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^
2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2
 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c...
```

3.36 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=534

$$-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2a}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx\right.}$$

```
[Out] 1/2*(6*A*a*c-2*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x^2-1/2*B*(-10*a*c+3*b^2)/a^2/
(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1
/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2
+a)-1/2*(2*A*(6*a^2*c^2-6*a*b^2*c+b^4)-a*b*(-6*a*c+b^2)*C)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-C*a)*ln(x)/a^3+1/4*(2
*A*b-C*a)*ln(c*x^4+b*x^2+a)/a^3-1/4*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-
10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.36, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1676, 1265, 836, 814, 648, 632, 212, 642, 12, 1135, 1295, 1180, 211}

$$\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}-\frac{B\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{B\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B(3b^2-10ac)}{2a^2x\left(b^2-4ac\right)}-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(-2ac+b^2+bcx^2\right)}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(2*A*b^2 - 6*a*A*c - a*b*C)/(a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^2 - 10*a
*c))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*
c)*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c
 + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*A*(b^4 - 6*a*b^2*c
+ 6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C
)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
```

```
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
```

```
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx &= \int \frac{B}{x^2\left(a+bx^2+cx^4\right)^2}\,dx+\int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx\\
&=\frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)+B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx\\
&=\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}-\\
&=-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\\
&=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\\
&=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\\
&=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\\
&=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B(3b^2-10ac)}{2a^2\left(b^2-4ac\right)x}+\frac{B(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+
\end{aligned}
$$

Mathematica [A]
time = 1.54, size = 655, normalized size = 1.23

$$
\frac{-\frac{2aA}{x^2}-\frac{4aB}{x}-\frac{2a\left(2a^2cC+b^2Bx(b+cx^2)+A(b^3-3abc+b^2cx^2-2ac^2x^2)-a(b^2C+2Bc^2x^3+bcx(3B+Cx))\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}aB\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}aB\sqrt{c}\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+4(-2Ab+aC)\log(x)+\frac{\left(2A\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)+a\left(-b^3+6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)C\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(2A\left(-b^4+6ab^2c-6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)+a\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)C\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3
- 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*B
+ C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3*b^
3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sq
rt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b^2*Sqrt[b^
2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c - b^2*Sqrt[b^2
- 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [A]
time = 0.14, size = 778, normalized size = 1.46

method	result
default	$-\frac{\frac{acB\left(2ac-b^{2}\right)x^{3}}{8ac-2b^{2}}+\frac{ac\left(2acA-A\,b^{2}+abC\right)x^{2}}{8ac-2b^{2}}+\frac{abB\left(3ac-b^{2}\right)x}{8ac-2b^{2}}+\frac{a\left(3Aabc-A\,b^{3}-2a^{2}cC+Ca\,b^{2}\right)}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(-\frac{\left(24A\sqrt{-4ac+b^{2}}\,a^{2}c^{2}-24A\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((1/2*a*c*B*(2*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*a*c*(2*A*a*c-A*b^2+C*a*b
)/(4*a*c-b^2)*x^2+1/2*a*b*B*(3*a*c-b^2)/(4*a*c-b^2)*x+1/2*a*(3*A*a*b*c-A*b^
3-2*C*a^2*c+C*a*b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*
c-4*b^2)*(-1/4*(24*A*(-4*a*c+b^2)^(1/2)*a^2*c^2-24*A*(-4*a*c+b^2)^(1/2)*a*b
^2*c+4*A*(-4*a*c+b^2)^(1/2)*b^4+64*A*a^2*b*c^2-32*A*a*b^3*c+4*A*b^5+12*C*(-
4*a*c+b^2)^(1/2)*a^2*b*c-2*C*(-4*a*c+b^2)^(1/2)*a*b^3-32*C*a^3*c^2+16*C*a^2
*b^2*c-2*C*a*b^4)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(16*B*(-4*a*c+b^2
)^(1/2)*a^2*b*c-3*B*(-4*a*c+b^2)^(1/2)*a*b^3-40*a^3*B*c^2+22*B*a^2*b^2*c-3*
```

```
B*a*b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(1/4*(24*A*(-4*a*c+b^2)^(1
/2)*a^2*c^2-24*A*(-4*a*c+b^2)^(1/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1/2)*b^4-64*A
*a^2*b*c^2+32*A*a*b^3*c-4*A*b^5+12*C*(-4*a*c+b^2)^(1/2)*a^2*b*c-2*C*(-4*a*c
+b^2)^(1/2)*a*b^3+32*C*a^3*c^2-16*C*a^2*b^2*c+2*C*a*b^4)/c*ln(b+2*c*x^2+(-4
*a*c+b^2)^(1/2))+1/2*(16*B*(-4*a*c+b^2)^(1/2)*a^2*b*c-3*B*(-4*a*c+b^2)^(1/2
)*a*b^3+40*a^3*B*c^2-22*B*a^2*b^2*c+3*B*a*b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))-1/2/a^
2*A/x^2-1/a^2*B/x+(-2*A*b+C*a)/a^3*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*B*b^2*c - 10*B*a*c^2)*x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^2)*c)*x^4
+ A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c)*x^3 - (C*a*b^2 - 2*A*b^3 - (
2*C*a^2 - 7*A*a*b)*c)*x^2 + 2*(B*a*b^2 - 4*B*a^2*c)*x)/((a^2*b^2*c - 4*a^3*
c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2) - 1/2*integ
rate((3*B*a*b^3 - 13*B*a^2*b*c - 2*(4*(C*a^2 - 2*A*a*b)*c^2 - (C*a*b^2 - 2*
A*b^3)*c)*x^3 + (3*B*a*b^2*c - 10*B*a^2*c^2)*x^2 + 2*(C*a*b^3 - 2*A*b^4 - 6
*A*a^2*c^2 - 5*(C*a^2*b - 2*A*a*b^2)*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2
 - 4*a^4*c) + (C*a - 2*A*b)*log(x)/a^3
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6939 vs. $2(470) = 940$.
time = 8.19, size = 6939, normalized size = 12.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3
+ 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2
+ 20*(b^2 - 4*a*c)*a*c^3)*B - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^2 - 6*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 - 6*a^6*b^9*c^2 + 300*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*b^7*c^3 + 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^4 - 37*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 - 600*a^8*b^5*c^4 + 832*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^3*c^5 + 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^9*b*c^6 - 1664*a^10*b*c^6 + 6*(b^2 - 4*a*c)*a^6*b^7*c^2 - 74*(b^2 -
 4*a*c)*a^7*b^5*c^3 + 304*(b^2 - 4*a*c)*a^8*b^3*c^4 - 416*(b^2 - 4*a*c)*a^9
*b*c^5)*B*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) + (6*a^12*b^12*c^4 -
128*a^13*b^10*c^5 + 1088*a^14*b^8*c^6 - 4608*a^15*b^6*c^7 + 9728*a^16*b^4*c
^8 - 8192*a^17*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^12*b^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^13*b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^12*b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^14*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^13*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^12*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^15*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^14*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^13*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^15*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^14*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*a
```

```
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^17*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^3*c^7 + 960*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^15*b^4*c^7 - 1024*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^2*c^8 - 6*(b^2 - 4*a*c)*
a^12*b^10*c^4 + 104*(b^2 - 4*a*c)*a^13*b^8*c^5 - 672*(b^2 - 4*a*c)*a^14*b^6
*c^6 + 1920*(b^2 - 4*a*c)*a^15*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^16*b^2*c^8)*B
)*arctan(2*sqrt(1/2)*x/sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 + sqr
t((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^2*c
^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c^2
- 8*a^7*b^2*c^3 + 16*a^8*c^4)))/((a^9*b^8*c - 16*a^10*b^6*c^2 - 2*a^9*b^7*c
^2 + 96*a^11*b^4*c^3 + 24*a^10*b^5*c^3 + a^9*b^6*c^3 - 256*a^12*b^2*c^4 - 9
6*a^11*b^3*c^4 - 12*a^10*b^4*c^4 + 256*a^13*c^5 + 128*a^12*b*c^5 + 48*a^11*
b^2*c^5 - 64*a^12*c^6)*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*abs(c))
+ 1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^
3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^
2 + 20*(b^2 - 4*a*c)*a*c^3)*B - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^2 - 6*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 + 6*a^6*b^9*c^2 + 300*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^6*b^7*c^3 - 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^4 - 37*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 + 600*a^8*b^5*c^4 + 832
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2...
```

Mupad [B]
time = 2.77, size = 2500, normalized size = 4.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 327680
*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*a^
12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^
4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A*
a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 12
```

2880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 -
12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 512*
A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 1794
048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7*
c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b^1
0*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a*b
^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576
*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z
^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^
3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 -
761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^1
0*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12*z
^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z -
432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z
+ 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5
*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^
4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 8294
4*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3
*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^
2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*
a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A
^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2
*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3
*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*
a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*
C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C
^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3
*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*
b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 +
1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*
a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5
*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*
a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 3
27680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 10485
76*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a
^8*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 31457
28*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3
+ 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z
^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 +
512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 -
1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4
*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^
3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^
2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 +
88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*

```
c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B
^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z
^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^
2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b
^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^
6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*
c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^
3*c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C
^2*a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z +
 82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^
7*c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B
^2*a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328
*C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104
448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 76
8*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 6553
6*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B
^2*C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 499...
```

3.37 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=399

$$\frac{a^3A(dx)^{1+m}}{d(1+m)}+\frac{a^3B(dx)^{2+m}}{d^2(2+m)}+\frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{3a^2bB(dx)^{4+m}}{d^4(4+m)}+\frac{3a(A(b^2+ac)+abC)\,(dx)^{5+m}}{d^5(5+m)}+\frac{3aB}{}$$

```
[Out] a^3*A*(d*x)^(1+m)/d/(1+m)+a^3*B*(d*x)^(2+m)/d^2/(2+m)+a^2*(3*A*b+C*a)*(d*x)
^(3+m)/d^3/(3+m)+3*a^2*b*B*(d*x)^(4+m)/d^4/(4+m)+3*a*(A*(a*c+b^2)+a*b*C)*(d
*x)^(5+m)/d^5/(5+m)+3*a*B*(a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(A*(6*a*b*c+b^3)+
3*a*(a*c+b^2)*C)*(d*x)^(7+m)/d^7/(7+m)+b*B*(6*a*c+b^2)*(d*x)^(8+m)/d^8/(8+m
)+(3*A*c*(a*c+b^2)+b*(6*a*c+b^2)*C)*(d*x)^(9+m)/d^9/(9+m)+3*B*c*(a*c+b^2)*(
d*x)^(10+m)/d^10/(10+m)+3*c*(A*b*c+(a*c+b^2)*C)*(d*x)^(11+m)/d^11/(11+m)+3*
b*B*c^2*(d*x)^(12+m)/d^12/(12+m)+c^2*(A*c+3*C*b)*(d*x)^(13+m)/d^13/(13+m)+B
*c^3*(d*x)^(14+m)/d^14/(14+m)+c^3*C*(d*x)^(15+m)/d^15/(15+m)
```

Rubi [A]
time = 0.28, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{1642\}$

$$\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}+\frac{3c(dx)^{m+11}\left(C(ac+b^2)+Abc\right)}{d^{11}(m+11)}+\frac{(dx)^{m+9}\left(3Ac(ac+b^2)+bC(6ac+b^2)\right)}{d^9(m+9)}+\frac{3a(dx)^{m+5}\left(A(ac+b^2)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A(6abc+b^3)+3aC(ac+b^2)\right)}{d^7(m+7)}+\frac{3Bc(ac+b^2)\,(dx)^{m+10}}{d^{10}(m+10)}+\frac{bB(6ac+b^2)\,(dx)^{m+8}}{d^8(m+8)}+\frac{3aB(ac+b^2)\,(dx)^{m+6}}{d^6(m+6)}+\frac{c^2(dx)^{m+13}(Ac+3bC)}{d^{13}(m+13)}+\frac{3bBc^2(dx)^{m+12}}{d^{12}(m+12)}+\frac{Bc^3(dx)^{m+14}}{d^{14}(m+14)}+\frac{c^3C(dx)^{m+15}}{d^{15}(m+15)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m))/
(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*
a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8
 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 +
m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m)) + (3
*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B*c^2*(d*
x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/(d^13*(13
 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(15 + m))/(d
^15*(15 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \int \left(a^3A(dx)^m + \frac{a^3B(dx)^{1+m}}{d} + \frac{a^2(3Ab+aC)(dx)^{2+m}}{d^2} + \cdots\right.$$
$$= \frac{a^3A(dx)^{1+m}}{d(1+m)} + \frac{a^3B(dx)^{2+m}}{d^2(2+m)} + \frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{3a\cdots}{\cdots}$$

Mathematica [A]
time = 2.11, size = 296, normalized size = 0.74

$$x(dx)^m \left(\frac{a^3A}{1+m} + \frac{a^3Bx}{2+m} + \frac{a^2(3Ab+aC)x^2}{3+m} + \frac{3a^2bBx^3}{4+m} + \frac{3a(A(b^2+ac)+abC)\,x^4}{5+m} + \frac{3aB(b^2+ac)\,x^5}{6+m} + \frac{(A(b^3+6abc)+3a(b^2+ac)\,C)\,x^6}{7+m} + \frac{bB(b^2+6ac)\,x^7}{8+m} + \frac{(3Ac(b^2+ac)+b(b^2+6ac)\,C)\,x^8}{9+m} + \frac{3Bc(b^2+ac)\,x^9}{10+m} + \frac{3c(Abc+(b^2+ac)\,C)\,x^{10}}{11+m} + \frac{3bBc^2x^{11}}{12+m} + \frac{c^2(Ac+3bC)x^{12}}{13+m} + \frac{Bc^3x^{13}}{14+m} + \frac{c^3Cx^{14}}{15+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(3
 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 + m)
 + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*
C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c) + b
*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3*c*(A
*b*c + (b^2 + a*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c^2*(A*
c + 3*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5519 vs. 2(399) = 798.
time = 0.04, size = 5520, normalized size = 13.83

method	result	size
gosper	Expression too large to display	5520
risch	Expression too large to display	5520

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.43, size = 611, normalized size = 1.53

$$\frac{Cc^3d^mx^{15}x^m}{m+15} + \frac{Bc^3d^mx^{14}x^m}{m+14} + \frac{3\,Cbc^2d^mx^{13}x^m}{m+13} + \frac{Ac^3d^mx^{13}x^m}{m+13} + \frac{3\,Bbc^2d^mx^{12}x^m}{m+12} + \frac{3\,Cb^2cd^mx^{11}x^m}{m+11} + \frac{3\,Cac^2d^mx^{11}x^m}{m+11} + \frac{3\,Abc^2d^mx^{11}x^m}{m+11} + \frac{3\,Bb^2cd^mx^{10}x^m}{m+10} + \frac{3\,Bac^2d^mx^{10}x^m}{m+10} + \frac{Cb^3d^mx^9x^m}{m+9} + \frac{6\,Cabcd^mx^9x^m}{m+9} + \frac{3\,Ab^2cd^mx^9x^m}{m+9} + \frac{3\,Aac^2d^mx^9x^m}{m+9} + \frac{Bb^3d^mx^8x^m}{m+8} + \frac{6\,Babcd^mx^8x^m}{m+8} + \frac{3\,Cab^2d^mx^7x^m}{m+7} + \frac{Ab^3d^mx^7x^m}{m+7} + \frac{3\,Ca^2cd^mx^7x^m}{m+7} + \frac{6\,Aabcd^mx^7x^m}{m+7} + \frac{3\,Bab^2d^mx^6x^m}{m+6} + \frac{3\,Ba^2cd^mx^6x^m}{m+6} + \frac{3\,Ca^2bd^mx^5x^m}{m+5} + \frac{3\,Aab^2d^mx^5x^m}{m+5} + \frac{3\,Aa^2cd^mx^5x^m}{m+5} + \frac{3\,Ba^2bd^mx^4x^m}{m+4} + \frac{Ca^3d^mx^3x^m}{m+3} + \frac{3\,Aa^2bd^mx^3x^m}{m+3} + \frac{Ba^3d^mx^2x^m}{m+2} + \frac{(dx)^{m+1}\,Aa^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] C*c^3*d^m*x^15*x^m/(m + 15) + B*c^3*d^m*x^14*x^m/(m + 14) + 3*C*b*c^2*d^m*x
^13*x^m/(m + 13) + A*c^3*d^m*x^13*x^m/(m + 13) + 3*B*b*c^2*d^m*x^12*x^m/(m
+ 12) + 3*C*b^2*c*d^m*x^11*x^m/(m + 11) + 3*C*a*c^2*d^m*x^11*x^m/(m + 11) +
 3*A*b*c^2*d^m*x^11*x^m/(m + 11) + 3*B*b^2*c*d^m*x^10*x^m/(m + 10) + 3*B*a*
c^2*d^m*x^10*x^m/(m + 10) + C*b^3*d^m*x^9*x^m/(m + 9) + 6*C*a*b*c*d^m*x^9*x
^m/(m + 9) + 3*A*b^2*c*d^m*x^9*x^m/(m + 9) + 3*A*a*c^2*d^m*x^9*x^m/(m + 9)
+ B*b^3*d^m*x^8*x^m/(m + 8) + 6*B*a*b*c*d^m*x^8*x^m/(m + 8) + 3*C*a*b^2*d^m
*x^7*x^m/(m + 7) + A*b^3*d^m*x^7*x^m/(m + 7) + 3*C*a^2*c*d^m*x^7*x^m/(m + 7
) + 6*A*a*b*c*d^m*x^7*x^m/(m + 7) + 3*B*a*b^2*d^m*x^6*x^m/(m + 6) + 3*B*a^2
*c*d^m*x^6*x^m/(m + 6) + 3*C*a^2*b*d^m*x^5*x^m/(m + 5) + 3*A*a*b^2*d^m*x^5*
x^m/(m + 5) + 3*A*a^2*c*d^m*x^5*x^m/(m + 5) + 3*B*a^2*b*d^m*x^4*x^m/(m + 4)
 + C*a^3*d^m*x^3*x^m/(m + 3) + 3*A*a^2*b*d^m*x^3*x^m/(m + 3) + B*a^3*d^m*x^
2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^3/(d*(m + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3898 vs. $2(399) = 798$.
time = 0.43, size = 3898, normalized size = 9.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^11 + 27497
47*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^8 + 2681453775*C*c^3
*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*c^3*m^5 + 159721605680*C*c^3*m
^4 + 310989260400*C*c^3*m^3 + 392156797824*C*c^3*m^2 + 283465647360*C*c^3*m
 + 87178291200*C*c^3)*x^15 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3*m^12
 + 147056*B*c^3*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 385081268*
B*c^3*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 59999485546*B*c^
3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 418753514880*B*c^
3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*x^14 + ((3*C*b*c^2 + A*c^
3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 + 5189*(3*C*b*c^2 + A*c^3)*m^12 + 15
0943*(3*C*b*c^2 + A*c^3)*m^11 + 2937363*(3*C*b*c^2 + A*c^3)*m^10 + 40372761
*(3*C*b*c^2 + A*c^3)*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 + 2965379989*(
3*C*b*c^2 + A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 + 63747744632*
(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^4 + 3017710080
00*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c^2 + A*c^3)*m^3 + 44
9213351040*(3*C*b*c^2 + A*c^3)*m^2 + 326044051200*(3*C*b*c^2 + A*c^3)*m)*x^
13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*m^13 + 5284*B*b*c^2*m^12 + 154992*B*b*c^
2*m^11 + 3039718*B*b*c^2*m^10 + 42081864*B*b*c^2*m^9 + 423113372*B*b*c^2*m^
8 + 3130267536*B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 67988181228*B*b*c^2*
m^5 + 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^2*m^3 + 484441814160*B*
b*c^2*m^2 + 352515844800*B*b*c^2*m + 108972864000*B*b*c^2)*x^12 + 3*((C*b^2
*c + (C*a + A*b)*c^2)*m^14 + 109*(C*b^2*c + (C*a + A*b)*c^2)*m^13 + 5381*(C
```

*b^2*c + (C*a + A*b)*c^2)*m^12 + 159209*(C*b^2*c + (C*a + A*b)*c^2)*m^11 + 3148323*(C*b^2*c + (C*a + A*b)*c^2)*m^10 + 43926927*(C*b^2*c + (C*a + A*b)*c^2)*m^9 + 444899543*(C*b^2*c + (C*a + A*b)*c^2)*m^8 + 3313733027*(C*b^2*c + (C*a + A*b)*c^2)*m^7 + 18180066256*(C*b^2*c + (C*a + A*b)*c^2)*m^6 + 72822481864*(C*b^2*c + (C*a + A*b)*c^2)*m^5 + 208624806576*(C*b^2*c + (C*a + A*b)*c^2)*m^4 + 118879488000*C*b^2*c + 411940473264*(C*b^2*c + (C*a + A*b)*c^2)*m^3 + 118879488000*(C*a + A*b)*c^2 + 525650497920*(C*b^2*c + (C*a + A*b)*c^2)*m^2 + 383662137600*(C*b^2*c + (C*a + A*b)*c^2)*m)*x^11 + 3*((B*b^2*c + B*a*c^2)*m^14 + 110*(B*b^2*c + B*a*c^2)*m^13 + 5480*(B*b^2*c + B*a*c^2)*m^12 + 163600*(B*b^2*c + B*a*c^2)*m^11 + 3263622*(B*b^2*c + B*a*c^2)*m^10 + 45922260*(B*b^2*c + B*a*c^2)*m^9 + 468873140*(B*b^2*c + B*a*c^2)*m^8 + 3518896600*(B*b^2*c + B*a*c^2)*m^7 + 19442163553*(B*b^2*c + B*a*c^2)*m^6 + 78381575150*(B*b^2*c + B*a*c^2)*m^5 + 225856355580*(B*b^2*c + B*a*c^2)*m^4 + 130767436800*B*b^2*c + 130767436800*B*a*c^2 + 448249789800*(B*b^2*c + B*a*c^2)*m^3 + 574497805824*(B*b^2*c + B*a*c^2)*m^2 + 420839556480*(B*b^2*c + B*a*c^2)*m)*x^10 + ((C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^14 + 111*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^13 + 5581*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^12 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^11 + 3386083*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^10 + 48083733*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^9 + 495342143*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^8 + 3749548713*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^7 + 20885191136*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^6 + 84836490456*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^5 + 246143692976*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^4 + 145297152000*C*b^3 + 435891456000*A*a*c^2 + 491520108816*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^3 + 633314724480*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^2 + 435891456000*(2*C*a*b + A*b^2)*c + 465985094400*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m)*x^9 + ((B*b^3 + 6*B*a*b*c)*m^14 + 112*(B*b^3 + 6*B*a*b*c)*m^13 + 5684*(B*b^3 + 6*B*a*b*c)*m^12 + 172928*(B*b^3 + 6*B*a*b*c)*m^11 + 3516198*(B*b^3 + 6*B*a*b*c)*m^10 + 50428896*(B*b^3 + 6*B*a*b*c)*m^9 + 524664572*(B*b^3 + 6*B*a*b*c)*m^8 + 4010311424*(B*b^3 + 6*B*a*b*c)*m^7 + 22548638161*(B*b^3 + 6*B*a*b*c)*m^6 + 92414105392*(B*b^3 + 6*B*a*b*c)*m^5 + 270359263944*(B*b^3 + 6*B*a*b*c)*m^4 + 163459296000*B*b^3 + 980755776000*B*a*b*c + 543939234048*(B*b^3 + 6*B*a*b*c)*m^3 + 705481831440*(B*b^3 + 6*B*a*b*c)*m^2 + 521962963200*(B*b^3 + 6*B*a*b*c)*m)*x^8 + ((3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^14 + 113*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^13 + 5789*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^12 + 177877*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^11 + 3654483*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^10 + 52977099*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^9 + 557256047*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^8 + 4306835671*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^7 + 24483279856*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^6 + 101420251688*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^5 + 299730345264*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^4 + 560431872000*C*a*b^2 + 186810624000*A*b^3 + 608700928752*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a...

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 47658 vs. $2(379) = 758$.
time = 3.36, size = 47658, normalized size = 119.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-A*a**3/(14*x**14) - A*a**2*b/(4*x**12) - 3*A*a**2*c/(10*x**10)
 - 3*A*a*b**2/(10*x**10) - 3*A*a*b*c/(4*x**8) - A*a*c**2/(2*x**6) - A*b**3/
(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c**3/(2*x**2) - B*a*
*3/(13*x**13) - 3*B*a**2*b/(11*x**11) - B*a**2*c/(3*x**9) - B*a*b**2/(3*x**
9) - 6*B*a*b*c/(7*x**7) - 3*B*a*c**2/(5*x**5) - B*b**3/(7*x**7) - 3*B*b**2*
c/(5*x**5) - B*b*c**2/x**3 - B*c**3/x - C*a**3/(12*x**12) - 3*C*a**2*b/(10*
x**10) - 3*C*a**2*c/(8*x**8) - 3*C*a*b**2/(8*x**8) - C*a*b*c/x**6 - 3*C*a*c
**2/(4*x**4) - C*b**3/(6*x**6) - 3*C*b**2*c/(4*x**4) - 3*C*b*c**2/(2*x**2)
+ C*c**3*log(x))/d**15, Eq(m, -15)), ((-A*a**3/(13*x**13) - 3*A*a**2*b/(11*
x**11) - A*a**2*c/(3*x**9) - A*a*b**2/(3*x**9) - 6*A*a*b*c/(7*x**7) - 3*A*a
*c**2/(5*x**5) - A*b**3/(7*x**7) - 3*A*b**2*c/(5*x**5) - A*b*c**2/x**3 - A*
c**3/x - B*a**3/(12*x**12) - 3*B*a**2*b/(10*x**10) - 3*B*a**2*c/(8*x**8) -
3*B*a*b**2/(8*x**8) - B*a*b*c/x**6 - 3*B*a*c**2/(4*x**4) - B*b**3/(6*x**6)
- 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2) + B*c**3*log(x) - C*a**3/(11*x*
*11) - C*a**2*b/(3*x**9) - 3*C*a**2*c/(7*x**7) - 3*C*a*b**2/(7*x**7) - 6*C*
a*b*c/(5*x**5) - C*a*c**2/x**3 - C*b**3/(5*x**5) - C*b**2*c/x**3 - 3*C*b*c*
*2/x + C*c**3*x)/d**14, Eq(m, -14)), ((-A*a**3/(12*x**12) - 3*A*a**2*b/(10*
x**10) - 3*A*a**2*c/(8*x**8) - 3*A*a*b**2/(8*x**8) - A*a*b*c/x**6 - 3*A*a*c
**2/(4*x**4) - A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3*A*b*c**2/(2*x**2)
+ A*c**3*log(x) - B*a**3/(11*x**11) - B*a**2*b/(3*x**9) - 3*B*a**2*c/(7*x**
7) - 3*B*a*b**2/(7*x**7) - 6*B*a*b*c/(5*x**5) - B*a*c**2/x**3 - B*b**3/(5*x
**5) - B*b**2*c/x**3 - 3*B*b*c**2/x + B*c**3*x - C*a**3/(10*x**10) - 3*C*a*
*2*b/(8*x**8) - C*a**2*c/(2*x**6) - C*a*b**2/(2*x**6) - 3*C*a*b*c/(2*x**4)
- 3*C*a*c**2/(2*x**2) - C*b**3/(4*x**4) - 3*C*b**2*c/(2*x**2) + 3*C*b*c**2*
log(x) + C*c**3*x**2/2)/d**13, Eq(m, -13)), ((-A*a**3/(11*x**11) - A*a**2*b
/(3*x**9) - 3*A*a**2*c/(7*x**7) - 3*A*a*b**2/(7*x**7) - 6*A*a*b*c/(5*x**5)
- A*a*c**2/x**3 - A*b**3/(5*x**5) - A*b**2*c/x**3 - 3*A*b*c**2/x + A*c**3*x
 - B*a**3/(10*x**10) - 3*B*a**2*b/(8*x**8) - B*a**2*c/(2*x**6) - B*a*b**2/(
2*x**6) - 3*B*a*b*c/(2*x**4) - 3*B*a*c**2/(2*x**2) - B*b**3/(4*x**4) - 3*B*
b**2*c/(2*x**2) + 3*B*b*c**2*log(x) + B*c**3*x**2/2 - C*a**3/(9*x**9) - 3*C
*a**2*b/(7*x**7) - 3*C*a**2*c/(5*x**5) - 3*C*a*b**2/(5*x**5) - 2*C*a*b*c/x*
*3 - 3*C*a*c**2/x - C*b**3/(3*x**3) - 3*C*b**2*c/x + 3*C*b*c**2*x + C*c**3*
x**3/3)/d**12, Eq(m, -12)), ((-A*a**3/(10*x**10) - 3*A*a**2*b/(8*x**8) - A*
a**2*c/(2*x**6) - A*a*b**2/(2*x**6) - 3*A*a*b*c/(2*x**4) - 3*A*a*c**2/(2*x*
*2) - A*b**3/(4*x**4) - 3*A*b**2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x*
*2/2 - B*a**3/(9*x**9) - 3*B*a**2*b/(7*x**7) - 3*B*a**2*c/(5*x**5) - 3*B*a*
```

```
b**2/(5*x**5) - 2*B*a*b*c/x**3 - 3*B*a*c**2/x - B*b**3/(3*x**3) - 3*B*b**2*
c/x + 3*B*b*c**2*x + B*c**3*x**3/3 - C*a**3/(8*x**8) - C*a**2*b/(2*x**6) -
3*C*a**2*c/(4*x**4) - 3*C*a*b**2/(4*x**4) - 3*C*a*b*c/x**2 + 3*C*a*c**2*log
(x) - C*b**3/(2*x**2) + 3*C*b**2*c*log(x) + 3*C*b*c**2*x**2/2 + C*c**3*x**4
/4)/d**11, Eq(m, -11)), ((-A*a**3/(9*x**9) - 3*A*a**2*b/(7*x**7) - 3*A*a**2
*c/(5*x**5) - 3*A*a*b**2/(5*x**5) - 2*A*a*b*c/x**3 - 3*A*a*c**2/x - A*b**3/
(3*x**3) - 3*A*b**2*c/x + 3*A*b*c**2*x + A*c**3*x**3/3 - B*a**3/(8*x**8) -
B*a**2*b/(2*x**6) - 3*B*a**2*c/(4*x**4) - 3*B*a*b**2/(4*x**4) - 3*B*a*b*c/x
**2 + 3*B*a*c**2*log(x) - B*b**3/(2*x**2) + 3*B*b**2*c*log(x) + 3*B*b*c**2*
x**2/2 + B*c**3*x**4/4 - C*a**3/(7*x**7) - 3*C*a**2*b/(5*x**5) - C*a**2*c/x
**3 - C*a*b**2/x**3 - 6*C*a*b*c/x + 3*C*a*c**2*x - C*b**3/x + 3*C*b**2*c*x
+ C*b*c**2*x**3 + C*c**3*x**5/5)/d**10, Eq(m, -10)), ((-A*a**3/(8*x**8) - A
*a**2*b/(2*x**6) - 3*A*a**2*c/(4*x**4) - 3*A*a*b**2/(4*x**4) - 3*A*a*b*c/x*
*2 + 3*A*a*c**2*log(x) - A*b**3/(2*x**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x
**2/2 + A*c**3*x**4/4 - B*a**3/(7*x**7) - 3*B*a**2*b/(5*x**5) - B*a**2*c/x*
*3 - B*a*b**2/x**3 - 6*B*a*b*c/x + 3*B*a*c**2*x - B*b**3/x + 3*B*b**2*c*x +
 B*b*c**2*x**3 + B*c**3*x**5/5 - C*a**3/(6*x**6) - 3*C*a**2*b/(4*x**4) - 3*
C*a**2*c/(2*x**2) - 3*C*a*b**2/(2*x**2) + 6*C*a*b*c*log(x) + 3*C*a*c**2*x**
2/2 + C*b**3*log(x) + 3*C*b**2*c*x**2/2 + 3*C*b*c**2*x**4/4 + C*c**3*x**6/6
)/d**9, Eq(m, -9)), ((-A*a**3/(7*x**7) - 3*A*a**2*b/(5*x**5) - A*a**2*c/x**
3 - A*a*b**2/x**3 - 6*A*a*b*c/x + 3*A*a*c**2*x - A*b**3/x + 3*A*b**2*c*x +
A*b*c**2*x**3 + A*c**3*x**5/5 - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B
*a**2*c/(2*x**2) - 3*B*a*b**2/(2*x**2) + 6*B*a*b*c*log(x) + 3*B*a*c**2*x**2
/2 + B*b**3*log(x) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6
- C*a**3/(5*x**5) - C*a**2*b/x**3 - 3*C*a**2*c/x - 3*C*a*b**2/x + 6*C*a*b*c
*x + C*a*c**2*x**3 + C*b**3*x + C*b**2*c*x**3 + 3*C*b*c**2*x**5/5 + C*c**3*
x**7/7)/d**8, Eq(m, -8)), ((-A*a**3/(6*x**6) - 3*A*a**2*b/(4*x**4) - 3*A*a*
*2*c/(2*x**2) - 3*A*a*b**2/(2*x**2) + 6*A*a*b*c*log(x) + 3*A*a*c**2*x**2/2
+ A*b**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/6 - B
*a**3/(5*x**5) - B*a**2*b/x**3 - 3*B*a**2*c/x - 3*B*a*b**2/x + 6*B*a*b*c*x
+ B*a*c**2*x**3 + B*b**3*x + B*b**2*c*x**3 + 3*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7808 vs. $2(399) = 798$.
time = 3.72, size = 7808, normalized size = 19.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c^3*m^14*x^15 + (d*x)^m*B*c^3*m^14*x^14 + 105*(d*x)^m*C*c^3*m^13
*x^15 + 3*(d*x)^m*C*b*c^2*m^14*x^13 + (d*x)^m*A*c^3*m^14*x^13 + 106*(d*x)^m
*B*c^3*m^13*x^14 + 5005*(d*x)^m*C*c^3*m^12*x^15 + 3*(d*x)^m*B*b*c^2*m^14*x^
12 + 321*(d*x)^m*C*b*c^2*m^13*x^13 + 107*(d*x)^m*A*c^3*m^13*x^13 + 5096*(d*
```

x)^m*B*c^3*m^12*x^14 + 143325*(d*x)^m*C*c^3*m^11*x^15 + 3*(d*x)^m*C*b^2*c*m
^14*x^11 + 3*(d*x)^m*C*a*c^2*m^14*x^11 + 3*(d*x)^m*A*b*c^2*m^14*x^11 + 324*
(d*x)^m*B*b*c^2*m^13*x^12 + 15567*(d*x)^m*C*b*c^2*m^12*x^13 + 5189*(d*x)^m*
A*c^3*m^12*x^13 + 147056*(d*x)^m*B*c^3*m^11*x^14 + 2749747*(d*x)^m*C*c^3*m^
10*x^15 + 3*(d*x)^m*B*b^2*c*m^14*x^10 + 3*(d*x)^m*B*a*c^2*m^14*x^10 + 327*(
d*x)^m*C*b^2*c*m^13*x^11 + 327*(d*x)^m*C*a*c^2*m^13*x^11 + 327*(d*x)^m*A*b*
c^2*m^13*x^11 + 15852*(d*x)^m*B*b*c^2*m^12*x^12 + 452829*(d*x)^m*C*b*c^2*m^
11*x^13 + 150943*(d*x)^m*A*c^3*m^11*x^13 + 2840838*(d*x)^m*B*c^3*m^10*x^14
+ 37312275*(d*x)^m*C*c^3*m^9*x^15 + (d*x)^m*C*b^3*m^14*x^9 + 6*(d*x)^m*C*a*
b*c*m^14*x^9 + 3*(d*x)^m*A*b^2*c*m^14*x^9 + 3*(d*x)^m*A*a*c^2*m^14*x^9 + 33
0*(d*x)^m*B*b^2*c*m^13*x^10 + 330*(d*x)^m*B*a*c^2*m^13*x^10 + 16143*(d*x)^m
*C*b^2*c*m^12*x^11 + 16143*(d*x)^m*C*a*c^2*m^12*x^11 + 16143*(d*x)^m*A*b*c^
2*m^12*x^11 + 464976*(d*x)^m*B*b*c^2*m^11*x^12 + 8812089*(d*x)^m*C*b*c^2*m^
10*x^13 + 2937363*(d*x)^m*A*c^3*m^10*x^13 + 38786748*(d*x)^m*B*c^3*m^9*x^14
 + 368411615*(d*x)^m*C*c^3*m^8*x^15 + (d*x)^m*B*b^3*m^14*x^8 + 6*(d*x)^m*B*
a*b*c*m^14*x^8 + 111*(d*x)^m*C*b^3*m^13*x^9 + 666*(d*x)^m*C*a*b*c*m^13*x^9
+ 333*(d*x)^m*A*b^2*c*m^13*x^9 + 333*(d*x)^m*A*a*c^2*m^13*x^9 + 16440*(d*x)
^m*B*b^2*c*m^12*x^10 + 16440*(d*x)^m*B*a*c^2*m^12*x^10 + 477627*(d*x)^m*C*b
^2*c*m^11*x^11 + 477627*(d*x)^m*C*a*c^2*m^11*x^11 + 477627*(d*x)^m*A*b*c^2*
m^11*x^11 + 9119154*(d*x)^m*B*b*c^2*m^10*x^12 + 121118283*(d*x)^m*C*b*c^2*m
^9*x^13 + 40372761*(d*x)^m*A*c^3*m^9*x^13 + 385081268*(d*x)^m*B*c^3*m^8*x^1
4 + 2681453775*(d*x)^m*C*c^3*m^7*x^15 + 3*(d*x)^m*C*a*b^2*m^14*x^7 + (d*x)^
m*A*b^3*m^14*x^7 + 3*(d*x)^m*C*a^2*c*m^14*x^7 + 6*(d*x)^m*A*a*b*c*m^14*x^7
+ 112*(d*x)^m*B*b^3*m^13*x^8 + 672*(d*x)^m*B*a*b*c*m^13*x^8 + 5581*(d*x)^m*
C*b^3*m^12*x^9 + 33486*(d*x)^m*C*a*b*c*m^12*x^9 + 16743*(d*x)^m*A*b^2*c*m^1
2*x^9 + 16743*(d*x)^m*A*a*c^2*m^12*x^9 + 490800*(d*x)^m*B*b^2*c*m^11*x^10 +
 490800*(d*x)^m*B*a*c^2*m^11*x^10 + 9444969*(d*x)^m*C*b^2*c*m^10*x^11 + 944
4969*(d*x)^m*C*a*c^2*m^10*x^11 + 9444969*(d*x)^m*A*b*c^2*m^10*x^11 + 126245
592*(d*x)^m*B*b*c^2*m^9*x^12 + 1209749541*(d*x)^m*C*b*c^2*m^8*x^13 + 403249
847*(d*x)^m*A*c^3*m^8*x^13 + 2816490248*(d*x)^m*B*c^3*m^7*x^14 + 1440932292
8*(d*x)^m*C*c^3*m^6*x^15 + 3*(d*x)^m*B*a*b^2*m^14*x^6 + 3*(d*x)^m*B*a^2*c*m
^14*x^6 + 339*(d*x)^m*C*a*b^2*m^13*x^7 + 113*(d*x)^m*A*b^3*m^13*x^7 + 339*(
d*x)^m*C*a^2*c*m^13*x^7 + 678*(d*x)^m*A*a*b*c*m^13*x^7 + 5684*(d*x)^m*B*b^3
*m^12*x^8 + 34104*(d*x)^m*B*a*b*c*m^12*x^8 + 168171*(d*x)^m*C*b^3*m^11*x^9
+ 1009026*(d*x)^m*C*a*b*c*m^11*x^9 + 504513*(d*x)^m*A*b^2*c*m^11*x^9 + 5045
13*(d*x)^m*A*a*c^2*m^11*x^9 + 9790866*(d*x)^m*B*b^2*c*m^10*x^10 + 9790866*(
d*x)^m*B*a*c^2*m^10*x^10 + 131780781*(d*x)^m*C*b^2*c*m^9*x^11 + 131780781*(
d*x)^m*C*a*c^2*m^9*x^11 + 131780781*(d*x)^m*A*b*c^2*m^9*x^11 + 1269340116*(
d*x)^m*B*b*c^2*m^8*x^12 + 8896139967*(d*x)^m*C*b*c^2*m^7*x^13 + 2965379989*
(d*x)^m*A*c^3*m^7*x^13 + 15200266081*(d*x)^m*B*c^3*m^6*x^14 + 56663366760*(
d*x)^m*C*c^3*m^5*x^15 + 3*(d*x)^m*C*a^2*b*m^14*x^5 + 3*(d*x)^m*A*a*b^2*m^14
x^5 + 3(d*x)^m*A*a^2*c*m^14*x^5 + 342*(d*x)^m*B*a*b^2*m^13*x^6 + 342*(d*x
)^m*B*a^2*c*m^13*x^6 + 17367*(d*x)^m*C*a*b^2*m^12*x^7 + 5789*(d*x)^m*A*b^3*
m^12*x^7 + 17367*(d*x)^m*C*a^2*c*m^12*x^7 + 34734*(d*x)^m*A*a*b*c*m^12*x^7
+ 172928*(d*x)^m*B*b^3*m^11*x^8 + 1037568*(d*x)^m*B*a*b*c*m^11*x^8 + 338608

```
3*(d*x)^m*C*b^3*m^10*x^9 + 20316498*(d*x)^m*C*a*b*c*m^10*x^9 + 10158249*(d*
x)^m*A*b^2*c*m^10*x^9 + 10158249*(d*x)^m*A*a*c^2*m^10*x^9 + 137766780*(d*x)
^m*B*b^2*c*m^9*x^10 + 137766780*(d*x)^m*B*a*c^2*m^9*x^10 + 1334698629*(d*x)
^m*C*b^2*c*m^8*x^11 + 1334698629*(d*x)^m*C*a*c^2*m^8*x^11 + 1334698629*(d*x
)^m*A*b*c^2*m^8*x^11 + 9390802608*(d*x)^m*B*b*c^2*m^7*x^12 + 48243569088*(d
*x)^m*C*b*c^2*m^6*x^13 + 16081189696*(d*x)^m*A*c^3*m^6*x^13 + 59999485546*(
d*x)^m*B*c^3*m^5*x^14 + 159721605680*(d*x)^m*C*c^3*m^4*x^15 + 3*(d*x)^m*B*a
^2*b*m^14*x^4 + 345*(d*x)^m*C*a^2*b*m^13*x^5 + 345*(d*x)^m*A*a*b^2*m^13*x^5
 + 345*(d*x)^m*A*a^2*c*m^13*x^5 + 17688*(d*x)^m*B*a*b^2*m^12*x^6 + 17688*(d
*x)^m*B*a^2*c*m^12*x^6 + 533631*(d*x)^m*C*a*b^2*m^11*x^7 + 177877*(d*x)^m*A
*b^3*m^11*x^7 + 533631*(d*x)^m*C*a^2*c*m^11*x^7 + 1067262*(d*x)^m*A*a*b*c*m
^11*x^7 + 3516198*(d*x)^m*B*b^3*m^10*x^8 + 21097188*(d*x)^m*B*a*b*c*m^10*x^
8 + 48083733*(d*x)^m*C*b^3*m^9*x^9 + 288502398*(d*x)^m*C*a*b*c*m^9*x^9 + 14
4251199*(d*x)^m*A*b^2*c*m^9*x^9 + 144251199*(d*x)^m*A*a*c^2*m^9*x^9 + 14066
19420*(d*x)^m*B*b^2*c*m^8*x^10 + 1406619420*(d*x)^m*B*a*c^2*m^8*x^10 + 9941
199081*(d*x)^m*C*b^2*c*m^7*x^11 + 9941199081*(d*x)^m*C*a*c^2*m^7*x^11 + 994
1199081*(d*x)^m*A*b*c^2*m^7*x^11 + 51203757363*(d*x)^m*B*b*c^2*m^6*x^12 + 1
91243233896*(d*x)^m*C*b*c^2*m^5*x^13 + 63747744...
```

Mupad [B]
time = 3.28, size = 2500, normalized size = 6.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^7*(d*x)^m*(A*b^3 + 3*C*a*b^2 + 3*C*a^2*c + 6*A*a*b*c)*(593193196800*m +
796089202560*m^2 + 608700928752*m^3 + 299730345264*m^4 + 101420251688*m^5 +
 24483279856*m^6 + 4306835671*m^7 + 557256047*m^8 + 52977099*m^9 + 3654483*
m^10 + 177877*m^11 + 5789*m^12 + 113*m^13 + m^14 + 186810624000))/(43391630
01600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 10096
72107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095
740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14
 + m^15 + 1307674368000) + (x^9*(d*x)^m*(C*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*
C*a*b*c)*(465985094400*m + 633314724480*m^2 + 491520108816*m^3 + 2461436929
76*m^4 + 84836490456*m^5 + 20885191136*m^6 + 3749548713*m^7 + 495342143*m^8
 + 48083733*m^9 + 3386083*m^10 + 168171*m^11 + 5581*m^12 + 111*m^13 + m^14
+ 145297152000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 +
 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7
 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m
^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*c^3*x^14*(d*x)^m*(3
03268406400*m + 418753514880*m^2 + 331303013496*m^3 + 169679309436*m^4 + 59
999485546*m^5 + 15200266081*m^6 + 2816490248*m^7 + 385081268*m^8 + 38786748
*m^9 + 2840838*m^10 + 147056*m^11 + 5096*m^12 + 106*m^13 + m^14 + 934053120
```

00))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 27068133456
00*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 820762800
0*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m
^13 + 120*m^14 + m^15 + 1307674368000) + (B*a^3*x^2*(d*x)^m*(1842662908800*
m + 2161577352960*m^2 + 1447709175432*m^3 + 629552085084*m^4 + 190060010998
*m^5 + 41371599841*m^6 + 6629764856*m^7 + 788931572*m^8 + 69582084*m^9 + 44
88198*m^10 + 205712*m^11 + 6344*m^12 + 118*m^13 + m^14 + 653837184000))/(43
39163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 +
 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 +
928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 12
0*m^14 + m^15 + 1307674368000) + (3*a*x^5*(d*x)^m*(A*b^2 + A*a*c + C*a*b)*(
815525625600*m + 1070058397824*m^2 + 797387461200*m^3 + 381885176880*m^4 +
125557386040*m^5 + 29449164928*m^6 + 5036392925*m^7 + 634247015*m^8 + 58769
745*m^9 + 3957747*m^10 + 188375*m^11 + 6005*m^12 + 115*m^13 + m^14 + 261534
873600))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813
345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 82076
28000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 65
80*m^13 + 120*m^14 + m^15 + 1307674368000) + (3*c*x^11*(d*x)^m*(C*b^2 + A*b
*c + C*a*c)*(383662137600*m + 525650497920*m^2 + 411940473264*m^3 + 2086248
06576*m^4 + 72822481864*m^5 + 18180066256*m^6 + 3313733027*m^7 + 444899543*
m^8 + 43926927*m^9 + 3148323*m^10 + 159209*m^11 + 5381*m^12 + 109*m^13 + m^
14 + 118879488000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^
3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*
m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 21840
0*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (a^2*x^3*(d*x)^m*(3
*A*b + C*a)*(1301090515200*m + 1621575699840*m^2 + 1145140001328*m^3 + 5205
57781424*m^4 + 163038108552*m^5 + 36588367376*m^6 + 6014254059*m^7 + 731124
647*m^8 + 65657031*m^9 + 4300483*m^10 + 199713*m^11 + 6229*m^12 + 117*m^13
+ m^14 + 435891456000))/(4339163001600*m + 6165817614720*m^2 + 505699570382
4*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129
553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 2
18400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (c^2*x^13*(d*x)
^m*(A*c + 3*C*b)*(326044051200*m + 449213351040*m^2 + 354444796368*m^3 + 18
0951426864*m^4 + 63747744632*m^5 + 16081189696*m^6 + 2965379989*m^7 + 40324
9847*m^8 + 40372761*m^9 + 2937363*m^10 + 150943*m^11 + 5189*m^12 + 107*m^13
 + m^14 + 100590336000))/(4339163001600*m + 6165817614720*m^2 + 50569957038
24*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 5463112
9553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 +
218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (A*a^3*x*(d*x)
^m*(3031488633600*m + 3134328981120*m^2 + 1922666722704*m^3 + 784146622896*
m^4 + 225525484184*m^5 + 47277726496*m^6 + 7353403057*m^7 + 854224943*m^8 +
 73870797*m^9 + 4687683*m^10 + 211939*m^11 + 6461*m^12 + 119*m^13 + m^14 +
1307674368000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 +
2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7
+ 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^

```
12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (C*c^3*x^15*(d*x)^m*(28
3465647360*m + 392156797824*m^2 + 310989260400*m^3 + 159721605680*m^4 + 566
63366760*m^5 + 14409322928*m^6 + 2681453775*m^7 + 368411615*m^8 + 37312275*
m^9 + 2749747*m^10 + 143325*m^11 + 5005*m^12 + ...
```

3.38 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=260

$$\frac{a^2A(dx)^{1+m}}{d(1+m)}+\frac{a^2B(dx)^{2+m}}{d^2(2+m)}+\frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{2abB(dx)^{4+m}}{d^4(4+m)}+\frac{\left(A(b^2+2ac)+2abC\right)(dx)^{5+m}}{d^5(5+m)}+\frac{B(b^2}{d}$$

```
[Out] a^2*A*(d*x)^(1+m)/d/(1+m)+a^2*B*(d*x)^(2+m)/d^2/(2+m)+a*(2*A*b+C*a)*(d*x)^(
3+m)/d^3/(3+m)+2*a*b*B*(d*x)^(4+m)/d^4/(4+m)+(A*(2*a*c+b^2)+2*a*b*C)*(d*x)^
(5+m)/d^5/(5+m)+B*(2*a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(2*A*b*c+(2*a*c+b^2)*C)
*(d*x)^(7+m)/d^7/(7+m)+2*b*B*c*(d*x)^(8+m)/d^8/(8+m)+c*(A*c+2*C*b)*(d*x)^(9
+m)/d^9/(9+m)+B*c^2*(d*x)^(10+m)/d^10/(10+m)+c^2*C*(d*x)^(11+m)/d^11/(11+m)
```

Rubi [A]

time = 0.16, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1642}

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+7}\left(C(2ac+b^2)+2Abc\right)}{d^7(m+7)}+\frac{(dx)^{m+5}\left(A(2ac+b^2)+2abC\right)}{d^5(m+5)}+\frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)}+\frac{B(2ac+b^2)(dx)^{m+6}}{d^6(m+6)}+\frac{2abB(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+9}(Ac+2bC)}{d^9(m+9)}+\frac{2bBc(dx)^{m+8}}{d^8(m+8)}+\frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d^4
*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*
(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(
d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A
*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx = \int \left(a^2A(dx)^m+\frac{a^2B(dx)^{1+m}}{d}+\frac{a(2Ab+aC)(dx)^{2+m}}{d^2}+\right.$$

$$=\frac{a^2A(dx)^{1+m}}{d(1+m)}+\frac{a^2B(dx)^{2+m}}{d^2(2+m)}+\frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{2a}{}$$

Mathematica [A]
time = 0.90, size = 187, normalized size = 0.72

$$(dx)^m\left(\frac{a^2Ax}{1+m}+\frac{a^2Bx^2}{2+m}+\frac{a(2Ab+aC)x^3}{3+m}+\frac{2abBx^4}{4+m}+\frac{(Ab^2+2aAc+2abC)\,x^5}{5+m}+\frac{B(b^2+2ac)\,x^6}{6+m}+\frac{(2Abc+b^2C+2acC)\,x^7}{7+m}+\frac{2bBcx^8}{8+m}+\frac{c(Ac+2bC)x^9}{9+m}+\frac{Bc^2x^{10}}{10+m}+\frac{c^2Cx^{11}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (d*x)^m*((a^2*A*x)/(1 + m) + (a^2*B*x^2)/(2 + m) + (a*(2*A*b + a*C)*x^3)/(3
 + m) + (2*a*b*B*x^4)/(4 + m) + ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^5)/(5 + m) +
 (B*(b^2 + 2*a*c)*x^6)/(6 + m) + ((2*A*b*c + b^2*C + 2*a*c*C)*x^7)/(7 + m)
+ (2*b*B*c*x^8)/(8 + m) + (c*(A*c + 2*b*C)*x^9)/(9 + m) + (B*c^2*x^10)/(10
+ m) + (c^2*C*x^11)/(11 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. 2(260) = 520.
time = 0.02, size = 2187, normalized size = 8.41

method	result	size
gosper	Expression too large to display	2187
risch	Expression too large to display	2187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^2
*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^10
*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m^10
*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m^10*
x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*m^9*x
^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8*x^7+1
67223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m^7*x^8+
902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^8*x^6+17
7765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*m^7*x^7+9
65328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^2*m^8*x^6+
355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61*A*b^2*m^9*
x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+3130*B*a*c*m
^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m^4*x^9+122*C*
a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C*b*c*m^5*x^8+8
409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+1620*A*b^2*m^8*x^
4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*x^3+46560*B*a*c*
m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*B*c^2*m^3*x^9+C*a
^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+202821*C*b^2*m^6*x^6+80
00956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^9*x^2+49140*A*a*c*m
^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+9991428*A*c^2*m^3*x^8+B*a^
```

```
2*m^10*x+3354*B*a*b*m^8*x^3+435486*B*a*c*m^6*x^5+217743*B*b^2*m^6*x^5+87427
18*B*b*c*m^4*x^7+13926276*B*c^2*m^2*x^9+63*C*a^2*m^9*x^2+49140*C*a*b*m^7*x^
4+2435622*C*a*c*m^5*x^6+1217811*C*b^2*m^5*x^6+19982856*C*b*c*m^3*x^8+106286
40*C*c^2*m*x^10+A*a^2*m^10+3472*A*a*b*m^8*x^2+469146*A*a*c*m^6*x^4+234573*A
*b^2*m^6*x^4+9629716*A*b*c*m^4*x^6+15335224*A*c^2*m^2*x^8+64*B*a^2*m^9*x+51
924*B*a*b*m^7*x^3+2662200*B*a*c*m^5*x^5+1331100*B*b^2*m^5*x^5+22049716*B*b*
c*m^3*x^7+11655216*B*c^2*m*x^9+1736*C*a^2*m^8*x^2+469146*C*a*b*m^6*x^4+9629
716*C*a*c*m^4*x^6+4814858*C*b^2*m^4*x^6+30670448*C*b*c*m^2*x^8+3628800*C*c^
2*x^10+65*A*a^2*m^9+54924*A*a*b*m^7*x^2+2929386*A*a*c*m^5*x^4+1464693*A*b^2
*m^5*x^4+24583448*A*b*c*m^3*x^6+12900960*A*c^2*m*x^8+1797*B*a^2*m^8*x+50715
0*B*a*b*m^6*x^3+10705870*B*a*c*m^4*x^5+5352935*B*b^2*m^4*x^5+34118424*B*b*c
*m^2*x^7+3991680*B*c^2*x^9+27462*C*a^2*m^7*x^2+2929386*C*a*b*m^5*x^4+245834
48*C*a*c*m^3*x^6+12291724*C*b^2*m^3*x^6+25801920*C*b*c*m*x^8+1860*A*a^2*m^8
+550074*A*a*b*m^6*x^2+12032140*A*a*c*m^4*x^4+6016070*A*b^2*m^4*x^4+38432016
*A*b*c*m^2*x^6+4435200*A*c^2*x^8+29076*B*a^2*m^7*x+3246516*B*a*b*m^5*x^3+27
756240*B*a*c*m^3*x^5+13878120*B*b^2*m^3*x^5+28888560*B*b*c*m*x^7+275037*C*a
^2*m^6*x^2+12032140*C*a*b*m^4*x^4+38432016*C*a*c*m^2*x^6+19216008*C*b^2*m^2
*x^6+8870400*C*b*c*x^8+30810*A*a^2*m^7+3624894*A*a*b*m^5*x^2+31830760*A*a*c
*m^3*x^4+15915380*A*b^2*m^3*x^4+32811840*A*b*c*m*x^6+299271*B*a^2*m^6*x+136
93006*B*a*b*m^4*x^3+43978712*B*a*c*m^2*x^5+21989356*B*b^2*m^2*x^5+9979200*B
*b*c*x^7+1812447*C*a^2*m^5*x^2+31830760*C*a*b*m^3*x^4+32811840*C*a*c*m*x^6+
16405920*C*b^2*m*x^6+326613*A*a^2*m^6+15804388*A*a*b*m^4*x^2+51362352*A*a*c
*m^2*x^4+25681176*A*b^2*m^2*x^4+11404800*A*b*c*x^6+2039016*B*a^2*m^5*x+3721
9436*B*a*b*m^3*x^3+37963680*B*a*c*m*x^5+18981840*B*b^2*m*x^5+7902194*C*a^2*
m^4*x^2+51362352*C*a*b*m^2*x^4+11404800*C*a*c*x^6+5702400*C*b^2*x^6+2310945
*A*a^2*m^5+44578296*A*a*b*m^3*x^2+45024192*A*a*c*m*x^4+22512096*A*b^2*m*x^4
+9261503*B*a^2*m^4*x+61638408*B*a*b*m^2*x^3+13305600*B*a*c*x^5+6652800*B*b^
2*x^5+22289148*C*a^2*m^3*x^2+45024192*C*a*b*m*x^4+11028590*A*a^2*m^4+767812
64*A*a*b*m^2*x^2+15966720*A*a*c*x^4+7983360*A*b^2*x^4+27472724*B*a^2*m^3*x+
55282320*B*a*b*m*x^3+38390632*C*a^2*m^2*x^2+15966720*C*a*b*x^4+34967140*A*a
^2*m^3+71492160*A*a*b*m*x^2+50312628*B*a^2*m^2*x+19958400*B*a*b*x^3+3574608
0*C*a^2*m*x^2+70290936*A*a^2*m^2+26611200*A*a*b*x^2+50292720*B*a^2*m*x+1330
5600*C*a^2*x^2+80627040*A*a^2*m+19958400*B*a^2*x+39916800*A*a^2)*(d*x)^m/(1
1+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [A]
time = 0.34, size = 344, normalized size = 1.32

$$\frac{Cc^2d^mx^{11}x^m}{m+11}+\frac{Bc^2d^mx^{10}x^m}{m+10}+\frac{2\,Cbcd^mx^9x^m}{m+9}+\frac{Ac^2d^mx^9x^m}{m+9}+\frac{2\,Bbcd^mx^8x^m}{m+8}+\frac{Cb^2d^mx^7x^m}{m+7}+\frac{2\,Cacd^mx^7x^m}{m+7}+\frac{2\,Abcd^mx^7x^m}{m+7}+\frac{Bb^2d^mx^6x^m}{m+6}+\frac{2\,Bacd^mx^6x^m}{m+6}+\frac{2\,Cabd^mx^5x^m}{m+5}+\frac{Ab^2d^mx^5x^m}{m+5}+\frac{2\,Aacd^mx^5x^m}{m+5}+\frac{2\,Babd^mx^4x^m}{m+4}+\frac{Ca^2d^mx^3x^m}{m+3}+\frac{2\,Aabd^mx^3x^m}{m+3}+\frac{Ba^2d^mx^2x^m}{m+2}+\frac{(dx)^{m+1}\,Aa^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*c^2*d^m*x^11*x^m/(m + 11) + B*c^2*d^m*x^10*x^m/(m + 10) + 2*C*b*c*d^m*x^9
*x^m/(m + 9) + A*c^2*d^m*x^9*x^m/(m + 9) + 2*B*b*c*d^m*x^8*x^m/(m + 8) + C*
```

```
b^2*d^m*x^7*x^m/(m + 7) + 2*C*a*c*d^m*x^7*x^m/(m + 7) + 2*A*b*c*d^m*x^7*x^m
/(m + 7) + B*b^2*d^m*x^6*x^m/(m + 6) + 2*B*a*c*d^m*x^6*x^m/(m + 6) + 2*C*a*
b*d^m*x^5*x^m/(m + 5) + A*b^2*d^m*x^5*x^m/(m + 5) + 2*A*a*c*d^m*x^5*x^m/(m
+ 5) + 2*B*a*b*d^m*x^4*x^m/(m + 4) + C*a^2*d^m*x^3*x^m/(m + 3) + 2*A*a*b*d^
m*x^3*x^m/(m + 3) + B*a^2*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^2/(d*(m +
 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(260) = 520$.
time = 0.41, size = 1603, normalized size = 6.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*c
^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 1275357
6*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*B*c
^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B*c^2
*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 1165521
6*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b*c + A
*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7 + 1777
65*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*(2*C*b*c
 + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 4435200*A*c
^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*m)*x^9 + 2
*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 + 189567*B*b
*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c*m^3 + 17059
212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^2 + 2*(C*a +
A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2 + 2*(C*a + A*
b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*b^2 + 2*(C*a +
A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 4814858*(C*b^2 + 2*(C
*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3 + 5702400*C*b^2 +
 19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a + A*b)*c + 16405920
*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m^10 + 60*(B*b^2 + 2
*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*b^2 + 2*B*a*c)*m^7 + 21
7743*(B*b^2 + 2*B*a*c)*m^6 + 1331100*(B*b^2 + 2*B*a*c)*m^5 + 5352935*(B*b^2
 + 2*B*a*c)*m^4 + 13878120*(B*b^2 + 2*B*a*c)*m^3 + 6652800*B*b^2 + 13305600
*B*a*c + 21989356*(B*b^2 + 2*B*a*c)*m^2 + 18981840*(B*b^2 + 2*B*a*c)*m)*x^6
 + ((2*C*a*b + A*b^2 + 2*A*a*c)*m^10 + 61*(2*C*a*b + A*b^2 + 2*A*a*c)*m^9 +
 1620*(2*C*a*b + A*b^2 + 2*A*a*c)*m^8 + 24570*(2*C*a*b + A*b^2 + 2*A*a*c)*m
^7 + 234573*(2*C*a*b + A*b^2 + 2*A*a*c)*m^6 + 1464693*(2*C*a*b + A*b^2 + 2*
A*a*c)*m^5 + 6016070*(2*C*a*b + A*b^2 + 2*A*a*c)*m^4 + 15915380*(2*C*a*b +
A*b^2 + 2*A*a*c)*m^3 + 15966720*C*a*b + 7983360*A*b^2 + 15966720*A*a*c + 25
681176*(2*C*a*b + A*b^2 + 2*A*a*c)*m^2 + 22512096*(2*C*a*b + A*b^2 + 2*A*a*
```

```
c)*m)*x^5 + 2*(B*a*b*m^10 + 62*B*a*b*m^9 + 1677*B*a*b*m^8 + 25962*B*a*b*m^7
 + 253575*B*a*b*m^6 + 1623258*B*a*b*m^5 + 6846503*B*a*b*m^4 + 18609718*B*a*
b*m^3 + 30819204*B*a*b*m^2 + 27641160*B*a*b*m + 9979200*B*a*b)*x^4 + ((C*a^
2 + 2*A*a*b)*m^10 + 63*(C*a^2 + 2*A*a*b)*m^9 + 1736*(C*a^2 + 2*A*a*b)*m^8 +
 27462*(C*a^2 + 2*A*a*b)*m^7 + 275037*(C*a^2 + 2*A*a*b)*m^6 + 1812447*(C*a^
2 + 2*A*a*b)*m^5 + 7902194*(C*a^2 + 2*A*a*b)*m^4 + 22289148*(C*a^2 + 2*A*a*
b)*m^3 + 13305600*C*a^2 + 26611200*A*a*b + 38390632*(C*a^2 + 2*A*a*b)*m^2 +
 35746080*(C*a^2 + 2*A*a*b)*m)*x^3 + (B*a^2*m^10 + 64*B*a^2*m^9 + 1797*B*a^
2*m^8 + 29076*B*a^2*m^7 + 299271*B*a^2*m^6 + 2039016*B*a^2*m^5 + 9261503*B*
a^2*m^4 + 27472724*B*a^2*m^3 + 50312628*B*a^2*m^2 + 50292720*B*a^2*m + 1995
8400*B*a^2)*x^2 + (A*a^2*m^10 + 65*A*a^2*m^9 + 1860*A*a^2*m^8 + 30810*A*a^2
*m^7 + 326613*A*a^2*m^6 + 2310945*A*a^2*m^5 + 11028590*A*a^2*m^4 + 34967140
*A*a^2*m^3 + 70290936*A*a^2*m^2 + 80627040*A*a^2*m + 39916800*A*a^2)*x)*(d*
x)^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13
339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 3
9916800)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 16323 vs. $2(245) = 490$.
time = 1.47, size = 16323, normalized size = 62.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-A*a**2/(10*x**10) - A*a*b/(4*x**8) - A*a*c/(3*x**6) - A*b**2/(
6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*a**2/(9*x**9) - 2*B*a*b/(7*x
**7) - 2*B*a*c/(5*x**5) - B*b**2/(5*x**5) - 2*B*b*c/(3*x**3) - B*c**2/x - C
*a**2/(8*x**8) - C*a*b/(3*x**6) - C*a*c/(2*x**4) - C*b**2/(4*x**4) - C*b*c/
x**2 + C*c**2*log(x))/d**11, Eq(m, -11)), ((-A*a**2/(9*x**9) - 2*A*a*b/(7*x
**7) - 2*A*a*c/(5*x**5) - A*b**2/(5*x**5) - 2*A*b*c/(3*x**3) - A*c**2/x - B
*a**2/(8*x**8) - B*a*b/(3*x**6) - B*a*c/(2*x**4) - B*b**2/(4*x**4) - B*b*c/
x**2 + B*c**2*log(x) - C*a**2/(7*x**7) - 2*C*a*b/(5*x**5) - 2*C*a*c/(3*x**3
) - C*b**2/(3*x**3) - 2*C*b*c/x + C*c**2*x)/d**10, Eq(m, -10)), ((-A*a**2/(
8*x**8) - A*a*b/(3*x**6) - A*a*c/(2*x**4) - A*b**2/(4*x**4) - A*b*c/x**2 +
A*c**2*log(x) - B*a**2/(7*x**7) - 2*B*a*b/(5*x**5) - 2*B*a*c/(3*x**3) - B*b
**2/(3*x**3) - 2*B*b*c/x + B*c**2*x - C*a**2/(6*x**6) - C*a*b/(2*x**4) - C*
a*c/x**2 - C*b**2/(2*x**2) + 2*C*b*c*log(x) + C*c**2*x**2/2)/d**9, Eq(m, -9
)), ((-A*a**2/(7*x**7) - 2*A*a*b/(5*x**5) - 2*A*a*c/(3*x**3) - A*b**2/(3*x*
*3) - 2*A*b*c/x + A*c**2*x - B*a**2/(6*x**6) - B*a*b/(2*x**4) - B*a*c/x**2
- B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2 - C*a**2/(5*x**5) - 2*C*
a*b/(3*x**3) - 2*C*a*c/x - C*b**2/x + 2*C*b*c*x + C*c**2*x**3/3)/d**8, Eq(m
, -8)), ((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*a*c/x**2 - A*b**2/(2*x**2)
+ 2*A*b*c*log(x) + A*c**2*x**2/2 - B*a**2/(5*x**5) - 2*B*a*b/(3*x**3) - 2*B
```

```
*a*c/x - B*b**2/x + 2*B*b*c*x + B*c**2*x**3/3 - C*a**2/(4*x**4) - C*a*b/x**
2 + 2*C*a*c*log(x) + C*b**2*log(x) + C*b*c*x**2 + C*c**2*x**4/4)/d**7, Eq(m
, -7)), ((-A*a**2/(5*x**5) - 2*A*a*b/(3*x**3) - 2*A*a*c/x - A*b**2/x + 2*A*
b*c*x + A*c**2*x**3/3 - B*a**2/(4*x**4) - B*a*b/x**2 + 2*B*a*c*log(x) + B*b
**2*log(x) + B*b*c*x**2 + B*c**2*x**4/4 - C*a**2/(3*x**3) - 2*C*a*b/x + 2*C
*a*c*x + C*b**2*x + 2*C*b*c*x**3/3 + C*c**2*x**5/5)/d**6, Eq(m, -6)), ((-A*
a**2/(4*x**4) - A*a*b/x**2 + 2*A*a*c*log(x) + A*b**2*log(x) + A*b*c*x**2 +
A*c**2*x**4/4 - B*a**2/(3*x**3) - 2*B*a*b/x + 2*B*a*c*x + B*b**2*x + 2*B*b*
c*x**3/3 + B*c**2*x**5/5 - C*a**2/(2*x**2) + 2*C*a*b*log(x) + C*a*c*x**2 +
C*b**2*x**2/2 + C*b*c*x**4/2 + C*c**2*x**6/6)/d**5, Eq(m, -5)), ((-A*a**2/(
3*x**3) - 2*A*a*b/x + 2*A*a*c*x + A*b**2*x + 2*A*b*c*x**3/3 + A*c**2*x**5/5
 - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*a*c*x**2 + B*b**2*x**2/2 + B*b*c*x*
*4/2 + B*c**2*x**6/6 - C*a**2/x + 2*C*a*b*x + 2*C*a*c*x**3/3 + C*b**2*x**3/
3 + 2*C*b*c*x**5/5 + C*c**2*x**7/7)/d**4, Eq(m, -4)), ((-A*a**2/(2*x**2) +
2*A*a*b*log(x) + A*a*c*x**2 + A*b**2*x**2/2 + A*b*c*x**4/2 + A*c**2*x**6/6
- B*a**2/x + 2*B*a*b*x + 2*B*a*c*x**3/3 + B*b**2*x**3/3 + 2*B*b*c*x**5/5 +
B*c**2*x**7/7 + C*a**2*log(x) + C*a*b*x**2 + C*a*c*x**4/2 + C*b**2*x**4/4 +
 C*b*c*x**6/3 + C*c**2*x**8/8)/d**3, Eq(m, -3)), ((-A*a**2/x + 2*A*a*b*x +
2*A*a*c*x**3/3 + A*b**2*x**3/3 + 2*A*b*c*x**5/5 + A*c**2*x**7/7 + B*a**2*lo
g(x) + B*a*b*x**2 + B*a*c*x**4/2 + B*b**2*x**4/4 + B*b*c*x**6/3 + B*c**2*x*
*8/8 + C*a**2*x + 2*C*a*b*x**3/3 + 2*C*a*c*x**5/5 + C*b**2*x**5/5 + 2*C*b*c
*x**7/7 + C*c**2*x**9/9)/d**2, Eq(m, -2)), ((A*a**2*log(x) + A*a*b*x**2 + A
*a*c*x**4/2 + A*b**2*x**4/4 + A*b*c*x**6/3 + A*c**2*x**8/8 + B*a**2*x + 2*B
*a*b*x**3/3 + 2*B*a*c*x**5/5 + B*b**2*x**5/5 + 2*B*b*c*x**7/7 + B*c**2*x**9
/9 + C*a**2*x**2/2 + C*a*b*x**4/2 + C*a*c*x**6/3 + C*b**2*x**6/6 + C*b*c*x*
*8/4 + C*c**2*x**10/10)/d, Eq(m, -1)), (A*a**2*m**10*x*(d*x)**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 65*A*a**2*m**9*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 1860*A*a**2*m**8*x*(d*x)**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 30810*A*a**2*m**7*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 326613*A*a**2*
m**6*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 2310945*A*a**2*m**5*x*(d*x)**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 11028590*A*a**2*m**4*x*(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m
**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525807
6*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*A*a**2*m**3*x*
(d*x)**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558
```

```
*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1
20543840*m + 39916800) + 70290936*A*a**2*m**2*x*(d*x)**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 263755...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3203 vs. $2(260) = 520$.
time = 5.22, size = 3203, normalized size = 12.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*x
^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^2*
m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*(d*
x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*x^10
 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C*a*c*
m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 2824*(d*x
)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2*m^7*x^
10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*x)^m*B*a
*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7 + 118*(d
*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*C*b*c*m^7*
x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^10 + 902055*
(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b^2*m^10*x^5
+ 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d*x)^m*B*a*c*m
^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*x^7 + 3024*(d*
x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d*x)^m*C*b*c*m^6
*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*m^5*x^10 + 34169
30*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122*(d*x)^m*C*a*b*m^
9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*x^5 + 1565*(d*x)^m
*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x)^m*C*b^2*m^7*x^7 +
 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m^7*x^7 + 379134*(d*x)^m
*B*b*c*m^6*x^8 + 2075346*(d*x)^m*C*b*c*m^5*x^9 + 1037673*(d*x)^m*A*c^2*m^5*
x^9 + 3686255*(d*x)^m*B*c^2*m^4*x^10 + 8409500*(d*x)^m*C*c^2*m^3*x^11 + (d*
x)^m*C*a^2*m^10*x^3 + 2*(d*x)^m*A*a*b*m^10*x^3 + 124*(d*x)^m*B*a*b*m^9*x^4
+ 3240*(d*x)^m*C*a*b*m^8*x^5 + 1620*(d*x)^m*A*b^2*m^8*x^5 + 3240*(d*x)^m*A*
a*c*m^8*x^5 + 23280*(d*x)^m*B*b^2*m^7*x^6 + 46560*(d*x)^m*B*a*c*m^7*x^6 + 2
02821*(d*x)^m*C*b^2*m^6*x^7 + 405642*(d*x)^m*C*a*c*m^6*x^7 + 405642*(d*x)^m
*A*b*c*m^6*x^7 + 2242044*(d*x)^m*B*b*c*m^5*x^8 + 8000956*(d*x)^m*C*b*c*m^4*
x^9 + 4000478*(d*x)^m*A*c^2*m^4*x^9 + 9133180*(d*x)^m*B*c^2*m^3*x^10 + 1275
3576*(d*x)^m*C*c^2*m^2*x^11 + (d*x)^m*B*a^2*m^10*x^2 + 63*(d*x)^m*C*a^2*m^9
*x^3 + 126*(d*x)^m*A*a*b*m^9*x^3 + 3354*(d*x)^m*B*a*b*m^8*x^4 + 49140*(d*x)
^m*C*a*b*m^7*x^5 + 24570*(d*x)^m*A*b^2*m^7*x^5 + 49140*(d*x)^m*A*a*c*m^7*x^
5 + 217743*(d*x)^m*B*b^2*m^6*x^6 + 435486*(d*x)^m*B*a*c*m^6*x^6 + 1217811*(
```

```
d*x)^m*C*b^2*m^5*x^7 + 2435622*(d*x)^m*C*a*c*m^5*x^7 + 2435622*(d*x)^m*A*b*
c*m^5*x^7 + 8742718*(d*x)^m*B*b*c*m^4*x^8 + 19982856*(d*x)^m*C*b*c*m^3*x^9
+ 9991428*(d*x)^m*A*c^2*m^3*x^9 + 13926276*(d*x)^m*B*c^2*m^2*x^10 + 1062864
0*(d*x)^m*C*c^2*m*x^11 + (d*x)^m*A*a^2*m^10*x + 64*(d*x)^m*B*a^2*m^9*x^2 +
1736*(d*x)^m*C*a^2*m^8*x^3 + 3472*(d*x)^m*A*a*b*m^8*x^3 + 51924*(d*x)^m*B*a
*b*m^7*x^4 + 469146*(d*x)^m*C*a*b*m^6*x^5 + 234573*(d*x)^m*A*b^2*m^6*x^5 +
469146*(d*x)^m*A*a*c*m^6*x^5 + 1331100*(d*x)^m*B*b^2*m^5*x^6 + 2662200*(d*x
)^m*B*a*c*m^5*x^6 + 4814858*(d*x)^m*C*b^2*m^4*x^7 + 9629716*(d*x)^m*C*a*c*m
^4*x^7 + 9629716*(d*x)^m*A*b*c*m^4*x^7 + 22049716*(d*x)^m*B*b*c*m^3*x^8 + 3
0670448*(d*x)^m*C*b*c*m^2*x^9 + 15335224*(d*x)^m*A*c^2*m^2*x^9 + 11655216*(
d*x)^m*B*c^2*m*x^10 + 3628800*(d*x)^m*C*c^2*x^11 + 65*(d*x)^m*A*a^2*m^9*x +
 1797*(d*x)^m*B*a^2*m^8*x^2 + 27462*(d*x)^m*C*a^2*m^7*x^3 + 54924*(d*x)^m*A
*a*b*m^7*x^3 + 507150*(d*x)^m*B*a*b*m^6*x^4 + 2929386*(d*x)^m*C*a*b*m^5*x^5
 + 1464693*(d*x)^m*A*b^2*m^5*x^5 + 2929386*(d*x)^m*A*a*c*m^5*x^5 + 5352935*
(d*x)^m*B*b^2*m^4*x^6 + 10705870*(d*x)^m*B*a*c*m^4*x^6 + 12291724*(d*x)^m*C
*b^2*m^3*x^7 + 24583448*(d*x)^m*C*a*c*m^3*x^7 + 24583448*(d*x)^m*A*b*c*m^3*
x^7 + 34118424*(d*x)^m*B*b*c*m^2*x^8 + 25801920*(d*x)^m*C*b*c*m*x^9 + 12900
960*(d*x)^m*A*c^2*m*x^9 + 3991680*(d*x)^m*B*c^2*x^10 + 1860*(d*x)^m*A*a^2*m
^8*x + 29076*(d*x)^m*B*a^2*m^7*x^2 + 275037*(d*x)^m*C*a^2*m^6*x^3 + 550074*
(d*x)^m*A*a*b*m^6*x^3 + 3246516*(d*x)^m*B*a*b*m^5*x^4 + 12032140*(d*x)^m*C*
a*b*m^4*x^5 + 6016070*(d*x)^m*A*b^2*m^4*x^5 + 12032140*(d*x)^m*A*a*c*m^4*x^
5 + 13878120*(d*x)^m*B*b^2*m^3*x^6 + 27756240*(d*x)^m*B*a*c*m^3*x^6 + 19216
008*(d*x)^m*C*b^2*m^2*x^7 + 38432016*(d*x)^m*C*a*c*m^2*x^7 + 38432016*(d*x)
^m*A*b*c*m^2*x^7 + 28888560*(d*x)^m*B*b*c*m*x^8 + 8870400*(d*x)^m*C*b*c*x^9
 + 4435200*(d*x)^m*A*c^2*x^9 + 30810*(d*x)^m*A*a^2*m^7*x + 299271*(d*x)^m*B
*a^2*m^6*x^2 + 1812447*(d*x)^m*C*a^2*m^5*x^3 + 3624894*(d*x)^m*A*a*b*m^5*x^
3 + 13693006*(d*x)^m*B*a*b*m^4*x^4 + 31830760*(d*x)^m*C*a*b*m^3*x^5 + 15915
380*(d*x)^m*A*b^2*m^3*x^5 + 31830760*(d*x)^m*A*a*c*m^3*x^5 + 21989356*(d*x)
^m*B*b^2*m^2*x^6 + 43978712*(d*x)^m*B*a*c*m^2*x^6 + 16405920*(d*x)^m*C*b^2*
m*x^7 + 32811840*(d*x)^m*C*a*c*m*x^7 + 32811840*(d*x)^m*A*b*c*m*x^7 + 99792
00*(d*x)^m*B*b*c*x^8 + 326613*(d*x)^m*A*a^2*m^6*x + 2039016*(d*x)^m*B*a^2*m
^5*x^2 + 7902194*(d*x)^m*C*a^2*m^4*x^3 + 15804388*(d*x)^m*A*a*b*m^4*x^3 + 3
7219436*(d*x)^m*B*a*b*m^3*x^4 + 51362352*(d*x)^m*C*a*b*m^2*x^5 + 25681176*(
d*x)^m*A*b^2*m^2*x^5 + 51362352*(d*x)^m*A*a*c*m^2*x^5 + 18981840*(d*x)^m*B*
b^2*m*x^6 + 37963680*(d*x)^m*B*a*c*m*x^6 + 5702...
```

Mupad [B]
time = 1.81, size = 1314, normalized size = 5.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(d*x)^m*(A*b^2 + 2*A*a*c + 2*C*a*b)*(22512096*m + 25681176*m^2 + 15915
380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m^6 + 24570*m^7 + 1620*m^8 + 6
```

```
1*m^9 + m^10 + 7983360))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 459
95730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9
+ 66*m^10 + m^11 + 39916800) + (x^7*(d*x)^m*(C*b^2 + 2*A*b*c + 2*C*a*c)*(16
405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821
*m^6 + 22086*m^7 + 1512*m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 1509
17976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357
423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*x^6*(d*x)^
m*(2*a*c + b^2)*(18981840*m + 21989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1
331100*m^5 + 217743*m^6 + 23280*m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/
(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
+ 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 399168
00) + (A*a^2*x*(d*x)^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590
*m^4 + 2310945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39
916800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1333
9535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
 + 39916800) + (c*x^9*(d*x)^m*(A*c + 2*C*b)*(12900960*m + 15335224*m^2 + 99
91428*m^3 + 4000478*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 +
 57*m^9 + m^10 + 4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4
5995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^
9 + 66*m^10 + m^11 + 39916800) + (a*x^3*(d*x)^m*(2*A*b + C*a)*(35746080*m +
 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1812447*m^5 + 275037*m^6 + 274
62*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600))/(120543840*m + 150917976*m^2
 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 +
 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*c^2*x^10*(d*x)^m*(1
1655216*m + 13926276*m^2 + 9133180*m^3 + 3686255*m^4 + 965328*m^5 + 167223*
m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 3991680))/(120543840*m + 15091
7976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 3574
23*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (C*c^2*x^11*(d
*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 + 902055*m^5 +
 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))/(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^
6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*a^2
*x^2*(d*x)^m*(50292720*m + 50312628*m^2 + 27472724*m^3 + 9261503*m^4 + 2039
016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m^10 + 19958400))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (2*B*b*c*x^8*(d*x)^m*(14444280*m + 17059212*m^2 + 11024858*m^3 + 437135
9*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^9 + m^10 + 4
989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1333
9535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
 + 39916800) + (2*B*a*b*x^4*(d*x)^m*(27641160*m + 30819204*m^2 + 18609718*m
^3 + 6846503*m^4 + 1623258*m^5 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9
 + m^10 + 9979200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730
*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*
m^10 + m^11 + 39916800)
```

3.39 $\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=137

$$\frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{bB(dx)^{4+m}}{d^4(4+m)}+\frac{(Ac+bC)(dx)^{5+m}}{d^5(5+m)}+\frac{Bc(dx)^{6+m}}{d^6(6+m)}+\frac{cC(dx)^{7+m}}{d^7(7+m)}$$

```
[Out] a*A*(d*x)^(1+m)/d/(1+m)+a*B*(d*x)^(2+m)/d^2/(2+m)+(A*b+C*a)*(d*x)^(3+m)/d^3
/(3+m)+b*B*(d*x)^(4+m)/d^4/(4+m)+(A*c+C*b)*(d*x)^(5+m)/d^5/(5+m)+B*c*(d*x)^
(6+m)/d^6/(6+m)+c*C*(d*x)^(7+m)/d^7/(7+m)
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1642}

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*b
 + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m)) +
((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6 + m
)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(aA(dx)^m+\frac{aB(dx)^{1+m}}{d}+\frac{(Ab+aC)(dx)^{2+m}}{d^2}+\frac{bB(dx\right.\\ &\quad\;\; d^3\\ &= \frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{bB(dx)^4}{d^4(4+}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 92, normalized size = 0.67

$$(dx)^m\left(\frac{aAx}{1+m}+\frac{aBx^2}{2+m}+\frac{(Ab+aC)x^3}{3+m}+\frac{bBx^4}{4+m}+\frac{(Ac+bC)x^5}{5+m}+\frac{Bcx^6}{6+m}+\frac{cCx^7}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*x)^m*((a*A*x)/(1 + m) + (a*B*x^2)/(2 + m) + ((A*b + a*C)*x^3)/(3 + m) +
(b*B*x^4)/(4 + m) + ((A*c + b*C)*x^5)/(5 + m) + (B*c*x^6)/(6 + m) + (c*C*x^
7)/(7 + m))
```

Maple [A]
time = 0.02, size = 136, normalized size = 0.99

method	result
norman	$\frac{(Ab+aC)x^3 e^{m\ln(dx)}}{3+m} + \frac{(Ac+bC)x^5 e^{m\ln(dx)}}{5+m} + \frac{aAx\, e^{m\ln(dx)}}{1+m} + \frac{aB\,x^2 e^{m\ln(dx)}}{2+m} + \frac{bB\,x^4 e^{m\ln(dx)}}{4+m} + \frac{cB\,x^6 e^{m\ln(dx)}}{6+m} + cC\,x$
gosper	$x(Ccm^6x^6+Bcm^6x^5+21Ccm^5x^6+Acm^6x^4+22Bcm^5x^5+Cbm^6x^4+175Ccm^4x^6+23Acm^5x^4+Bbm^6x^3+190Bcm^4x^5+23Cbm$
risch	$x(Ccm^6x^6+Bcm^6x^5+21Ccm^5x^6+Acm^6x^4+22Bcm^5x^5+Cbm^6x^4+175Ccm^4x^6+23Acm^5x^4+Bbm^6x^3+190Bcm^4x^5+23Cbm$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (A*b+C*a)/(3+m)*x^3*exp(m*ln(d*x))+(A*c+C*b)/(5+m)*x^5*exp(m*ln(d*x))+a*A/(
1+m)*x*exp(m*ln(d*x))+a*B/(2+m)*x^2*exp(m*ln(d*x))+b*B/(4+m)*x^4*exp(m*ln(d
*x))+c*B/(6+m)*x^6*exp(m*ln(d*x))+c*C/(7+m)*x^7*exp(m*ln(d*x))
```

Maxima [A]
time = 0.31, size = 155, normalized size = 1.13

$$\frac{Ccd^mx^7x^m}{m+7} + \frac{Bcd^mx^6x^m}{m+6} + \frac{Cbd^mx^5x^m}{m+5} + \frac{Acd^mx^5x^m}{m+5} + \frac{Bbd^mx^4x^m}{m+4} + \frac{Cad^mx^3x^m}{m+3} + \frac{Abd^mx^3x^m}{m+3} + \frac{Bad^mx^2x^m}{m+2} + \frac{(dx)^{m+1}Aa}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] C*c*d^m*x^7*x^m/(m + 7) + B*c*d^m*x^6*x^m/(m + 6) + C*b*d^m*x^5*x^m/(m + 5)
 + A*c*d^m*x^5*x^m/(m + 5) + B*b*d^m*x^4*x^m/(m + 4) + C*a*d^m*x^3*x^m/(m +
 3) + A*b*d^m*x^3*x^m/(m + 3) + B*a*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a
/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(137) = 274$.
time = 0.38, size = 444, normalized size = 3.24

$$\frac{\left((Ccm^6 + 21\,Ccm^5 + 175\,Ccm^4 + 735\,Ccm^3 + 1624\,Ccm^2 + 1764\,Ccm + 720\,Cc)x^7 + (Bcm^6 + 22\,Bcm^5 + 190\,Bcm^4 + 820\,Bcm^3 + 1849\,Bcm^2 + 2038\,Bcm + 840\,Bc)x^6 + ((Cb+Ac)m^6 + 23\,(Cb+Ac)m^5 + 207\,(Cb+Ac)m^4 + 925\,(Cb+Ac)m^3 + 2144\,(Cb+Ac)m^2 + 1008\,Cb + 1008\,Ac + 2412\,(Cb+Ac)m)x^5 + (Bbm^6 + 24\,Bbm^5 + 226\,Bbm^4 + 1056\,Bbm^3 + 2545\,Bbm^2 + 2952\,Bbm + 1260\,Bb)x^4 + ((Ca+Ab)m^6 + 25\,(Ca+Ab)m^5 + 247\,(Ca+Ab)m^4 + 1219\,(Ca+Ab)m^3 + 3112\,(Ca+Ab)m^2 + 1680\,Ca + 1680\,Ab + 3796\,(Ca+Ab)m)x^3 + (Bam^6 + 26\,Bam^5 + 270\,Bam^4 + 1420\,Bam^3 + 3929\,Bam^2 + 5274\,Bam + 2520\,Ba)x^2 + (Aam^6 + 27\,Aam^5 + 295\,Aam^4 + 1665\,Aam^3 + 5104\,Aam^2 + 8028\,Aam + 5040\,Aa)x\right)(dx)^m}{m^7 + 28\,m^6 + 322\,m^5 + 1960\,m^4 + 6769\,m^3 + 13132\,m^2 + 13068\,m + 5040}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C*
c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 + 18
49*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*c)*
m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2 + 10
08*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 + 226*B
*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + ((C*a +
 A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A*b)*m^3
 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)*x^3 + (
B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 + 5274*B*a
*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A*a*m^3 + 5
104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 + 322*m^5 + 1
960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3628 vs. $2(122) = 244$.

time = 0.58, size = 3628, normalized size = 26.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B*
b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m,
-7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**2)
+ B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(4*x*
*4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(2*x**
2) + C*b*log(x) + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x + A
*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*x**3/3)
/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/x + B*b*
x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m, -3)), ((
-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*a*x
+ C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*x**2/2 + A*
c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x**4/4 + C*c*
x**6/6)/d, Eq(m, -1)), (A*a*m**6*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*m**5*x*(d*x)**m
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) + 295*A*a*m**4*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*m**3*x*(d*x)**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
+ 5104*A*a*m**2*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m*
*3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*m*x*(d*x)**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 5040*A*a
*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + A*b*m**6*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 19
```

```
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 25*A*b*m**5*x**3*(d*x)
**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068
*m + 5040) + 247*A*b*m**4*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*A*b*m**3*x**3*(d*x)**
m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 3112*A*b*m**2*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3796*A*b*m*x**3*(d*x)**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) + 1680*A*b*x**3*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769
*m**3 + 13132*m**2 + 13068*m + 5040) + A*c*m**6*x**5*(d*x)**m/(m**7 + 28*m*
*6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23*A
*c*m**5*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 207*A*c*m**4*x**5*(d*x)**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*A*c*
m**3*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 131
32*m**2 + 13068*m + 5040) + 2144*A*c*m**2*x**5*(d*x)**m/(m**7 + 28*m**6 + 3
22*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*A*c*m
*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m
**2 + 13068*m + 5040) + 1008*A*c*x**5*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*a*m**6*x**2*(d*x)
**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068
*m + 5040) + 26*B*a*m**5*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m*
*4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 270*B*a*m**4*x**2*(d*x)**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 1420*B*a*m**3*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3929*B*a*m**2*x**2*(d*x)**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 5274*B*a*m*x**2*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + 2520*B*a*x**2*(d*x)**m/(m**7 + 28
*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B
*b*m**6*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 24*B*b*m**5*x**4*(d*x)**m/(m**7 + 28*m**6 +
322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 226*B*b*m
**4*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 1313
2*m**2 + 13068*m + 5040) + 1056*B*b*m**3*x**4*(d*x)**m/(m**7 + 28*m**6 + 32
2*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2545*B*b*m*
*2*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132
*m**2 + 13068*m + 5040) + 2952*B*b*m*x**4*(d*x)**m/(m**7 + 28*m**6 + 322*m*
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1260*B*b*x**4*(
d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 1
3068*m + 5040) + B*c*m**6*x**6*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 22*B*c*m**5*x**6*(d*x)**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 190*B*c*m**4*x**6*(d*x)**m/(m**7 + 28*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(137) = 274$.
time = 6.07, size = 914, normalized size = 6.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x)
^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)^m
*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m*A*
c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m*C*a
*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*C*b*m
^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*x)^m*C
*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)^m*A*b*
m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(d*x)^m*A
*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (d*x)^m*A*
a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*(d*x)^m*A*
b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5 + 2144*(d*x
)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7 + 27*(d*x)^m
*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^3 + 1219*(d*x
)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b*m*x^5 + 2412*
(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^4*x + 1420*(d*x
)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A*b*m^2*x^3 + 295
2*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*A*c*x^5 + 1665*(d
*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m*C*a*m*x^3 + 3796*
(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*A*a*m^2*x + 5274*(d
*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^m*A*b*x^3 + 8028*(d*x)^
m*A*a*m*x + 2520*(d*x)^m*B*a*x^2 + 5040*(d*x)^m*A*a*x)/(m^7 + 28*m^6 + 322*
m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

Mupad [B]
time = 1.07, size = 527, normalized size = 3.85

$$\frac{x^3\,{\left(d\,x\right)}^m\,\left(A\,b+C\,a\right)\,\left(m^6+25\,m^5+247\,m^4+1219\,m^3+3112\,m^2+3796\,m+1680\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{x^5\,{\left(d\,x\right)}^m\,\left(A\,c+C\,b\right)\,\left(m^6+23\,m^5+207\,m^4+925\,m^3+2144\,m^2+2412\,m+1008\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{A\,a\,x\,{\left(d\,x\right)}^m\,\left(m^6+27\,m^5+295\,m^4+1665\,m^3+5104\,m^2+8028\,m+5040\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{B\,a\,x^2\,{\left(d\,x\right)}^m\,\left(m^6+26\,m^5+270\,m^4+1420\,m^3+3929\,m^2+5274\,m+2520\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{B\,b\,x^4\,{\left(d\,x\right)}^m\,\left(m^6+24\,m^5+226\,m^4+1056\,m^3+2545\,m^2+2952\,m+1260\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{B\,c\,x^6\,{\left(d\,x\right)}^m\,\left(m^6+22\,m^5+190\,m^4+820\,m^3+1849\,m^2+2038\,m+840\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{C\,c\,x^7\,{\left(d\,x\right)}^m\,\left(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (x^3*(d*x)^m*(A*b + C*a)*(3796*m + 3112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5 +
 m^6 + 1680))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6
 + m^7 + 5040) + (x^5*(d*x)^m*(A*c + C*b)*(2412*m + 2144*m^2 + 925*m^3 + 20
7*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 +
322*m^5 + 28*m^6 + m^7 + 5040) + (A*a*x*(d*x)^m*(8028*m + 5104*m^2 + 1665*m
```

```
^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/(13068*m + 13132*m^2 + 6769*m^3 + 1960
*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*a*x^2*(d*x)^m*(5274*m + 3929*m^2
 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6 + 2520))/(13068*m + 13132*m^2 + 6769*m
^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*b*x^4*(d*x)^m*(2952*m +
 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m + 13132*m^2
 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*c*x^6*(d*x)^m*
(2038*m + 1849*m^2 + 820*m^3 + 190*m^4 + 22*m^5 + m^6 + 840))/(13068*m + 13
132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (C*c*x^7*(
d*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068
*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
```

3.40 $\int \frac{(dx)^m(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=368

$$\frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\right.}{\left(b+\sqrt{b^2-4ac}\right)d(1+}$$

```
[Out] (d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))+2*
B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+(d*x)^(1+m)*hype
rgeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(C+(-2*A*
c+C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))-2*B*c*(d*x)^(2+m)
*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d^2/(2+m
)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]

time = 0.43, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1676, 1299, 371, 12, 1145}

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+\frac{2Bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2Bc(dx)^{m+2}{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d^2(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometr
ic2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m)*Hyp
ergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1145

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \frac{\int \frac{B(dx)^{1+m}}{a+bx^2+cx^4}\,dx}{d}+\int \frac{(dx)^m\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right.$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}+\frac{\left(C-\right.}{}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)}+\frac{\left(C-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 1.15, size = 438, normalized size = 1.19

$$\frac{(dx)^m \left(A(2+3m+m^2)\,\text{RootSum}\left[a+b\#1^2+c\#1^4\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1+2c\#1^3}\&\right]+B(2+m)\,\text{RootSum}\left[a+b\#1^2+c\#1^4\&,\frac{mx+{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1+m\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1}{b\#1+2c\#1^3}\&\right]+C\,\text{RootSum}\left[a+b\#1^2+c\#1^4\&,\frac{mx^2+m^2x^2+2mx\#1+m^2x\#1+2\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1^2+3m\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1^2+m^2\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1^2+m\left(\frac{x}{\#1}\right)^{-m}\#1^2}{b\#1+2c\#1^3}\&\right]\right)}{2m(1+m)(2+m)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d*x)^m*(A*(2 + 3*m + m^2)*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2
F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ] +
B*(2 + m)*RootSum[a + b*#1^2 + c*#1^4 & , (m*x + (Hypergeometric2F1[-m, -m,
 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m + (m*Hypergeometric2F1[-m, -m, 1
 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m)/(b*#1 + 2*c*#1^3) & ] + C*RootSum
[a + b*#1^2 + c*#1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hyperg
eometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hype
rgeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hy
pergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1
^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ]))/(2*m*(1 + m)*(2 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m \left(C\,x^2+Bx+A\right)}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.36, size = 32, normalized size = 0.09

$$\text{integral}\left(\frac{(Cx^2+Bx+A)(dx)^m}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m\,(C\,x^2 + B\,x + A)}{c\,x^4 + b\,x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
[Out] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x)
```

3.41 $\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=685

$$\frac{B(dx)^{2+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d^2\left(a+bx^2+cx^4\right)}+\frac{(dx)^{1+m}\left(A(b^2-2ac)-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)}+\frac{c\Big(2aC\Big(2b-\sqrt{b^2-4ac}\,(}{}$$

```
[Out] 1/2*B*(d*x)^(2+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)+1/
2*(d*x)^(1+m)*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/d/(c*
x^4+b*x^2+a)+1/2*B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+
b^2)^(3/2)/d^2/(2+m)/(b+(-4*a*c+b^2)^(1/2))-1/2*B*c*(d*x)^(2+m)*hypergeom([
1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b+
(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))-
1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))*(2*a*C*(2*b+(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*a*c*(3-m)
-b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b+(-4*a*c+b^2)^
(1/2))+1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))*(2*a*C*(2*b-(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*a*
c*(3-m)+b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*
c+b^2)^(1/2))
```

Rubi [A]

time = 1.58, antiderivative size = 670, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1676, 1291, 1299, 371, 12, 1135}

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-(1-m)\sqrt{b^2-4ac}\,(Ab-2aC)-4aAc(3-m)+4abC+Ab^2(1-m)\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(A(b^2-2ac)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{Bc(dx)^{m+2}\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}+\frac{Bc(dx)^{m+2}\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right){}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad^2(m+2)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{B(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*(a + b*x^2
 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
)/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) + (c*(2*a*C*(2*b - Sqrt[b^2 - 4
*a*c]*(1 - m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 -
m)))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (c*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*C)*(1
 - m) - 4*a*A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)) - (B*c*(4*a*c*(2 - m) + b*(b + Sqrt[b^2 - 4*a*
```

```
c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*
d^2*(2 + m)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{\int \frac{B(dx)^{1+m}}{(a+bx^2+cx^4)^2}\,dx}{d} + \int \frac{(dx)^m (A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{(dx)^{1+m} (A(b^2-2ac)-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)\,d\,(a+bx^2+cx^4)} - \frac{\int \frac{(dx)^m (-Ab^2(1-m)+2aAc(3-\cdots}{a-\cdots}}{2a\,(\cdots}$$

$$= \frac{B(dx)^{2+m} (b^2-2ac+bcx^2)}{2a(b^2-4ac)\,d^2\,(a+bx^2+cx^4)} + \frac{(dx)^{1+m} (A(b^2-2ac)-abC+c(Ab-2a\cdots}{2a(b^2-4ac)\,d\,(a+bx^2+cx^4)}$$

$$= \frac{B(dx)^{2+m} (b^2-2ac+bcx^2)}{2a(b^2-4ac)\,d^2\,(a+bx^2+cx^4)} + \frac{(dx)^{1+m} (A(b^2-2ac)-abC+c(Ab-2a\cdots}{2a(b^2-4ac)\,d\,(a+bx^2+cx^4)}$$

$$= \frac{B(dx)^{2+m} (b^2-2ac+bcx^2)}{2a(b^2-4ac)\,d^2\,(a+bx^2+cx^4)} + \frac{(dx)^{1+m} (A(b^2-2ac)-abC+c(Ab-2a\cdots}{2a(b^2-4ac)\,d\,(a+bx^2+cx^4)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.18, size = 242, normalized size = 0.35

$$\frac{x(dx)^m \left(A(6+5m+m^2)\,F_1\left(\frac{1+m}{2};2,2;\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(1+m)x\left(B(3+m)F_1\left(\frac{2+m}{2};2,2;\frac{4+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+C(2+m)xF_1\left(\frac{3+m}{2};2,2;\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{a^2(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*(A*(6 + 5*m + m^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x
*(B*(3 + m)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + C*(2 + m)*x*AppellF1[(3 + m
```

```
)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])))/(a^2*(1 + m)*(2 + m)*(3 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m \left(C\,x^2+Bx+A\right)}{\left(c\,x^4+b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.36, size = 57, normalized size = 0.08

$$\text{integral}\left(\frac{(Cx^2+Bx+A)(dx)^m}{c^2x^8+2\,bcx^6+(b^2+2\,ac)x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4
 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m\,(C\,x^2+B\,x+A)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
[Out] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

3.42 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{\ }}{\sqrt{b-}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4}}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac-}{a+bx^2+cx^4}}{2(b^2-4a}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) -$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\right.}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\right.}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.68, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]

[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4

Maple [A]
time = 0.04, size = 456, normalized size = 1.28

method	result
risch	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-bC)\_R^2}{4ac-b^2}-\frac{2\_RbB}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln(x-\_R}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{B\sqrt{-4ac+b^2}\,b\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-4bcA\sqrt{-4ac+\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*b*B+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x-1/(4*a*c-b^2)*a*B)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/4/c/(4*a*c-b^2)*(B*(-4*a*c+b^2)^(1/2)*b*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*(-4*a*c+b^2)^(1/2)-8*c^2*a*A+2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C*b^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^2)*(-B*(-4*a*c+b^2)^(1/2)*b*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*(-4*a*c+b^2)^(1/2)+8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4440 vs. 2(306) = 612.
time = 7.20, size = 4440, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
```

```
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
```

```
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4...
```

Mupad [B]
time = 0.00, size = 2500, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
```

4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2

```
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 32768...
```

3.43 $\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$\frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{b-\ldots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4a\ldots}}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1599, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+}{a+bx^2+cx^4}}{2(b^2-4a}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) -$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\right.}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\right.}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\right.}{2\sqrt{2}}$$

Mathematica [A]

time = 0.10, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]

time = 0.05, size = 456, normalized size = 1.28

method	result
risch	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-bC)\_R^2}{4ac-b^2}-\frac{2\_RbB}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln(x-\_\dots}{2c\_R^3+\_Rb}\right.\dots}{4}$
default	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{B\sqrt{-4ac+b^2}\,b\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-4bcA\sqrt{-4ac-\dots\right.}{\dots}}{\dots}\right.}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*b*B+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-1/(4*a*c-b^2)*a*B)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/4/c/(4*a*
c-b^2)*(B*(-4*a*c+b^2)^(1/2)*b*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*
c*A*(-4*a*c+b^2)^(1/2)-8*c^2*a*A+2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4
```

```
*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C*b^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^
2)*(-B*(-4*a*c+b^2)^(1/2)*b*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*
(-4*a*c+b^2)^(1/2)+8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c
+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4440 vs. $2(306) = 612$.
time = 7.46, size = 4440, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
```

```
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4...
```

Mupad [B]
time = 1.55, size = 2500, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
```

- 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
+ 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +

```
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 32768...
```

3.44 $\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4}}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {1608, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \int \frac{Bx^3}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x^2(A+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + B\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+\cdots}{a+bx^2+cx^4}}{2(b^2-4ac\cdots}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right) - \cdots$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\cdots\right.}{2\sqrt{2}\cdots}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\cdots\right.}{2\sqrt{2}\cdots}$$

$$= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\cdots\right.}{2\sqrt{2}\cdots}$$

Mathematica [A]
time = 0.10, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.05, size = 456, normalized size = 1.28

method	result
risch	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-bC)\_R^2}{4ac-b^2}-\frac{2\_RbB}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\_R\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{B\sqrt{-4ac+b^2}\,b\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-4bcA\sqrt{-4ac+\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*b*B+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-1/(4*a*c-b^2)*a*B)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/4/c/(4*a*
c-b^2)*(B*(-4*a*c+b^2)^(1/2)*b*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*
c*A*(-4*a*c+b^2)^(1/2)-8*c^2*a*A+2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4
```

```
*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C*b^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^
2)*(-B*(-4*a*c+b^2)^(1/2)*b*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*
(-4*a*c+b^2)^(1/2)+8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c
+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4440 vs. 2(306) = 612.
time = 8.15, size = 4440, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^

```
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4...
```

Mupad [B]
time = 1.47, size = 2500, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
```

- 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +

```
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 32768...
```

3.45 $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{b-\ldots}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4a\ldots}}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = $\{1599, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212\}$

$$-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2(A+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

$$= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2\right.}{}$$

$$= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4A}{\ }\right.}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4A}{\ }\right.}{2\sqrt{2}}$$

$$= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4A}{\ }\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.10, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.06, size = 456, normalized size = 1.28

method	result
risch	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-bC)\_R^2}{4ac-b^2}-\frac{2\_RbB}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{cx^4+bx^2+a}+\frac{2c\left(\frac{B\sqrt{-4ac+b^2}\,b\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-4bcA\sqrt{-4ac-\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*b*B+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-1/(4*a*c-b^2)*a*B)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/4/c/(4*a*
c-b^2)*(B*(-4*a*c+b^2)^(1/2)*b*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*
c*A*(-4*a*c+b^2)^(1/2)-8*c^2*a*A+2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4
```

```
*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C*b^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^
2)*(-B*(-4*a*c+b^2)^(1/2)*b*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*
(-4*a*c+b^2)^(1/2)+8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c
+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4440 vs. 2(306) = 612.
time = 7.39, size = 4440, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
```

```
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4...
```

Mupad [B]
time = 1.39, size = 2500, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
```

- 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +

```
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 32768...
```

3.46 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=356

$$\frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{\ }}{\sqrt{b-\ }}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4}}}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1599, 1676, 1289, 1180, 211, 12, 1128, 652, 632, 212}

$$-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx &= \int \frac{x^2(A+Bx+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2(A+Cx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac \dots}{a+bx^2+cx^4 \dots}}{2\left(b^2-4a \dots\right.} \\
&= -\frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right) - \dots \\
&= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC- \dots\right.}{2\sqrt{2 \dots}} \\
&= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC- \dots\right.}{2\sqrt{2 \dots}} \\
&= \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC- \dots\right.}{2\sqrt{2 \dots}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.05, size = 456, normalized size = 1.28

method	result
risch	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-bC)\_R^2}{4ac-b^2}-\frac{2\_RbB}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\_R\right.}{2c\_R^3+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-bC)x^3}{8ac-2b^2}-\frac{x^2bB}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{aB}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{B\sqrt{-4ac+b^2}\,b\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-4bcA\sqrt{-4ac+\right.}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2/(4*a*c-b^2)*x^2*b*B+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-1/(4*a*c-b^2)*a*B)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/4/c/(4*a*
c-b^2)*(B*(-4*a*c+b^2)^(1/2)*b*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*
c*A*(-4*a*c+b^2)^(1/2)-8*c^2*a*A+2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4
```

```
*a*c+b^2)^(1/2)*b^2+4*C*a*b*c-C*b^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^
2)*(-B*(-4*a*c+b^2)^(1/2)*b*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*b*c*A*
(-4*a*c+b^2)^(1/2)+8*c^2*a*A-2*A*b^2*c+4*C*(-4*a*c+b^2)^(1/2)*a*c+C*(-4*a*c
+b^2)^(1/2)*b^2-4*C*a*b*c+C*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4440 vs. 2(306) = 612.
time = 7.19, size = 4440, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
```

```
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4...
```

Mupad [B]
time = 1.41, size = 2500, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
```

 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +

```
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 32768...
```

3.47 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$\frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4}+\frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3}+\frac{(ce-bf)x^6}{6c^2}+\frac{fx^8}{8c}-\frac{(b^4ce-4ab^2c^2e+\cdots}{\cdots}$$

```
[Out] 1/2*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*x^2/c^4+1/4*(c^2*d+b^2*f-c*(a*
f+b*e))*x^4/c^3+1/6*(-b*f+c*e)*x^6/c^2+1/8*f*x^8/c-1/4*(b^3*c*e-2*a*b*c^2*e
-b^4*f-b^2*c*(-3*a*f+c*d)+a*c^2*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/c^5-1/2*(b^4*
c*e-4*a*b^2*c^2*e+2*a^2*c^3*e-b^5*f-b^3*c*(-5*a*f+c*d)+a*b*c^2*(-5*a*f+3*c*
d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1642, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4(-c(af+be)+b^2f+c^2d)}{4c^3}+\frac{x^2(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce)}{2c^4}-\frac{\log\left(a+bx^2+cx^4\right)(-b^2c(cd-3af)-2abc^2e+ac^2(cd-af)+b^4(-f)+b^3ce)}{4c^5}+\frac{x^6(ce-bf)}{6c^2}+\frac{fx^8}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + ((c^2*d + b
^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/(6*c^2) + (f*x^8)/(8
*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex+fx^2)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2ce-ac^2e-b^3f-bc(cd-2af)}{c^4} + \frac{(c^2d+b^2f-c(be+af))\,x}{c^3} + \cdots\right.\right.$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4} + \frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3} + \frac{(ce-b\cdots}{6c^2\cdots}$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4} + \frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3} + \frac{(ce-b\cdots}{6c^2\cdots}$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4} + \frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3} + \frac{(ce-b\cdots}{6c^2\cdots}$$

$$= \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4} + \frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3} + \frac{(ce-b\cdots}{6c^2\cdots}$$

Mathematica [A]
time = 0.13, size = 260, normalized size = 0.95

$$\frac{-12c(-b^2ce+ac^2e+b^3f+bc(cd-2af))\,x^2+6c^2(c^2d+b^2f-c(be+af))\,x^4+4c^3(ce-bf)x^6+3c^4fx^8-\frac{12(-b^4ce+4ab^2c^2e-2a^2c^3e+b^5f+b^3c(cd-5af)+abc^2(-3cd+5af))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+6(-b^3ce+2abc^2e+b^4f+b^2c(cd-3af)+ac^2(-cd+af))\log(a+bx^2+cx^4)}{24c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2*
d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (12*
(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f) + a
*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^
2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f) + a*
c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

Maple [A]
time = 0.11, size = 330, normalized size = 1.21

method	result
default	$\frac{\frac{1}{4} f\,x^{8} c^{3}-\frac{1}{3} b\,c^{2} f\,x^{6}+\frac{1}{3} c^{3} e\,x^{6}-\frac{1}{2} a\,c^{2} f\,x^{4}+\frac{1}{2} b^{2} c f\,x^{4}-\frac{1}{2} b\,c^{2} e\,x^{4}+\frac{1}{2} c^{3} d\,x^{4}+2 a b c f\,x^{2}-a\,c^{2} e\,x^{2}-b^{3} f\,x^{2}+b^{2} c e\,x^{2}-b\,c^{2} d\,x^{2}}{2 c^{4}}+\frac{\left(a^{2} c^{2} f-3a\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^4*(1/4*f*x^8*c^3-1/3*b*c^2*f*x^6+1/3*c^3*e*x^6-1/2*a*c^2*f*x^4+1/2*b^
2*c*f*x^4-1/2*b*c^2*e*x^4+1/2*c^3*d*x^4+2*a*b*c*f*x^2-a*c^2*e*x^2-b^3*f*x^2
+b^2*c*e*x^2-b*c^2*d*x^2)+1/2/c^4*(1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*e-a
*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(-2*a^2*b*c*f+a^2*c^2
*e+a*b^3*f-a*b^2*c*e+a*b*c^2*d-1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*e-a*c^3
*d+b^4*f-b^3*c*e+b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.62, size = 900, normalized size = 3.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 -
4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f
)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x
^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c
^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d -
 (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*
((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c -
 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((b^3*c^2 - 3*a*
b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 6*((b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e +
(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x^2 + a))/(
b^2*c^5 - 4*a*c^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.45, size = 306, normalized size = 1.12

$$\frac{3\,c^3fx^8-4\,bc^2fx^6+4\,c^3x^6e+6\,c^3dx^4+6\,b^2cfx^4-6\,ac^2fx^4-6\,bc^2x^4e-12\,bc^2dx^2-12\,b^3fx^2+24\,abcfx^2+12\,b^2cx^2e-12\,ac^2x^2e}{24\,c^4}+\frac{(b^2c^2d-ac^3d+b^4f-3\,ab^2cf+a^2c^2f-b^3ce+2\,abc^2e)\log\left(cx^4+bx^2+a\right)}{4\,c^5}-\frac{(b^3c^2d-3\,abc^3d+b^5f-5\,ab^3cf+5\,a^2bc^2f-b^4ce+4\,ab^2c^2e-2\,a^2c^3e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*c^3*f*x^8 - 4*b*c^2*f*x^6 + 4*c^3*x^6*e + 6*c^3*d*x^4 + 6*b^2*c*f*x
^4 - 6*a*c^2*f*x^4 - 6*b*c^2*x^4*e - 12*b*c^2*d*x^2 - 12*b^3*f*x^2 + 24*a*b
*c*f*x^2 + 12*b^2*c*x^2*e - 12*a*c^2*x^2*e)/c^4 + 1/4*(b^2*c^2*d - a*c^3*d
+ b^4*f - 3*a*b^2*c*f + a^2*c^2*f - b^3*c*e + 2*a*b*c^2*e)*log(c*x^4 + b*x^
```

```
2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c
^2*f - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^
2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

Mupad [B]
time = 1.60, size = 2972, normalized size = 10.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*(e/(6*c) - (b*f)/(6*c^2)) - x^4*((b*(e/c - (b*f)/c^2))/(4*c) - d/(4*c)
+ (a*f)/(4*c^2)) - x^2*((a*(e/c - (b*f)/c^2))/(2*c) - (b*((b*(e/c - (b*f)/c
^2))/c - d/c + (a*f)/c^2))/(2*c)) + (f*x^8)/(8*c) - (log(a + b*x^2 + c*x^4)
*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^
2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))
/(2*(16*a*c^6 - 4*b^2*c^5)) + (atan((2*c^8*(4*a*c - b^2)*(x^2*(((((4*a^2*c^
8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d - 16*a*b^2*c^7
*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*f + 8*a^2*c^4*d
 + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f
- 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5)
)*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f +
4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) - (b*(b^5*f - 2
*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*
e + 5*a^2*b*c^2*f)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b
^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e
- 16*a^2*b*c^3*e))/(2*c^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a -
(b*((((4*a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d
 - 16*a*b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*
f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f
- 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c
^6 - 4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5
*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e -
16*a^2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (b^9*f^2 + b^5*c^4*d^2 + b^7*
c^2*e^2 - 3*a*b^3*c^5*d^2 + 2*a^2*b*c^6*d^2 - 5*a*b^5*c^3*e^2 - 2*a^3*b*c^5
*e^2 + 3*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 7*a^2*b^3*c^4*e^2 + 16*a^2*b^5*c^2*f
^2 - 13*a^3*b^3*c^3*f^2 - 7*a*b^7*c*f^2 + a^3*c^6*d*e - 2*b^6*c^3*d*e - a^4
*c^5*e*f + 2*b^7*c^2*d*f + 8*a*b^4*c^4*d*e - 10*a*b^5*c^3*d*f - 5*a^3*b*c^5
*d*f + 12*a*b^6*c^2*e*f - 8*a^2*b^2*c^5*d*e + 14*a^2*b^3*c^4*d*f - 22*a^2*b
^4*c^3*e*f + 12*a^3*b^2*c^4*e*f)/c^8 + (b*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d
- b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)^2)/(
2*c^8*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a^3*c^7*f - 8*a^2
*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d - 8*a*b^3*c^6*e + 16*a^2*b*c^7*e
+ 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^
```

```
3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12
*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5))*(b^5*f - 2*a^2*c^3*
e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2
*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) + (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)*(2
*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c
^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(c
^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a + (b*((a*b^8*f^2 + a^3*c^
6*d^2 + a^5*c^4*f^2 + a*b^4*c^4*d^2 + a*b^6*c^2*e^2 - 6*a^2*b^6*c*f^2 - 2*a
^2*b^2*c^5*d^2 - 4*a^2*b^4*c^3*e^2 + 4*a^3*b^2*c^4*e^2 + 11*a^3*b^4*c^2*f^2
 - 6*a^4*b^2*c^3*f^2 - 2*a^4*c^5*d*f - 2*a*b^5*c^3*d*e - 4*a^3*b*c^5*d*e +
2*a*b^6*c^2*d*f + 4*a^4*b*c^4*e*f + 6*a^2*b^3*c^4*d*e - 8*a^2*b^4*c^3*d*f +
 8*a^3*b^2*c^4*d*f + 10*a^2*b^5*c^2*e*f - 14*a^3*b^3*c^3*e*f - 2*a*b^7*c*e*
f)/c^8 + (((8*a^3*c^7*f - 8*a^2*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d -
8*a*b^3*c^6*e + 16*a^2*b*c^7*e + 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8
*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*
a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 -
4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e
+ 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^
2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)^2)
/(c^8*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^10*f^2 + 4*a^4*c^6*e^
2 + b^6*c^4*d^2 + b^8*c^2*e^2 - 6*a*b^4*c^5*d^2 - 8*a*b^6*c^3*e^2 - 2*b^9*c
*e*f + 9*a^2*b^2*c^6*d^2 + 20*a^2*b^4*c^4*e^2 - 16*a^3*b^2*c^5*e^2 + 35*a^2
*b^6*c^2*f^2 - 50*a^3*b^4*c^3*f^2 + 25*a^4*b^2*c^4*f^2 - 10*a*b^8*c*f^2 - 2
*b^7*c^3*d*e + 2*b^8*c^2*d*f + 14*a*b^5*c^4*d*e + 12*a^3*b*c^6*d*e - 16*a*b
^6*c^3*d*f + 18*a*b^7*c^2*e*f - 20*a^4*b*c^5*e*f - 28*a^2*b^3*c^5*d*e + 40*
a^2*b^4*c^4*d*f - 30*a^3*b^2*c^5*d*f - 54*a^2*b^5*c^3*e*f + 60*a^3*b^3*c^4*
e*f))*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*
f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c^5*(4*a*c - b^2)^(1/2))
```

3.48 $\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=203

$$\frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\tan}{2c^4\sqrt{b^2-4ac}}$$

```
[Out] 1/2*(c^2*d+b^2*f-c*(a*f+b*e))*x^2/c^3+1/4*(-b*f+c*e)*x^4/c^2+1/6*f*x^6/c+1/
4*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*ln(c*x^4+b*x^2+a)/c^4+1/2*(b^3*c
*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))*arctanh((2*c*x^
2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\frac{x^2(-c(af+be)+b^2f+c^2d)}{2c^3}+\frac{\log(a+bx^2+cx^4)(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce)}{2c^4\sqrt{b^2-4ac}}+\frac{x^4(ce-bf)}{4c^2}+\frac{fx^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)/(4*c^2) +
 (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*
a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^
2 - 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^
2 + c*x^4])/(4*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+ex+fx^2)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x}{c^2}+\frac{fx^2}{c}-\frac{a(c^2d+b^2f-c(be$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}-\frac{\text{Subst}\left(\int \frac{a(c^2d+b^2f-c(be+af))}{}\right.}{}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{(b^2ce-ac^2e-b^3f-bc(cd}{}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{(b^2ce-ac^2e-b^3f-bc(cd}{4}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{(b^3ce-3abc^2e-b^4f-b^2c}{}$$

Mathematica [A]

time = 0.10, size = 193, normalized size = 0.95

$$\frac{6c(c^2d+b^2f-c(be+af))\,x^2+3c^2(ce-bf)x^4+2c^3fx^6+\frac{6(-b^3ce+3abc^2e+b^4f+b^2c(cd-4af)+2ac^2(-cd+af))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-3(-b^2ce+ac^2e+b^3f+bc(cd-2af))\log\left(a+bx^2+cx^4\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f*
x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^2*
(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*
x^4])/(12*c^4)
```

Maple [A]
time = 0.09, size = 224, normalized size = 1.10

method	result
default	$-\frac{-\frac{1}{3} f\,x^{6} c^{2}+\frac{1}{2} b c f\,x^{4}-\frac{1}{2} c^{2} e\,x^{4}+a c f\,x^{2}-b^{2} f\,x^{2}+b c e\,x^{2}-c^{2} d\,x^{2}}{2 c^{3}}+\frac{\frac{\left(2 a b c f-a\,c^{2} e-b^{3} f+b^{2} c e-b\,c^{2} d\right) \ln \left(c\,x^{4}+b\,x^{2}+a\right)}{2 c}+\frac{2 \left(a^{2} c f-a\,b^{2} f+a \cdots\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^3*(-1/3*f*x^6*c^2+1/2*b*c*f*x^4-1/2*c^2*e*x^4+a*c*f*x^2-b^2*f*x^2+b*
c*e*x^2-c^2*d*x^2)+1/2/c^3*(1/2*(2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/c
*ln(c*x^4+b*x^2+a)+2*(a^2*c*f-a*b^2*f+a*b*c*e-a*c^2*d-1/2*(2*a*b*c*f-a*c^2*
e-b^3*f+b^2*c*e-b*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.56, size = 677, normalized size = 3.33

$$\left[\frac{2 \, {\left(b^{2} c^{3} - 4 \, a c^{4}\right)} f x^{6} + 3 \, {\left({\left(b^{2} c^{3} - 4 \, a c^{4}\right)} e - {\left(b^{3} c^{2} - 4 \, a b c^{3}\right)} f\right)} x^{4} + 6 \, {\left({\left(b^{2} c^{3} - 4 \, a c^{4}\right)} d - {\left(b^{3} c^{2} - 4 \, a b c^{3}\right)} e + {\left(b^{4} c - 5 \, a b^{2} c^{2} + 4 \, a^{2} c^{3}\right)} f\right)} x^{2} + 3 \, \sqrt{b^{2} - 4 \, a c} {\left({\left(b^{2} c^{2} - 2 \, a c^{3}\right)} d - {\left(b^{3} c - 3 \, a b c^{2}\right)} e + {\left(b^{4} - 4 \, a b^{2} c + 2 \, a^{2} c^{2}\right)} f\right)} \log\left(\frac{2 \, c^{2} x^{4} + 2 \, b c x^{2} + b^{2} - 2 \, a c - {\left(2 \, c x^{2} + b\right)} \sqrt{b^{2} - 4 \, a c}}{c x^{4} + b x^{2} + a}\right) - 3 \, {\left({\left(b^{3} c^{2} - 4 \, a b c^{3}\right)} d - {\left(b^{4} c - 5 \, a b^{2} c^{2} + 4 \, a^{2} c^{3}\right)} e + {\left(b^{5} - 6 \, a b^{3} c + 8 \, a^{2} b c^{2}\right)} f\right)} \log\left(c x^{4} + b x^{2} + a\right)}{12 \, {\left(b^{2} c^{4} - 4 \, a c^{5}\right)}}, \frac{2 \, {\left(b^{2} c^{3} - 4 \, a c^{4}\right)} f x^{6} + 3 \, {\left({\left(b^{2} c^{3} - 4 \, a c^{4}\right)} e - {\left(b^{3} c^{2} - 4 \, a b c^{3}\right)} f\right)} x^{4} + 6 \, {\left({\left(b^{2} c^{3} - 4 \, a c^{4}\right)} d - {\left(b^{3} c^{2} - 4 \, a b c^{3}\right)} e + {\left(b^{4} c - 5 \, a b^{2} c^{2} + 4 \, a^{2} c^{3}\right)} f\right)} x^{2} - 6 \, \sqrt{-b^{2} + 4 \, a c} {\left({\left(b^{2} c^{2} - 2 \, a c^{3}\right)} d - {\left(b^{3} c - 3 \, a b c^{2}\right)} e + {\left(b^{4} - 4 \, a b^{2} c + 2 \, a^{2} c^{2}\right)} f\right)} \arctan\left(-\frac{{\left(2 \, c x^{2} + b\right)} \sqrt{-b^{2} + 4 \, a c}}{b^{2} - 4 \, a c}\right) - 3 \, {\left({\left(b^{3} c^{2} - 4 \, a b c^{3}\right)} d - {\left(b^{4} c - 5 \, a b^{2} c^{2} + 4 \, a^{2} c^{3}\right)} e + {\left(b^{5} - 6 \, a b^{3} c + 8 \, a^{2} b c^{2}\right)} f\right)} \log\left(c x^{4} + b x^{2} + a\right)}{12 \, {\left(b^{2} c^{4} - 4 \, a c^{5}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 -
4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 - 2*
a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log((2*
c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^
4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e
- (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*
b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2 + 4*a*c)*
((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c
^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^3*c^
2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.28, size = 214, normalized size = 1.05

$$\frac{2\,c^2fx^6-3\,bcfx^4+3\,c^2x^4e+6\,c^2dx^2+6\,b^2fx^2-6\,acfx^2-6\,bcx^2e}{12\,c^3}-\frac{(bc^2d+b^3f-2\,abcf-b^2ce+ac^2e)\log\left(cx^4+bx^2+a\right)}{4\,c^4}+\frac{(b^2c^2d-2\,ac^3d+b^4f-4\,ab^2cf+2\,a^2c^2f-b^3ce+3\,abc^2e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*c^2*f*x^6 - 3*b*c*f*x^4 + 3*c^2*x^4*e + 6*c^2*d*x^2 + 6*b^2*f*x^2 -
 6*a*c*f*x^2 - 6*b*c*x^2*e)/c^3 - 1/4*(b*c^2*d + b^3*f - 2*a*b*c*f - b^2*c*
e + a*c^2*e)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d + b^4*
f - 4*a*b^2*c*f + 2*a^2*c^2*f - b^3*c*e + 3*a*b*c^2*e)*arctan((2*c*x^2 + b)
/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [**B**]
time = 1.63, size = 2295, normalized size = 11.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4*(e/(4*c) - (b*f)/(4*c^2)) - x^2*((b*(e/c - (b*f)/c^2))/(2*c) - d/(2*c)
+ (a*f)/(2*c^2)) + (log(a + b*x^2 + c*x^4)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c
^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c
^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) + (f*x^6)/(6*c) + (atan((2*c^6*(4*a*c - b
^2)*(x^2*(((((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c
^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e
 + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e +
16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f -
 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/
2)) + (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2
*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*
c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*c^2*(4*a*c - b^
2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((b^7*f^2 + b^3*c^4*d^2 + b^5*c^2*
e^2 - 3*a*b^3*c^3*e^2 + 2*a^2*b*c^4*e^2 - 2*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 7
*a^2*b^3*c^2*f^2 - a*b*c^5*d^2 - 5*a*b^5*c*f^2 - a^2*c^5*d*e - 2*b^4*c^3*d*
e + a^3*c^4*e*f + 2*b^5*c^2*d*f + 4*a*b^2*c^4*d*e - 6*a*b^3*c^3*d*f + 3*a^2
*b*c^4*d*f + 8*a*b^4*c^2*e*f - 8*a^2*b^2*c^3*e*f)/c^6 + (((6*b^2*c^6*d + 4*
a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2
*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8
*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4
*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d -
 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)
) - (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e
 - 4*a*b^2*c*f)^2)/(2*c^6*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
(8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e + 8*a*b^3*c^4*f - 16*a^2*b*c^5*f
)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c
^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c
^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e -
 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/2)) + (a*(b^4*f + b^2*c^2*d + 2*a^2*
c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c
^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*
e + 16*a^2*b*c^2*f))/(c^2*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a -
(b*((a*b^6*f^2 + a^3*c^4*e^2 + a*b^2*c^4*d^2 + a*b^4*c^2*e^2 - 4*a^2*b^4*c*
f^2 - 2*a^2*b^2*c^3*e^2 + 4*a^3*b^2*c^2*f^2 - 2*a*b^3*c^3*d*e + 2*a^2*b*c^4
*d*e + 2*a*b^4*c^2*d*f - 4*a^3*b*c^3*e*f - 4*a^2*b^2*c^3*d*f + 6*a^2*b^3*c^
2*e*f - 2*a*b^5*c*e*f)/c^6 + (((8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e +
 8*a*b^3*c^4*f - 16*a^2*b*c^5*f)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*
b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^
2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d -
2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/
(2*(16*a*c^5 - 4*b^2*c^4)) - (a*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*
```

```
d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)^2)/(c^6*(4*a*c - b^2))))/(2*a*(4*a
*c - b^2)^(1/2))))/(b^8*f^2 + 4*a^2*c^6*d^2 + b^4*c^4*d^2 + 4*a^4*c^4*f^2 +
 b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*
c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 8*a^3*c
^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 12*a^2*b*c^5*d*
e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 12*a^3*b*c^4*e*f + 20*a^2*b^2*c^4
*d*f - 28*a^2*b^3*c^3*e*f))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d -
b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*c^4*(4*a*c - b^2)^(1/2))
```

3.49 $\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=144

$$\frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(c^2d+b^2f-c(be+af))\,\mathrm{lo}}{4c^3}$$

```
[Out] 1/2*(-b*f+c*e)*x^2/c^2+1/4*f*x^4/c+1/4*(c^2*d+b^2*f-c*(a*f+b*e))*ln(c*x^4+b
*x^2+a)/c^3-1/2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d))*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\frac{\log(a+bx^2+cx^4)(-c(af+be)+b^2f+c^2d)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(ce-bf)}{2c^2}+\frac{fx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b
^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*
c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex+fx^2)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{ce-bf}{c^2}+\frac{fx}{c}-\frac{a(ce-bf)-(c^2d-bce+b^2f-acf)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx, x, x$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\text{Subst}\left(\int \frac{a(ce-bf)-(c^2d-bce+b^2f-acf)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{(-c^2d+bce-b^2f+acf)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}+\frac{(c^2d-bce+b^2f-acf)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{(b^2ce-2a$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2c^3\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.07, size = 136, normalized size = 0.94

$$\frac{2c(ce-bf)x^2+c^2fx^4-\frac{2(-b^2ce+2ac^2e+b^3f+bc(cd-3af))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2d+b^2f-c(be+af))\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*c
*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [**A**]
time = 0.11, size = 146, normalized size = 1.01

method	result
default	$-\frac{-\frac{1}{2}cf\,x^{4}+bf\,x^{2}-ce\,x^{2}}{2c^{2}}+\frac{\frac{\left(-acf+b^{2}f-bce+c^{2}d\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(abf-ace-\frac{\left(-acf+b^{2}f-bce+c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c^{2}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*(-1/2*c*f*x^4+b*f*x^2-c*e*x^2)+1/2/c^2*(1/2*(-a*c*f+b^2*f-b*c*e+c^
2*d)/c*ln(c*x^4+b*x^2+a)+2*(a*b*f-a*c*e-1/2*(-a*c*f+b^2*f-b*c*e+c^2*d)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.49, size = 473, normalized size = 3.28

$$\left[\frac{(b^2c^2-4\,ac^3)fx^4+2\,((b^2c^2-4\,ac^3)e-(b^3c-4\,abc^2)f)x^2-(bc^2d-(b^2c-2\,ac^2)e+(b^3-3\,abc)f)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+((b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-5\,ab^2c+4\,a^2c^2)f)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^3-4\,ac^4)},\ \frac{(b^2c^2-4\,ac^3)fx^4+2\,((b^2c^2-4\,ac^3)e-(b^3c-4\,abc^2)f)x^2+2\,(bc^2d-(b^2c-2\,ac^2)e+(b^3-3\,abc)f)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+((b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-5\,ab^2c+4\,a^2c^2)f)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^3-4\,ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b
*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c
^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 -
 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3
*c - 4*a*b*c^2)*f)*x^2 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)
*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*
c)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*b^2*c + 4
*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.65, size = 141, normalized size = 0.98

$$\frac{cfx^4 - 2\,bfx^2 + 2\,cx^2e}{4\,c^2} + \frac{(c^2d + b^2f - acf - bce)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{(bc^2d + b^3f - 3\,abcf - b^2ce + 2\,ac^2e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*f*x^4 - 2*b*f*x^2 + 2*c*x^2*e)/c^2 + 1/4*(c^2*d + b^2*f - a*c*f - b*
c*e)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d + b^3*f - 3*a*b*c*f - b^2*c*
e + 2*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*c^3)
```

Mupad [**B**]
time = 1.30, size = 1689, normalized size = 11.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(e/(2*c) - (b*f)/(2*c^2)) + (f*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 1
0*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (atan((2*c^4*(4*a*c - b^2)*(x^2*
```

```
(((((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^4*f)/c^4
+ (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8
*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*e + b*
c^2*d - b^2*c*e - 3*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b*(b^3*f + 2*a
*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*
f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(2*c*(4*a*c - b^2)
^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((b^5*f^2 + b*c^4*d^2 + b^3*c^2*e^2
+ 2*a^2*b*c^2*f^2 + a*c^4*d*e - 2*b^4*c*e*f - a*b*c^3*e^2 - 3*a*b^3*c*f^2 -
 2*b^2*c^3*d*e - a^2*c^3*e*f + 2*b^3*c^2*d*f + 4*a*b^2*c^2*e*f - 3*a*b*c^3*
d*f)/c^4 + (((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^
4*f)/c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^
3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*f + 2*b
^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f
))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e -
 3*a*b*c*f)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a
^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f)/c^4 - (8*a*c^2*(2*b^4*f
 + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b
^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3
*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(b^3*f + 2*a*c^2*e + b*c^2*d -
b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b
^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*
b^2*c^3)))/a + (b*((((8*a^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f
)/c^4 - (8*a*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c
*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*f + 2*b^2*
c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/
(2*(16*a*c^4 - 4*b^2*c^3)) - (a*c^4*d^2 + a*b^4*f^2 + a^3*c^2*f^2 + a*b^2*c
^2*e^2 - 2*a^2*b^2*c*f^2 - 2*a^2*c^3*d*f + 2*a*b^2*c^2*d*f + 2*a^2*b*c^2*e*
f - 2*a*b*c^3*d*e - 2*a*b^3*c*e*f)/c^4 + (a*(b^3*f + 2*a*c^2*e + b*c^2*d -
b^2*c*e - 3*a*b*c*f)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(
b^6*f^2 + 4*a^2*c^4*e^2 + b^2*c^4*d^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b
^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 - 2*b^3*c^3*d*e + 2*b^4*c^2*d*
f - 6*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 12*a^2*b*c^3*e*f + 4*a*b*c^4*d*e))
*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^3*(4*a*c - b^2)^
(1/2))
```

3.50 $\int \frac{x\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=103

$$\frac{fx^2}{2c} - \frac{(2c^2d - bce + b^2f - 2acf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

[Out] 1/2*f*x^2/c+1/4*(-b*f+c*e)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1677, 1671, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2acf + b^2f - bce + 2c^2d)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{fx^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]

[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x}{c\left(a+bx+cx^2\right)}\right)\,dx, x, x^2\right) \\
&= \frac{fx^2}{2c}+\frac{\text{Subst}\left(\int \frac{cd-af+(ce-bf)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} \\
&= \frac{fx^2}{2c}+\frac{(ce-bf)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}+\frac{\left(2c^2d-bce+b^2f-2acf\right)\text{Subst}\left(\int\right.}{4c^2} \\
&= \frac{fx^2}{2c}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(2c^2d-bce+b^2f-2acf\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-}\right.}{2c^2} \\
&= \frac{fx^2}{2c}-\frac{\left(2c^2d-bce+b^2f-2acf\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2\right.}{4c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 0.97

$$
\frac{2cfx^2+\frac{2\left(2c^2d+b^2f-c(be+2af)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^2)
```

Maple [A]
time = 0.06, size = 101, normalized size = 0.98

method	result	size
default	$\frac{f\,x^{2}}{2c}+\frac{\frac{\left(-bf+ce\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(-fa+cd-\frac{\left(-bf+ce\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c}$	101
risch	Expression too large to display	1690

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/c+1/2/c*(1/2*(-b*f+c*e)/c*ln(c*x^4+b*x^2+a)+2*(-f*a+c*d-1/2*(-b*f
+c*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 318, normalized size = 3.09

$$\left[\frac{2\,(b^2c-4\,ac^2)fx^2-(2\,c^2d-bce+(b^2-2\,ac)f)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+((b^2c-4\,ac^2)e-(b^3-4\,abc)f)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^2-4\,ac^3)},\ \frac{2\,(b^2c-4\,ac^2)fx^2-2\,(2\,c^2d-bce+(b^2-2\,ac)f)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+((b^2c-4\,ac^2)e-(b^3-4\,abc)f)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^2-4\,ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
```

```
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)
*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^2)*f
*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e - (b^3
- 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.48, size = 99, normalized size = 0.96

$$\frac{fx^2}{2\,c} - \frac{(bf - ce)\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(2\,c^2d + b^2f - 2\,acf - bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c - 1/4*(b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d + b
^2*f - 2*a*c*f - b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*c^2)
```

Mupad [**B**]
time = 1.83, size = 1081, normalized size = 10.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e -
8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*(x^2*((
(((4*c^4*d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f +
 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2
*f - 2*a*c*f - b*c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b*(2*c^2*d + b^2*f -
2*a*c*f - b*c*e)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(4*a*c -
 b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - (b*((b^3*f^2 + b*c^2*e^2 - c^3*d*e
```

```
 - a*b*c*f^2 + a*c^2*e*f + b*c^2*d*f - 2*b^2*c*e*f)/c^2 + (((4*c^4*d + 6*b^
2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^
2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*
e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b*(2*c^2*d + b^2*f - 2*a*c*f
 - b*c*e)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a*c^
3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*
c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(
4*a*c - b^2)^(1/2)) - (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a
*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2
)))/a + (b*((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e
 - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2
*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*b^2*f^2 + a*c^2*e^2
 - 2*a*b*c*e*f)/c^2 + (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)^2)/(c^2*(4*a*c
 - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(4*c^4*d^2 + b^4*f^2 + 4*a^2*c^2*f^2
 + b^2*c^2*e^2 - 8*a*c^3*d*f - 4*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 4
*b^2*c^2*d*f + 4*a*b*c^2*e*f))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*
(4*a*c - b^2)^(1/2))
```

3.51 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=97

$$\frac{(bcd-2ace+abf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

```
[Out] d*ln(x)/a-1/4*(-a*f+c*d)*ln(c*x^4+b*x^2+a)/a/c+1/2*(a*b*f-2*a*c*e+b*c*d)*ar
ctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1642, 648, 632, 212, 642}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4])/(
4*a*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-(cd-af)x}{a\left(a+bx+cx^2\right)}\right)\,dx,x,x^2\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-(cd-af)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{d\log(x)}{a}-\frac{(cd-af)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4ac}-\frac{(bcd-2ace+abf)\text{Subst}\left(\int \ldots\right.}{4ac}\\
&= \frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{(bcd-2ace+abf)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\ldots\right.}{2ac}\\
&= \frac{(bcd-2ace+abf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+\ldots\right.}{4ac}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 178, normalized size = 1.84

$$\frac{4c\sqrt{b^2-4ac}\,d\log(x)-\left(bcd+c\sqrt{b^2-4ac}\,d-2ace+abf-a\sqrt{b^2-4ac}\,f\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+\left(bcd-c\sqrt{b^2-4ac}\,d-2ace+abf+a\sqrt{b^2-4ac}\,f\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4ac\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
+ a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b*
c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Log[
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*c*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.05, size = 99, normalized size = 1.02

method	result
default	$\frac{\frac{(fa-cd)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(ae-bd-\frac{(fa-cd)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}+\frac{d\ln(x)}{a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c^{2}-a\,b^{2}c\right)\_Z^{2}+\left(-4a^{2}cf+a\,b^{2}f+4a\,c^{2}d-b^{2}cd\right)\_Z+a^{2}f^{2}-abef-2acdf+ac\,e^{2}+b^{2}df-bcde+c^{2}d^{2}\right)}\_R\ln\left(\left(\left(10\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(1/2*(a*f-c*d)/c*ln(c*x^4+b*x^2+a)+2*(a*e-b*d-1/2*(a*f-c*d)*b/c)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+d*ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.63, size = 309, normalized size = 3.19

$$\left[\frac{4\,(b^{2}c-4\,ac^{2})d\log(x)+(bcd-2\,ace+abf)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-((b^{2}c-4\,ac^{2})d-(ab^{2}-4\,a^{2}c)f)\log(cx^{4}+bx^{2}+a)}{4\,(ab^{2}c-4\,a^{2}c^{2})},\ \frac{4\,(b^{2}c-4\,ac^{2})d\log(x)+2\,(bcd-2\,ace+abf)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-((b^{2}c-4\,ac^{2})d-(ab^{2}-4\,a^{2}c)f)\log(cx^{4}+bx^{2}+a)}{4\,(ab^{2}c-4\,a^{2}c^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2), 1/4*(4*(b^2*c - 4*a*c^2)*d*lo
g(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*
c)*f)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.10, size = 97, normalized size = 1.00

$$\frac{d\log\left(x^2\right)}{2\,a}-\frac{(cd-af)\log\left(cx^4+bx^2+a\right)}{4\,ac}-\frac{(bcd+abf-2\,ace)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)/a - 1/4*(c*d - a*f)*log(c*x^4 + b*x^2 + a)/(a*c) - 1/2*(b*c*
d + a*b*f - 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*a*c)
```

Mupad [**B**]
time = 8.88, size = 2500, normalized size = 25.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d*log(x))/a - (log((b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2
 - b*e*f - c*d*f) + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^
2*(4*a*c - b^2)))^(1/2))*(a*b^2*f^2 - x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*
e + 11*a*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) + a*c^2*e^2 - 4
*b*c^2*d*e + 4*b^2*c*d*f + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/
(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d
```

- 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (b*c*
(c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/
2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) - 2*a*b*c*e*f))/(4*a*c) - 2
*b*c*d*e*f)*(b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2 - b*e*f
 - c*d*f) + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c
 - b^2)))^(1/2))*(x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a*b*c*f^2 - 9
*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) - a*b^2*f^2 - a*c^2*e^2 + 4*b*c^2*d
*e - 4*b^2*c*d*f + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2
*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d - 4*b^2*
c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f - (b*c*(a*f - c
*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(a*b
 + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) + 2*a*b*c*e*f))/(4*a*c) - 2*b*c*d*e
f))(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*
b^2*c)) + (atan(((4*a*c - b^2)*(((((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d -
 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*
d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^(1/2)) + (b
^2*c*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c
f))/(2(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(8*a*c^2*d + 2*a*b^2
*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c
*e + b*c*d)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*
c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2
*c^2 - 4*a*b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*
a^2*c^2 - 4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(4*a*c*(4
*a*c - b^2)^(1/2)) - (b^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(16*a^2*c*(4*a*c - b
^2)^(3/2)))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 4*a^3*c*f -
 28*a*b^2*c*d + 6*a^2*b*c*e))/(c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2
 - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 +
 a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d
*e)) + (16*a^3*c*x^2*(((3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*d)*
(c^2*e^3 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(3*b^3*f^2 - b*
c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(((12*b^3*c^2 -
40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2
 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a*
b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d*e - 11*a*b*c*f^2 + 9*a*c^2
*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + b^2*e*f^2
 - a*b*f^3 + a*c*e*f^2 + b*c*d*f^2 - 2*b*c*e^2*f - c^2*d*e*f - ((((a*b*f -
2*a*c*e + b*c*d)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2
*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e
+ 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^(1/2)) + ((
12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f -
 2*b^2*c*d - 8*a^2*c*f))/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2
)))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c - b^2)^(1/2)) - ((12*b^3*c^2 -
 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d
 - 8*a^2*c*f))/(32*a^2*c^2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2))))/(8*a^3
c^2(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*

```
b*e*f - 10*a^2*c*d*f - a*b*c*d*e)) + ((((((a*b*f - 2*a*c*e + b*c*d)*(((12*b
^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(1
6*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*
f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^(1/2)) + ((12*b^3*c^2 - 40*a*b*c^3)
*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))
/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(8*a*c^2*d + 2*a*b^2
*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c
*e + b*c*d)*(3*b^3*f^2 - b*c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d -
8*a^2*c*f)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d -
 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b
*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d
*e - 11*a*b*c*f^2 + 9*a*c^2*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(4*a*c*(4*a*c
 - b^2)^(1/2)) - ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^3)/(6
4*a^3*c^3*(4*a*c - b^2)^(3/2)))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a
*b^3*e - 4*a^3*c*f - 28*a*b^2*c*d + 6*a^2*b*c*e))/(16*a^3*c^2*(4*a*c - b^2)
^(1/2)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*...
```

3.52 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=118

$$-\frac{d}{2ax^2}-\frac{(b^2d-abe-2a(cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2*d/a/x^2-(-a*e+b*d)*ln(x)/a^2+1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/a^2-1/2*
(b^2*d-a*b*e-2*a*(-a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1642, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-abe-2a(cd-af)+b^2d)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*d/(a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + (
(b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)\,dx, x,\right.\\
&= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}\\
&= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{(b^2d-abe}{}\\
&= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{(b^2d-abe-2a(c}{}\\
&= -\frac{d}{2ax^2}-\frac{(b^2d-abe-2a(cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 203, normalized size = 1.72

$$\frac{-\frac{2ad}{x^2}+4(-bd+ae)\log(x)+\frac{\left(b^2d+b\left(\sqrt{b^2-4ac}\,d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}\,e+2af\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2d+b\left(\sqrt{b^2-4ac}\,d+ae\right)-a\left(-2cd+\sqrt{b^2-4ac}\,e+2af\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d
- a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e
) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A]
time = 0.05, size = 132, normalized size = 1.12

method	result
default	$\frac{\frac{(-ace+bcd)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(a^2f-abe-acd+b^2d-\frac{(-ace+bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}-\frac{d}{2a\,x^2}+\frac{(ae-bd)\ln(x)}{a^2}$
risch	$-\frac{d}{2a\,x^2}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(4a^2ce-a\,b^2e-4abcd+b^3d\right)\_Z+a^2f^2-abef-2acdf+ac\,e^2+b^2df-bcde+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*(1/2*(-a*c*e+b*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*f-a*b*e-a*c*d+b^2*d-
1/2*(-a*c*e+b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2)))-1/2*d/a/x^2+(a*e-b*d)/a^2*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.60, size = 399, normalized size = 3.38

$$\left[-\frac{(abe-2a^2f-(b^2-2ac)d)\sqrt{b^2-4ac}\,x^2\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-((b^3-4abc)d-(ab^2-4a^2c)e)x^2\log\left(cx^4+bx^2+a\right)+4((b^3-4abc)d-(ab^2-4a^2c)e)x^2\log\left(x\right)+2(ab^2-4a^2c)d}{4(a^2b^2-4a^3c)x^2}, \frac{2(abe-2a^2f-(b^2-2ac)d)\sqrt{-b^2+4ac}\,x^2\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+((b^3-4abc)d-(ab^2-4a^2c)e)x^2\log\left(cx^4+bx^2+a\right)-4((b^3-4abc)d-(ab^2-4a^2c)e)x^2\log\left(x\right)-2(ab^2-4a^2c)d}{4(a^2b^2-4a^3c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2
*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
b*x^2 + a)) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b*x
^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) + 2*(a*b^2
 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b^2 -
2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b
*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) - 2*(a*b
^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.39, size = 135, normalized size = 1.14

$$\frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{(bd-ae)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d-2\,acd+2\,a^2f-abe\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bdx^2-ax^2e-ad}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 +
 1/2*(b^2*d - 2*a*c*d + 2*a^2*f - a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*x^2*e - a*d)/(a^2*x^2)
```

Mupad [**B**]
time = 7.86, size = 2500, normalized size = 21.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - d/(2*a*x^2) - (log(((c^2*(a*e - b*d)*(a*f - c*d)
^2)/a^3 - ((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4
```

```
*a*c - b^2)))^(1/2))*(((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*
d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f + b^2*
c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a*c*d)
)/a + (b*c^2*(b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*
(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2) + (c^2
*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c*d))/a^2 - (c^2*x^2*(a*f - c*d
)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d)^3)/a^3)
*((c^2*(a*e - b*d)*(a*f - c*d)^2)/a^3 - ((a*e - b*d + a^2*(-(b^2*d + 2*a^2*
f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(((a*e - b*d + a^2*(-(b^
2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*
(10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(
b^2*d + a^2*f - a*b*e - a*c*d))/a - (b*c^2*(a*e - b*d + a^2*(-(b^2*d + 2*a^
2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*
a*c*x^2))/a^2))/(4*a^2) - (c^2*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c
*d))/a^2 + (c^2*x^2*(a*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2)
+ (c^2*x^2*(a*f - c*d)^3)/a^3))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*
d))/(2*(16*a^3*c - 4*a^2*b^2)) - (atan((16*a^6*(4*a*c - b^2)^(3/2)*(x^2*(((
(c^5*d^3 - a^3*c^2*f^3 + 3*a^2*c^3*d*f^2 - 3*a*c^4*d^2*f)/a^3 + (((a^3*b*c^
2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2*b*c^3*d*f)/a^
3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10
*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2*a*b^2*e +
 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2
*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2
*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((((20*a^3*c^4*d
 - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10*a^3*b*c^3*e)/a^3 +
 ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*
c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/
(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(b^2*d + 2*a
^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(8*a
^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e -
2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(b
^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*
b*c*d))/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))*(3*b^4*d + a^2*c^2*d
 + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9*a*b^2*c*d + 8*a^2*b*c*e))/(8*a^3*c^2
*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b
^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*
c*d*e)) - ((((((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*
c^2*f - 10*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2
*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(b^2*d +
 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 -
 12*a^3*b^3*c^2)*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e +
 8*a^2*c*e - 8*a*b*c*d))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))
)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2))
 + (((a^3*b*c^2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2
*b*c^3*d*f)/a^3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*
```

```
b^2*c^2*f - 10*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d
 - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*b
^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(b^2
*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c
^3 - 12*a^3*b^3*c^2)*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3)/(64*a^9*(4*a*c
- b^2)^(3/2)))*(6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e - 30*a*b^3
*c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e))/(16*a^3*c^2*(4*a*c -
 b^2)^(1/2)*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d
^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f
- 49*a^2*b*c*d*e))) + (((b*c^4*d^3 - a^3*c^2*e*f^2 - a*c^4*d^2*e - 2*a*b*c^
3*d^2*f + 2*a^2*c^3*d*e*f + a^2*b*c^2*d*f^2)/a^3 - (((a^2*c^4*d^2 + a^4*c^2
*f^2 - 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f
+ 4*a^2*b^2*c^2*d*f)/a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c
^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e -
 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a
*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a
*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*
e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e
+ 8*a^2*c*e - 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(b^2*d + 2*a^2*f - a*b*e
- 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*...
```

3.53 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{(b^2d-abe-a(cd-af))\,\mathrm{lo}}{a^3}$$

```
[Out] -1/4*d/a/x^4+1/2*(-a*e+b*d)/a^2/x^2+(b^2*d-a*b*e-a*(-a*f+c*d))*ln(x)/a^3-1/
4*(b^2*d-a*b*e-a*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/a^3+1/2*(b^3*d-a*b^2*e+2*a^2
*c*e-a*b*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1677, 1642, 648, 632, 212, 642}

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\log(x)\left(-abe-a(cd-af)+b^2d\right)}{a^3}+\frac{bd-ae}{2a^2x^2}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)}{2a^3\sqrt{b^2-4ac}}-\frac{d}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*d/(a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e -
a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^
2 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x])/a^3 - ((b^2*d - a*b*
e - a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^3}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)}{a^3x}+\frac{-b^3d+ab^2e-a^2ce\cdots}{\cdots}\right.\right.\\
&= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b^3d+ab^2e-a^2ce+\cdots}{\cdots}\right.}{\cdots}\\
&= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}-\frac{\left(b^2d-abe-a(cd-af\cdots\right.}{\cdots}\\
&= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}-\frac{\left(b^2d-abe-a(cd-af\cdots\right.}{4a^3\cdots}\\
&= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^3d-ab^2e+2a^2ce-ab(3cd-af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\cdots}{2a^3\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 314, normalized size = 1.80

$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-bd+ae)}{x^2}-4\left(b^2d-abe+a(-cd+af)\right)\log(x)+\frac{\left(b^3d+b^2\left(\sqrt{b^2-4ac}\,d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}\,e+af\right)+a\left(-c\sqrt{b^2-4ac}\,d+2ace+a\sqrt{b^2-4ac}\,f\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^3d+b^2\left(\sqrt{b^2-4ac}\,d+ae\right)-ab\left(-3cd+\sqrt{b^2-4ac}\,e+af\right)+a\left(-c\left(\sqrt{b^2-4ac}\,d+2ae\right)+a\sqrt{b^2-4ac}\,f\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d)
 + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d -
 Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*Sqr
t[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
+ ((-(b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^2 - 4
*a*c]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 - 4*a*c
]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/a^3
```

Maple [A]
time = 0.07, size = 203, normalized size = 1.17

method	result
default	$-\frac{\frac{\left(a^2cf-abce-a\,c^2d+b^2cd\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(a^2bf+a^2ce-a\,b^2e-2abcd+b^3d-\frac{\left(a^2cf-abce-a\,c^2d+b^2cd\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^3}-$
risch	$\frac{-\frac{(ae-bd)x^2}{2a^2}-\frac{d}{4a}}{x^4}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^2}-\frac{\ln(x)cd}{a^2}+\frac{\ln(x)b^2d}{a^3}+\frac{\Big(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^4-b^2a^3\right)\_Z^2+\left(4a^3cf-a^2b^2f-4a^2bce-4a^2c^2d+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*(1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*
b*f+a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d-1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)*b
/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/4*d/x^4/a-1/
2*(a*e-b*d)/a^2/x^2+(a^2*f-a*b*e-a*c*d+b^2*d)/a^3*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.91, size = 609, normalized size = 3.50

$$\left[\frac{\left(a^2bf+(b^3-3abc)d-(ab^2-2a^2c)e\right)\sqrt{b^2-4ac}\,x^4\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left((b^4-5ab^2c+4a^2c^2)d-(ab^3-4a^2bc)e+(a^2b^2-4a^3c)f\right)x^4\log\left(cx^4+bx^2+a\right)+4\left((b^4-5ab^2c+4a^2c^2)d-(ab^3-4a^2bc)e+(a^2b^2-4a^3c)f\right)x^4\log(x)+2\left((ab^3-4a^2bc)d-(a^2b^2-4a^3c)e\right)x^2-(a^2b^2-4a^3c)d}{4(a^3b^2-4a^4c)x^4},\ \frac{2\left(a^2bf+(b^3-3abc)d-(ab^2-2a^2c)e\right)\sqrt{-b^2+4ac}\,x^4\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-\left((b^4-5ab^2c+4a^2c^2)d-(ab^3-4a^2bc)e+(a^2b^2-4a^3c)f\right)x^4\log\left(cx^4+bx^2+a\right)+4\left((b^4-5ab^2c+4a^2c^2)d-(ab^3-4a^2bc)e+(a^2b^2-4a^3c)f\right)x^4\log(x)+2\left((ab^3-4a^2bc)d-(a^2b^2-4a^3c)e\right)x^2-(a^2b^2-4a^3c)d}{4(a^3b^2-4a^4c)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c)
*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*
a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^4 -
5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x
^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^2*b^
2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 - 3*a*b*c
)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2 + b)*sqr
t(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3
- 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*
f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^
2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.70, size = 212, normalized size = 1.22

$$-\frac{(b^2d-acd+a^2f-abe)\log\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{(b^2d-acd+a^2f-abe)\log\left(x^2\right)}{2\,a^3}-\frac{(b^3d-3\,abcd+a^2bf-ab^2e+2\,a^2ce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^3}-\frac{3\,b^2dx^4-3\,acdx^4+3\,a^2fx^4-3\,abx^4e-2\,abdx^2+2\,a^2x^2e+a^2d}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d - a*c*d + a^2*f - a*b*e)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2*
d - a*c*d + a^2*f - a*b*e)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d + a^2*b*f
- a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 + 3*a^2*f*x^4 - 3*a*b*x^4*e
- 2*a*b*d*x^2 + 2*a^2*x^2*e + a^2*d)/(a^3*x^4)
```

Mupad [**B**]
time = 9.92, size = 2500, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(b^2*d + a^2*f - a*b*e - a*c*d))/a^3 - (d/(4*a) + (x^2*(a*e - b*d))
/(2*a^2))/x^4 + (log((((((((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*
c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a
*b*c*d))/a^2 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^2*d + a^2*f + a^3*(
-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))
^(1/2) - a*b*e - a*c*d))/a^3)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2
*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d)
)/(4*a^3) + (c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5
*a*b*c*d))/a^4 + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*
d))/a^4)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*
a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d))/(4*a^3) + (c^4*(a*e
 - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 - (c^5*x^2*(a*e - b*d)^3)/a^
6)*((((c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5*a*b*c
*d))/a^4 - (((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*c*e + 5*a*b*c
*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^2
- (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f +
 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*
e + a*c*d))/a^3)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)
^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) + (
c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*d))/a^4)*(a^3*(-(b
^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1
/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) - (c^4*(a*e - b*d)^2*(b^2*d +
 a^2*f - a*b*e - a*c*d))/a^6 + (c^5*x^2*(a*e - b*d)^3)/a^6))*(2*b^4*d + 8*a
^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e
))/(2*(16*a^4*c - 4*a^3*b^2)) - (atan((16*a^9*(4*a*c - b^2)^(3/2)*(x^2*((((
a^3*c^5*e^3 - b^3*c^5*d^3 + 3*a*b^2*c^5*d^2*e - 3*a^2*b*c^5*d*e^2)/a^6 - ((
(6*a^4*b*c^4*e^2 - 5*a^3*b*c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*
a^5*c^4*e*f + 5*a^4*b*c^4*d*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*
d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 +
 ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*
a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3
*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a
*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d
 + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(1
6*a^4*c - 4*a^3*b^2)) + (((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3
*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2
)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c
*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b
*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 -
 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)*(2*b^4
*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a
```

```
^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a
*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3
*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f +
 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a
^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^
2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 1
2*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a
^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2
*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12
*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f
- 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b
^2*c*d + 8*a^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*
(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d
 + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)) + (((6*a^4*b*c^4*e^2 - 5*a^3*b*
c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*a^5*c^4*e*f + 5*a^4*b*c^4*d
*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^
2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^
3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*
b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^
2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*...
```

3.54 $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=244

$$-\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af))\tanh^{-1}\left(\right.}{2a^4\sqrt{b^2-4ac}}$$

```
[Out] -1/6*d/a/x^6+1/4*(-a*e+b*d)/a^2/x^4+1/2*(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x^2
-(b^3*d-a*b^2*e+a^2*c*e-a*b*(-a*f+2*c*d))*ln(x)/a^4+1/4*(b^3*d-a*b^2*e+a^2*
c*e-a*b*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^4-1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+
2*a^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1642, 648, 632, 212, 642}

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{bd-ae}{4a^2x^4}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\log(x)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{a^4}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d\right)}{2a^4\sqrt{b^2-4ac}}-\frac{d}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*d/(a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f))/
(2*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2
*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*Sqrt[b^2 -
 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x])/a^4 + (
(b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*
a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^4\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^4} + \frac{-bd+ae}{a^2x^3} + \frac{b^2d-abe-a(cd-af)}{a^3x^2} + \frac{-b^3d+ab^2e-a^2ce}{a^4x}\right.\right.$$

$$= -\frac{d}{6ax^6} + \frac{bd-ae}{4a^2x^4} - \frac{b^2d-abe-a(cd-af)}{2a^3x^2} - \frac{(b^3d-ab^2e+a^2ce-ab(2cd-a}{a^4}$$

$$= -\frac{d}{6ax^6} + \frac{bd-ae}{4a^2x^4} - \frac{b^2d-abe-a(cd-af)}{2a^3x^2} - \frac{(b^3d-ab^2e+a^2ce-ab(2cd-a}{a^4}$$

$$= -\frac{d}{6ax^6} + \frac{bd-ae}{4a^2x^4} - \frac{b^2d-abe-a(cd-af)}{2a^3x^2} - \frac{(b^3d-ab^2e+a^2ce-ab(2cd-a}{a^4}$$

$$= -\frac{d}{6ax^6} + \frac{bd-ae}{4a^2x^4} - \frac{b^2d-abe-a(cd-af)}{2a^3x^2} - \frac{(b^4d-ab^3e+3a^2bce+2a^2c(cd}{}$$

Mathematica [A]

time = 0.22, size = 416, normalized size = 1.70

$$\frac{-\frac{2a^3d}{x^6}+\frac{3a^2(bd-ae)}{x^4}+\frac{6a(-b^2d+abe+a(cd-af))}{x^2}-12(b^3d-ab^2e+a^2ce+ab(-2cd+af))\log(x)+\frac{3\left(b^4d+b^3\left(\sqrt{b^2-4ac}\,d-ae\right)+a^2c\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)+ab^2\left(-4cd-\sqrt{b^2-4ac}\,e+af\right)+ab\left(-2c\sqrt{b^2-4ac}\,d+3ace+a\sqrt{b^2-4ac}\,f\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{3\left(-b^4d+b^3\left(\sqrt{b^2-4ac}\,d+ae\right)-ab^2\left(-4cd+\sqrt{b^2-4ac}\,e+af\right)+a^2c\left(-2cd+\sqrt{b^2-4ac}\,e+2af\right)+ab\left(-2c\sqrt{b^2-4ac}\,d-3ace+a\sqrt{b^2-4ac}\,f\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*d
 - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x]
+ (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2 - 4
*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2*c*S
qrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 - 4*a*c]
*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e + a*Sq
rt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
)/(12*a^4)
```

Maple [A]
time = 0.08, size = 294, normalized size = 1.20

method	result
default	$-\frac{\frac{\left(-a^2bcf-a^2c^2e+a\,b^2ce+2ab\,c^2d-b^3cd\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(a^3cf-a^2b^2f-2a^2bce-a^2c^2d+a\,b^3e+3a\,b^2cd-b^4d-\frac{\left(-a^2bcf-a^2c^2e+a\,b^2ce+2a\right.}{2c}\right.}{\sqrt{4ac-b^2}}}{2a^4}$
risch	$\frac{-\frac{\left(a^2f-abe-acd+b^2d\right)x^4}{2a^3}-\frac{(ae-bd)x^2}{4a^2}-\frac{d}{6a}}{x^6}-\frac{\ln(x)bf}{a^2}-\frac{\ln(x)ce}{a^2}+\frac{\ln(x)b^2e}{a^3}+\frac{2\ln(x)bcd}{a^3}-\frac{\ln(x)b^3d}{a^4}+\frac{\left(\_R=\text{RootOf}\left(\left(4c\,a^5-\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4*(1/2*(-a^2*b*c*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)/c*ln(c*x
^4+b*x^2+a)+2*(a^3*c*f-a^2*b^2*f-2*a^2*b*c*e-a^2*c^2*d+a*b^3*e+3*a*b^2*c*d-
b^4*d-1/2*(-a^2*b*c*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/6*d/a/x^6-1/4*(a*e-b*d)
/a^2/x^4-1/2*(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x^2+1/a^4*(-a^2*b*f-a^2*c*e+a*b^
2*e+2*a*b*c*d-b^3*d)*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 1.64, size = 834, normalized size = 3.42

$$\left[-\frac{3\sqrt{b^2-4ac}\left((b^4-4ab^2c+2a^2c^2)d-(ab^3-3a^2bc)e+(a^2b^2-2a^3c)f\right)x^6\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-3\left((b^5-6ab^3c+8a^2bc^2)d-(ab^4-5a^2b^2c+4a^3c^2)e+(a^2b^3-4a^3bc)f\right)x^6\log(cx^4+bx^2+a)+12\left((b^5-6ab^3c+8a^2bc^2)d-(ab^4-5a^2b^2c+4a^3c^2)e+(a^2b^3-4a^3bc)f\right)x^6\log(x)+6\left((ab^4-5a^2b^2c+4a^3c^2)d-(a^2b^3-4a^3bc)e+(a^3b^2-4a^4c)f\right)x^4-3\left((a^2b^3-4a^3bc)d-(a^3b^2-4a^4c)e\right)x^2+2(a^3b^2-4a^4c)d}{12(a^4b^2-4a^5c)x^6},\ -\frac{6\sqrt{-b^2+4ac}\left((b^4-4ab^2c+2a^2c^2)d-(ab^3-3a^2bc)e+(a^2b^2-2a^3c)f\right)x^6\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-3\left((b^5-6ab^3c+8a^2bc^2)d-(ab^4-5a^2b^2c+4a^3c^2)e+(a^2b^3-4a^3bc)f\right)x^6\log(cx^4+bx^2+a)+12\left((b^5-6ab^3c+8a^2bc^2)d-(ab^4-5a^2b^2c+4a^3c^2)e+(a^2b^3-4a^3bc)f\right)x^6\log(x)+6\left((ab^4-5a^2b^2c+4a^3c^2)d-(a^2b^3-4a^3bc)e+(a^3b^2-4a^4c)f\right)x^4-3\left((a^2b^3-4a^3bc)d-(a^3b^2-4a^4c)e\right)x^2+2(a^3b^2-4a^4c)d}{12(a^4b^2-4a^5c)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a
^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 -
 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*
c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6
*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e
+ (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^
4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6), -1/12*(6*
sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e
 + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c +
4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^
2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d -
 (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3
*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 -
 4*a^5*c)*x^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.81, size = 313, normalized size = 1.28

$$\frac{(b^3d-2\,abcd+a^2bf-ab^2e+a^2ce)\log(cx^4+bx^2+a)}{4\,a^4}-\frac{(b^3d-2\,abcd+a^2bf-ab^2e+a^2ce)\log(x^2)}{2\,a^4}+\frac{(b^4d-4\,ab^2cd+2\,a^2c^2d+a^2b^2f-2\,a^3cf-ab^3e+3\,a^2bce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^4}+\frac{11\,b^3dx^6-22\,abcdx^6+11\,a^2bfx^6-11\,ab^2x^6e+11\,a^2cx^6e-6\,ab^2dx^4+6\,a^2cdx^4-6\,a^3fx^4+6\,a^2bx^4e+3\,a^2bdx^2-3\,a^3x^2e-2\,a^3d}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a
)/a^4 - 1/2*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(x^2)/a^4
+ 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d + a^2*b^2*f - 2*a^3*c*f - a*b^3*e
+ 3*a^2*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 + 11*a^2*b*f*x^6 - 11*a*b^2*x^6
*e + 11*a^2*c*x^6*e - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 - 6*a^3*f*x^4 + 6*a^2*b
*x^4*e + 3*a^2*b*d*x^2 - 3*a^3*x^2*e - 2*a^3*d)/(a^4*x^6)
```

Mupad [B]
time = 13.83, size = 2500, normalized size = 10.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^12*(4*a*c - b^2)^(3/2)*(x^2*((((a^3*c^8*d^3 - b^6*c^5*d^3 - a^6
*c^5*f^3 + 3*a*b^4*c^6*d^3 - 3*a^4*c^7*d^2*f + 3*a^5*c^6*d*f^2 - 3*a^2*b^2*
c^7*d^3 + a^3*b^3*c^5*e^3 + 3*a*b^5*c^5*d^2*e + 3*a^3*b*c^7*d^2*e + 3*a^5*b
*c^5*e*f^2 - 6*a^2*b^3*c^6*d^2*e - 3*a^2*b^4*c^5*d*e^2 + 3*a^3*b^2*c^6*d*e^
2 - 3*a^2*b^4*c^5*d^2*f + 6*a^3*b^2*c^6*d^2*f - 3*a^4*b^2*c^5*d*f^2 - 3*a^4
*b^2*c^5*e^2*f - 6*a^4*b*c^6*d*e*f + 6*a^3*b^3*c^5*d*e*f)/a^9 - (((11*a^5*b
*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*a^4*b
^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^6*b*c
^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f - 12*
a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8
*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 +
((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*
a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2
*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^
4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16
*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12
*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c -
 4*a^4*b^2)) + (((((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8*a^7*b
^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 + ((40*a
^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*
e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*a^9*(
16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*
c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) + ((40*a^10*b
*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c
*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*
b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(8*a
```

```
^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2
*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*
b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(32*a^17*(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b
^6*d - a^3*c^3*d + 3*a^2*b^4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d -
 15*a*b^4*c*d + 12*a^2*b^3*c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2
*(a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2
+ a^6*c^2*f^2 + 36*a^3*b^4*c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^
2*b^4*c^2*d^2 + 96*a^3*b^2*c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 -
12*a^2*b^6*d*f + 12*a^3*b^5*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4
*b*c^3*d*e + 72*a^3*b^4*c*d*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a
^3*b^3*c^2*d*e - 95*a^4*b^2*c^2*d*f)) + (((((((20*a^9*c^4*f - 20*a^8*c^5*d
+ 2*a^6*b^4*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 1
0*a^8*b*c^4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3
*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d
+ 10*a^2*b^2*c*e))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a
^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c
- b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^
2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(8*a^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))
*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*
c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) + (((11*
a^5*b*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*
a^4*b^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^
6*b*c^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f
- 12*a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*
d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a
^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e
 - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e
))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2
*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(
2*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a
^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) - ((40*a^1
0*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^
3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^3)/(64*a^21*(4*a*c - b^2)^(3/2)))*(6*b^7
*d + 6*a^2*b^5*f + 20*a^4*c^3*e - 6*a*b^6*e + 84*a^2*b^3*c^2*d - 54*a^3*b^2
*c^2*e - 42*a*b^5*c*d - 46*a^3*b*c^3*d + 36*a^2*b^4*c*e - 30*a^3*b^3*c*f +
26*a^4*b*c^2*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2...
```

3.55 $\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=369

$$\frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4\cdots}{\sqrt{b^2-4a\cdots}}\right.}{\sqrt{2}\,c^{7/2}\sqrt{b-\sqrt{b^2-\cdots}}}$$

```
[Out] (c^2*d+b^2*f-c*(a*f+b*e))*x/c^3+1/3*(-b*f+c*e)*x^3/c^2+1/5*f*x^5/c+1/2*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b^3*f-b
*c*(-2*a*f+c*d)+(-b^3*c*e+3*a*b*c^2*e+b^4*f+b^2*c*(-4*a*f+c*d)-2*a*c^2*(-a*
f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b
^3*f-b*c*(-2*a*f+c*d)+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2
*(-a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A]

time = 3.12, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1678, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}\,c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}\,c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(-c(af+be)+b^2f+c^2d)}{c^3}+\frac{x^3(ce-bf)}{3c^2}+\frac{fx^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3*a
*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d
- 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \int \left(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x^2}{c^2}+\frac{fx^4}{c}-\frac{a(c^2d+b^2f-c(be+af))}{c^3} \ldots\right.$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}-\frac{\int \frac{a(c^2d+b^2f-c(be+af))+(-b^2ce+\ldots}{a+bx^2+cx\ldots}}{c^3}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd\ldots\right.}{\ldots}$$

$$= \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd\ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.32, size = 456, normalized size = 1.24

$$\frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}-\frac{\left(-b^4f-b^2c\left(cd+\sqrt{b^2-4ac}\,e-4af\right)+ac^2\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)+b^3\left(ce+\sqrt{b^2-4ac}\,f\right)+bc\left(c\sqrt{b^2-4ac}\,d-3ace-2a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^4f+b^2c\left(cd-\sqrt{b^2-4ac}\,e-4af\right)+ac^2\left(-2cd+\sqrt{b^2-4ac}\,e+2af\right)+b^3\left(-ce+\sqrt{b^2-4ac}\,f\right)+bc\left(c\sqrt{b^2-4ac}\,d+3ace-2a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2*(
2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) + b*
c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 - 4*a*c
```

```
]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-(c*e) +
 Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*Sqrt[b^2
 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.06, size = 453, normalized size = 1.23

method	result
risch	$\frac{f x^5}{5c} - \frac{bf x^3}{3c^2} + \frac{e x^3}{3c} - \frac{afx}{c^2} + \frac{b^2 fx}{c^3} - \frac{bex}{c^2} + \frac{dx}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\left(2abcf-a c^2 e-b^3 f+b^2 ce-b c^2 d\right)\_R^2+a^2 c \ldots\right.}{2c\_R^3+\_ \ldots}}{2c^3}$
	$-\frac{\left(2\sqrt{-4ac+b^2}\, abcf-a c^2 e\sqrt{-4ac+b^2}-b^3 f\sqrt{-4ac+\ldots\right.}{\ldots}$
default	$-\frac{-\frac{1}{5} f x^5 c^2+\frac{1}{3} bcf x^3-\frac{1}{3} c^2 e x^3+acfx-b^2 fx+bcex-c^2 dx}{c^3} + \ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/5*f*x^5*c^2+1/3*b*c*f*x^3-1/3*c^2*e*x^3+a*c*f*x-b^2*f*x+b*c*e*x-
c^2*d*x)+4/c^2*(-1/8*(2*(-4*a*c+b^2)^(1/2)*a*b*c*f-a*c^2*e*(-4*a*c+b^2)^(1/
2)-b^3*f*(-4*a*c+b^2)^(1/2)+b^2*c*e*(-4*a*c+b^2)^(1/2)-b*c^2*d*(-4*a*c+b^2)
^(1/2)+2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*c^3*a*d+b^4*f-b^3*c*e+b^2*c^2*
d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(2*(-4*a*c+b^2)^(1/2)*a*b
*c*f-a*c^2*e*(-4*a*c+b^2)^(1/2)-b^3*f*(-4*a*c+b^2)^(1/2)+b^2*c*e*(-4*a*c+b^
2)^(1/2)-b*c^2*d*(-4*a*c+b^2)^(1/2)-2*a^2*c^2*f+4*a*b^2*c*f-3*a*b*c^2*e+2*c
^3*a*d-b^4*f+b^3*c*e-b^2*c^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*f*x^5 - 5*(b*c*f - c^2*e)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c)
*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - b^2*c*e + a*c^2*e +
 (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 15467 vs. $2(331) = 662$.
time = 40.75, size = 15467, normalized size = 41.92

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*c^2*f*x^5 - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(
b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c
^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^
2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2
*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^1
0)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*
a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 +
7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^
6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c
^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^
7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3
*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6
- 19*a^3*b^2*c^7 + 3*a^4* ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7243 vs. $2(335) = 670$.
time = 6.12, size = 7243, normalized size = 19.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
```

```
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d +
(2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 -
 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*f - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2
*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2
 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3
 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c
^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*e - 2*(sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^5 + 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c^5 + 8*(b^2 - 4
*a*c)*a^2*c^6)*d*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*
c^2 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 18*a
^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^5 + 48*a^3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a*
c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*f*abs(c) + 2*(sqrt(2)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5
*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*
a^2*b*c^5)*abs(c)*e - (2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*a
*b*c^7)*d - (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4...
```

Mupad [B]
time = 4.91, size = 2500, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*(e/(3*c) - (b*f)/(3*c^2)) - x*((b*(e/c - (b*f)/c^2))/c - d/c + (a*f)/c^
2) + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*
d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 -
 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^
2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*
c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4
*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a
^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*
c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6
*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5
*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*
c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f
+ 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^
```

3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*
e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/
2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*
f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c
^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*
d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2
*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e
 + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*
d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*
b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) -
 (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^
2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b
^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^
3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5
*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*
c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2
+ b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a
*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28
*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 -
 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e -
16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a
^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*
a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c -
 b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*
f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c
^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*
c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5
 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b
^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^
5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^

```
4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63
*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*
a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*
b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4...
```

3.56 $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=282

$$\frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(c^2d\right.}{}$$

```
[Out] (-b*f+c*e)*x/c^2+1/3*f*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a*c*f+(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+
c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a
*c*f+(-b^2*c*e+2*a*c^2*e+b^3*f+b*c*(-3*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2
)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.42, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1678, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(ce-bf)}{c^2}+\frac{fx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b^
2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx &= \int \left(\frac{ce-bf}{c^2}+\frac{fx^2}{c}-\frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x^2}{c^2\left(a+bx^2+cx^4\right)}\right)dx\\ &= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}-\frac{\int \frac{a(ce-bf)+\left(-c^2d+bce-b^2f+acf\right)x^2}{a+bx^2+cx^4}\,dx}{c^2}\\ &= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+\frac{1}{2}}}{2c^2}\\ &= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 365, normalized size = 1.29

$$\frac{6\sqrt{c}\,(ce-bf)x+2c^{3/2}fx^3+\frac{3\sqrt{2}\left(-b^3f-bc\left(cd+\sqrt{b^2-4ac}\,e-3af\right)+b^2\left(ce+\sqrt{b^2-4ac}\,f\right)+c\left(c\sqrt{b^2-4ac}\,d-2ace-a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\left(b^3f+bc\left(cd-\sqrt{b^2-4ac}\,e-3af\right)+b^2\left(-ce+\sqrt{b^2-4ac}\,f\right)+c\left(c\sqrt{b^2-4ac}\,d+2ace-a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f) - b*c*(c*
d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) + c*(c*S
qrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (3*Sqrt[2]*(b^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b
^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [A]
time = 0.06, size = 333, normalized size = 1.18

method	result
risch	$\frac{f\,x^{3}}{3c}-\frac{bfx}{c^{2}}+\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\left(-acf+b^{2}f-bce+c^{2}d\right)\_R^{2}+abf-ace\right)\ln\left(x-\_R\right)}{2c\_R^{3}+\_Rb}}{2c^{2}}$
default	$-\frac{-\frac{1}{3}c\,x^{3}f+bfx-cex}{c^{2}}+\frac{-\frac{\left(-acf\sqrt{-4ac+b^{2}}+b^{2}f\sqrt{-4ac+b^{2}}-bce\sqrt{-4ac+b^{2}}+c^{2}d\sqrt{-4ac+b^{2}}+3abcf-2a\,c^{2}\cdots\right.}{2\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\sqrt{-4ac\cdots\right.}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/3*c*x^3*f+b*f*x-c*e*x)+4/c*(-1/8*(-a*c*f*(-4*a*c+b^2)^(1/2)+b^2*
f*(-4*a*c+b^2)^(1/2)-b*c*e*(-4*a*c+b^2)^(1/2)+c^2*d*(-4*a*c+b^2)^(1/2)+3*a*
b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))+1/8*(-a*c*f*(-4*a*c+b^2)^(1/2)+b^2*f*(-4*a*c+b^2)^(1/2)-b*c*e*(-4*a*
c+b^2)^(1/2)+c^2*d*(-4*a*c+b^2)^(1/2)-3*a*b*c*f+2*a*c^2*e+b^3*f-b^2*c*e+b*c
^2*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*f*x^3 - 3*(b*f - c*e)*x)/c^2 - integrate(-(a*b*f + (c^2*d - b*c*e +
(b^2 - a*c)*f)*x^2 - a*c*e)/(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9364 vs. $2(246) = 492$.
time = 8.23, size = 9364, normalized size = 33.21

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*f*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d
*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((
b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5
- 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 -
4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 -
 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^
3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2
- 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3
 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c
^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^
3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4
*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*
b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3
*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^
5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 +
 ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2
- a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + sqrt(1/2)*((b^
2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2
*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^
3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 +
29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*
c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c
^4)*e^2)*f + (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5
- 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6
- a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a
^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*
f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a
*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^
5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6
*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*
c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c
^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c
^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)
*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 -
 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^
3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^
3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c
^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^
5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3
*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*
```

```
c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^
2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2
+ (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*
c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (
b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 -
a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2
*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 +
5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^
2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*
c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*
e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c
^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2
*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a
^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((
b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3
)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3
*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c
^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3
*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 -
7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5461 vs. 2(252) = 504.
time = 5.95, size = 5461, normalized size = 19.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
```

(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d + (2*b^6*c^2
 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 -
 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2
*c^4)*c^2*f - (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c
^2*e + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)
*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(c) - 2*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c
^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*a
bs(c)*e - (2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*
a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6
*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 1
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6

```
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c
)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*f + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a
^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^
3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^
5 + 4*(b^2 - 4*a*c)*a*b*c^6)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2
*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c
^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/8*((2*b^4*c^4 - 16*a*b^2
*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq...
```

Mupad [B]
time = 3.36, size = 2500, normalized size = 8.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(e/c - (b*f)/c^2) - atan(((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f
 - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4
*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
+ 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
```

12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^
3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2
 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*
c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7
*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^
2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e +
 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*
a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d
f(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*a^2*c^5*e - 4*a*b^2*c
^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*
(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 -
 b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
 + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4
*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
+ 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2
+ b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f
 + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^

```
4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c
^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*
b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) ...
```

3.57 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=219

$$\frac{fx}{c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\tan^{-1}(\ldots}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4\ldots}}}$$

```
[Out] f*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*e-b*f+(
2*c^2*d+b^2*f-c*(2*a*f+b*e))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(c*e-b*f+(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1690, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f -
2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x^2}{c\,(a+bx^2+cx^4)}\right)dx$$

$$= \frac{fx}{c}+\frac{\int \frac{cd-af+(ce-bf)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{fx}{c}+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-}{\sqrt{b^2-}}\right.}{}$$

$$= \frac{fx}{c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(ce-bf-\frac{2c^2}{}\right.}{}$$

Mathematica [A]
time = 0.20, size = 258, normalized size = 1.18

$$\frac{2\sqrt{c}\,fx+\frac{\sqrt{2}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)f+c\left(-be+\sqrt{b^2-4ac}\,e-2af\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right)f-c\left(be+\sqrt{b^2-4ac}\,e+2af\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e
) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*
(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [A]
time = 0.04, size = 224, normalized size = 1.02

method	result

risch	$\frac{fx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left((-bf+ce)\_R^2-fa+cd\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{fx}{c}-\frac{\left(-bf\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,ce-2acf+b^2f-bce+2c^2d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] f*x/c-1/2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e-2*a*c*f+b^2*f-b*c
*e+2*c^2*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*(-b*f*(-4*a*c+b^
2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(-4*a*c+b^2)^(
1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/c - integrate(((b*f - c*e)*x^2 - c*d + a*f)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5788 vs. $2(185) = 370$.
time = 4.06, size = 5788, normalized size = 26.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3
*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*
a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b
^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2
)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^
2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*
```

b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c
^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4
- ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^
2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2
+ (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c
^4)*d^3)*f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4
)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c
^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4
*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)
*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a
^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4
+ (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a
^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)
*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e -
a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*
d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d -
(a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^
5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b
^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e +
(a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^
5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*
a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c
^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c -
2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 +
a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^
3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3
- 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b
*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/
(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2
*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (
a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2
- (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3
*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(1/2)*((b^2*c^4 - 4*a*
c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c
^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*
b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3
)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2
*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4
- 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 +
4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*
c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2
- 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2
*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 -
3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 -
4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3

```
*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c
^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*
c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^
2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqr
t(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)
*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*
sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3
- 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c
^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3
)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 -
b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4
 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e
^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*
c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*
f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4086 vs. $2(189) = 378$.
time = 7.59, size = 4086, normalized size = 18.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/c + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*f -
(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
```

(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*d*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*f*abs(c) - 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*f + (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b

```
*c^3)*c^2*f - (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 +...
```

Mupad [B]
time = 3.36, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x)/c - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)
/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 +
12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*
a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*
c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b
^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3
 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b
^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^
3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c
^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d
*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*
e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*
a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*
f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2
 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2
 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 +
16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2
*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*
a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
```

```
c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*
d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3
*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 +
 a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*
c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2
- c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2
*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*
c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d
*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^
2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^
5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^
3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e
*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*
c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^
2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b
^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((((4*b^2*c^3*d + 16*a^2
*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a
*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 -
 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*
c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*
f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^
2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d
*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2
+ b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c
^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(
-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - ...
```

3.58 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=213

$$-\frac{d}{ax}-\frac{\left(cd-af+\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-a*f+
(a*b*f-2*a*c*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c
*d-a*f+(-a*b*f+2*a*c*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1678, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{d}{ax^2}+\frac{-bd+ae-(cd-af)x^2}{a\left(a+bx^2+cx^4\right)}\right)dx$$

$$= -\frac{d}{ax}+\frac{\int \frac{-bd+ae+(-cd+af)x^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{d}{ax}-\frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}+\frac{\left(-cd+af+\frac{2ace-\cdots}{\sqrt{b^2\cdots}}\right.\cdots}{\cdots}$$

$$= -\frac{d}{ax}-\frac{\left(cd-af-\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(cd-af-\frac{b\cdots}{\cdots}\right.\cdots}{\cdots}$$

Mathematica [A]
time = 0.19, size = 253, normalized size = 1.19

$$\frac{-\frac{2d}{x}-\frac{\sqrt{2}\left(bcd+c\sqrt{b^2-4ac}\,d-2ace+abf-a\sqrt{b^2-4ac}\,f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(bcd-c\sqrt{b^2-4ac}\,d-2ace+abf+a\sqrt{b^2-4ac}\,f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*S
qrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*c*d
 - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [A]
time = 0.06, size = 220, normalized size = 1.03

method	result
default	$\frac{4c\left(-\frac{\left(af\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}-abf+2ace-bcd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(af\sqrt{-4\right.}{}\right)}{a}$
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^4d^4-2bc^3ed^3+b^2c^2d^2e^2+2ac^3d^2e^2+\left(16a^5c^3-8b^2c^2a^4+b^4ca^3\right)\_Z^4+\left(-4a^4bcf^2+16a^4c^2ef+a^3b^3f^2-4a^3b^2cef-\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(-1/8*(a*f*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/2)-a*b*f+2*a*c*e-b*
c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(a*f*(-4*a*c+b^2)^(1/2)
-c*d*(-4*a*c+b^2)^(1/2)+a*b*f-2*a*c*e+b*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)))-d/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(177) = 354$.
time = 1.68, size = 5930, normalized size = 27.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^
2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^3*b^2*c
```

- 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f
^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*
e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a
^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^
2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4
*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*
f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b
^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 +
(3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 -
3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x + sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a
^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3
*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b
*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^
3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d -
(a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c
^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^
2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/
(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b
*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^
3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3
 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c
^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*
e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e
^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c
 - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3
*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f +
 (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d
*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2
*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^
5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2
*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b
^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^
2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*
b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)
*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2
*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - sqrt(1/2)*((b^5*c - 5*a*b
^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e
+ 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b
^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^
2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a
^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt
(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2
*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*

```
c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*
c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c
^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b
^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*
c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4
*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c
^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e -
 a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3))
)/(a^3*b^2*c - 4*a^4*c^2))) + sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2
+ (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*
a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4
 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b
^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*
b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2
*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c
^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*
e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d
^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c
- 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3988 vs. 2(179) = 358.
time = 8.51, size = 3988, normalized size = 18.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -d/(a*x) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
```

```
2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*a^2*d - (
2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*a^2*f + 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*c)*a*b^3*c^2 -
8*(b^2 - 4*a*c)*a^2*b*c^3)*d*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 + 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 -
 32*a^4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*abs(a)
*e + (2*a^2*b^4*c^3 - 8*a^3*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^3)*d + (2*a^3*b^4*c^2
 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2
 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*f - 2*(2*a^3*b^3*c^3 - 8*a^4*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a^3*
b*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))
/((a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a
^3*b^2*c^3 - 4*a^4*c^4)*abs(a)*abs(c)) + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 3
2*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b
```

```
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*
(b^2 - 4*a*c)*a*c^4)*a^2*d - (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2
- 4*a*c)*a^2*c^3)*a^2*f - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*
c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 16*a^2*b^
3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*...
```

Mupad [B]
time = 3.52, size = 2500, normalized size = 11.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
```

c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2)*1i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d - 4*a^5*b^2*c^2*e - 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2)*1i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^

```
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2...
```

3.59 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=267

$$-\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}\right.\right.}{}$$

```
[Out] -1/3*d/a/x^3+(-a*e+b*d)/a^2/x+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*c^(1/2)*(b*d-a*e+(b^2*d-a*b*e-2*a*(-a*f+c*d))/(-4*a*c+b^2)^(1
/2))/a^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-b*(a*e+d*(-4*a*c+b^2)^(1/2))-a
*(2*c*d-2*a*f-e*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1678, 1180, 211}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{b^2-4ac}+ae\right)+b^2d\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bd-ae}{a^2x}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*d/(a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*e
 - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(
b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*
a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2
*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a^2\left(a+bx^2+cx^4\right)}\right)dx$$

$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a+bx^2+cx^4}\,dx}{a^2}$$

$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\left(c\left(bd-ae-\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}$$

$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4a}}}\right.}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.22, size = 284, normalized size = 1.06

$$\frac{-\frac{2ad}{x^3}+\frac{6bd-6ae}{x}+\frac{3\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}\,d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}\,e+2af\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2d+b\left(\sqrt{b^2-4ac}\,d+ae\right)-a\left(-2cd+\sqrt{b^2-4ac}\,e+2af\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2
 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d +
 a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(6*a^2)
```

Maple [A]
time = 0.07, size = 244, normalized size = 0.91

method	result
default	$\frac{4c\left(-\frac{\left(-\sqrt{-4ac+b^2}\,ae+bd\sqrt{-4ac+b^2}+2a^2f-abe-2acd+b^2d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-\sqrt{\right.}}{}\right.}{a^2}$
risch	$\frac{-\frac{(ae-bd)x^2}{a^2}-\frac{d}{3a}}{x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c^5d^4+2ac^4d^2e^2+\left(16c^2a^7-8b^2ca^6+b^4a^5\right)\_Z^4+2a^2bc^2def^2-2ab^3cdef^2-2a^3bcef^3+2a^2b^2cdf^3+a^2b^2c\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*c*(-1/8*(-(-4*a*c+b^2)^(1/2)*a*e+b*d*(-4*a*c+b^2)^(1/2)+2*a^2*f-a*b*e
-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-(-4*a*c+b^2)
^(1/2)*a*e+b*d*(-4*a*c+b^2)^(1/2)-2*a^2*f+a*b*e+2*a*c*d-b^2*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)))-1/3*d/a/x^3-(a*e-b*d)/a^2/x
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((a^2*f + (b*c*d - a*c*e)*x^2 - a*b*e + (b^2 - a*c)*d)/(c*x^4 + b*
x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*x^3)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9850 vs. $2(226) = 452$.
time = 11.02, size = 9850, normalized size = 36.89

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2
```

```
 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6
*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2
*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5
 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4
 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*
a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2
 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 -
9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^
2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)
)*log(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b
^3*c^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^
2*b^3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e
 - (3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3
*b^2*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c
- 5*a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2
 + 3*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^
4*b*c^2)*e^3)*f)*x + sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*
b^2*c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^
4*b*c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*
e^2 - (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 +
 3*((a^4*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 +
((3*a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^
5 - 16*a^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^
2)*e^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^
2*c + 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a
^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e
^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)
*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a
^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*
a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e
+ (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(
a^10*b^2 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 +
2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)
*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3
*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 -
3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 -
4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6
*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 +
4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a
^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 -
```

```
 (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))
- 3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^
2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2
*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*
sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*
a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3
*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4
*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*
c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4
*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^
4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 -
(a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*lo
g(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c
^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^
3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (
3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2
*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*
a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3
*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3813 vs. $2(231) = 462$.
time = 7.06, size = 3813, normalized size = 14.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
```

2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*
c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32
*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c
)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4
*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*d + (sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
2 + 16*a^3*b^2*c^2 + 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*c^3 - 32*a^4*c^3 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2 -
2*(b^2 - 4*a*c)*a^2*b*c^2)*f - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2
 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a
^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)
*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b +
 sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c +
 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*((sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c
+ 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b

```
^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 2
4*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10
*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^
3 - 6*(b^2 - 4*a*c)*a*b*c^3)*d + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*a^3*b
^2*c^2 - 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3
+ 32*a^4*c^3 + 8*a^3*b*c^3 + sqrt(2)*sqrt(b^2 -...
```

Mupad [B]
time = 4.76, size = 2500, normalized size = 9.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^
2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*
e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 +
 a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e
^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2
+ a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3
*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b
^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2
)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*
b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(
4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-
(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a
^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a
^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^
3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d
*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
```

+ 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i + (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f + 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f + 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7

```
*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*
a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-...
```

3.60 $\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=329

$$-\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\,a^3\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/5*d/a/x^5+1/3*(-a*e+b*d)/a^2/x^3+(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x-1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-
a*(-a*f+c*d)+(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))
/a^3*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-a*(-a*f+c*d)+(-b^3*d+a*b^2*e
-2*a^2*c*e+a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a^3*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A]

time = 1.30, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1678, 1180, 211}

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}\,a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}\,a^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*d/(a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))/
(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*a^
2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(
3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{d}{ax^6}+\frac{-bd+ae}{a^2x^4}+\frac{b^2d-abe-a(cd-af)}{a^3x^2}+\frac{-b^3d+ab^2e-a^2ce+ab(2cd \cdots}{a^3(a \cdots}\right.$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}+\frac{\int \frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c(b^2d-ab \cdots}{a+bx^2+cx^4}}{a^3}$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\left(c\left(b^2d-abe-a(cd-af)-\frac{b^3d \cdots}{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3 \cdots}{\cdots}\right.}{\sqrt{2 \cdots}}$$

Mathematica [A]
time = 0.36, size = 394, normalized size = 1.20

$$\frac{-\frac{6a^2d}{x^5}+\frac{10a(bd-ae)}{x^3}+\frac{30(-b^2d+abe+a(cd-af))}{x}-\frac{15\sqrt{2}\sqrt{c}\left(b^3d+b^2\left(\sqrt{b^2-4ac}\,d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}\,e+af\right)+a\left(-c\sqrt{b^2-4ac}\,d+2ace+a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{15\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ab\left(-3cd+\sqrt{b^2-4ac}\,e+af\right)+a\left(c\sqrt{b^2-4ac}\,d+2ace-a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{30a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d -
 a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) +
a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*
a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sqrt[2]
*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqrt[b^2
- 4*a*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2 - 4*a*c
```

```
]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

Maple [A]
time = 0.08, size = 360, normalized size = 1.09

method	result
default	$\frac{4c\left(-\frac{\left(-a^2f\sqrt{-4ac+b^2}+abe\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,acd-\sqrt{-4ac+b^2}\,b^2d-a^2bf-2a^2ce+a\,b^2e+3abcd-b^3d\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$
risch	$\frac{-\frac{\left(a^2f-abe-acd+b^2d\right)x^4}{a^3}-\frac{(ae-bd)x^2}{3a^2}-\frac{d}{5a}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(-4a\,c^6d^3f+2b^2c^5d^3f-2a^2b\,c^4e^3f-4a^2c^5d\,e^2f-2b^3c^4d^2ef-4a^3c^4d\,f^3+2a^3c\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^3*c*(-1/8*(-a^2*f*(-4*a*c+b^2)^(1/2)+a*b*e*(-4*a*c+b^2)^(1/2)+(-4*a*c+b
^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1/2)*b^2*d-a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*
d-b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a^2*f*(-4*a*c+b^2)^
(1/2)+a*b*e*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1/2)*
b^2*d+a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))-1/5*d/a/x^5-1/3*(a*e-b*d)/a^2/x^3-(a^2*f-a*b*e-a*c*d+b^2*d)/a^
3/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b*f - a*b^2*e + a^2*c*e + (a^2*c*f - a*b*c*e + (b^2*c - a*c
^2)*d)*x^2 + (b^3 - 2*a*b*c)*d)/(c*x^4 + b*x^2 + a), x)/a^3 - 1/15*(15*(a^2
*f - a*b*e + (b^2 - a*c)*d)*x^4 + 3*a^2*d - 5*(a*b*d - a^2*e)*x^2)/(a^3*x^5
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15830 vs. $2(289) = 578$.

time = 42.36, size = 15830, normalized size = 48.12

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)
*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 -
 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c
^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3
+ 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5
*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^
6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^
2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5
- 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6
*b^4*c^2 - 19*a^7*b^2*c^3 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6718 vs. $2(297) = 594$.
time = 5.53, size = 6718, normalized size = 20.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
```

b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4)*a^2*d + (2*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 32*a^4*c^4
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^
2 + 8*(b^2 - 4*a*c)*a^3*c^3)*a^2*f - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3
*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b
^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 32*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 12*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c
^2 + 20*a^2*b^5*c^2 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3
 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 6*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 64*a^3*b^3*c^3 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c
 - 12*(b^2 - 4*a*c)*a^2*b^3*c^2 + 16*(b^2 - 4*a*c)*a^3*b*c^3)*d*abs(a) + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^4*c - 2*a^3*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^2
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 16*a^4*b^3*c^2 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 32*a^5*b*c^3 + 2*(b^2 - 4*a*c)*a^3*b^3*c -
 8*(b^2 - 4*a*c)*a^4*b*c^2)*f*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*a^2*b^6*c + 24*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
*c^2 + 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3

```
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 48*a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^2*b^4*c - 10*
(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^4*c^3)*abs(a)*e + (2*a^2*b^6*
c^2 - 14*a^3*b^4*c^3 + 24*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^4*c^2 + 6*(b^2 - 4*a*c)*a^3*b^2
*c^3)*d + (2*a^4*b^4*c^2 - 8*a^5*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^2 - 2*(b^2 - 4*a*c)*a^4*b^2*c^2)*f - (2*a^3*b^5*c^2
- 12*a^4*b^3*c^3 + 16*a^5*b*c^4 - sqrt(2)*sqrt(...
```

Mupad [B]
time = 6.25, size = 2500, normalized size = 7.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*
d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^
12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e +
12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c
^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e
^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*
a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5
*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2)
 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f -
16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a
^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b
^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^
2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c -
```

b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c
^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f
^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^
2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 4
2*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/
2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^
7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f -
2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f
- 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*
d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2
)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a
^15*c^4*e + 4*a^12*b^5*c^2*d - 24*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^
14*b^2*c^3*e + 4*a^14*b^3*c^2*f + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^
9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a
^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^
6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5
*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*
c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 +
2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5
*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*
c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a
^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b
*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i + (x*(4*a^13
*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^
4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 +
2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*
f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12
*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(
4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 -
20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a

```
^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e -
 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(...
```

3.61 $\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)}+\frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)}+\frac{x^6(2ace-b(cd+af)-(2c}{2c\,(b^2-4ac)\,(a}$$

```
[Out] 1/2*(2*b^2*c*e-6*a*c^2*e-3*b^3*f-b*c*(-11*a*f+c*d))*x^2/c^3/(-4*a*c+b^2)+1/
4*(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*x^4/c^2/(-4*a*c+b^2)+1/2*x^6*(2*a*c*e-b
*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/2*(2*b^4*c*e-12*a*b^2*c^2*e+12*a^2*c^3*e-3*b^5*f-b^3*c*(-20*a*f+c*d)+6
*a*b*c^2*(-5*a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+
b^2)^(3/2)+1/4*(c^2*d+3*b^2*f-2*c*(a*f+b*e))*ln(c*x^4+b*x^2+a)/c^4
```

Rubi [A]
time = 0.81, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1677, 1658, 814, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^4(-2c(4af+be)+3b^2f+4c^2d)}{4c^2\left(b^2-4ac\right)}+\frac{x^6(-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)(-2c(af+be)+3b^2f+c^2d)}{4c^4}+\frac{x^2(-bc(cd-11af)-6ac^2e-3b^3f+2b^2ce)}{2c^3\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c^3*(b^2 -
4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)/(4*c^2*(b^2 - 4*a*c
)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e
+ 12*a^2*c^3*e - 3*b^5*f - b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c
^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex+fx^2)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3\left(2ae-\frac{b(cd}{}\right.\right.}{}\right.}{}$$

$$= \frac{x^6(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int\left(-\frac{2b^2ce-6ac^2e}{}\right.\right.}{}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)}$$

$$= \frac{(2b^2ce-6ac^2e-3b^3f-bc(cd-11af))\,x^2}{2c^3\,(b^2-4ac)} + \frac{(4c^2d+3b^2f-2c(be+4af))\,x^4}{4c^2\,(b^2-4ac)}$$

Mathematica [A]
time = 0.32, size = 309, normalized size = 0.97

$$\frac{2c(ce-2bf)x^2+c^2fx^4+\frac{2(2a^3c^2f+b^3(c^2d-bce+b^2f)x^2+ab(b^3f-3c^3dx^2+bc^2(d+4ex^2)-b^2c(e+5fx^2))+a^2c(-4b^2f-2c^2(d+ex^2)+bc(3e+5fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2(-2b^4ce+12ab^2c^2e-12a^2c^3e+3b^5f+b^3c(cd-20af)+6abc^2(-cd+5af))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+(c^2d+3b^2f-2c(be+af))\log(a+bx^2+cx^4)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e +
 b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e + 5
*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2
*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*ArcTa
n[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d + 3*b^2*
f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [A]
time = 0.15, size = 432, normalized size = 1.35

method	result
default	$\frac{(-c x^2 f+2bf-ce)^2}{4c^4 f}+\frac{-\frac{(5a^2 b c^2 f-2a^2 c^3 e-5a b^3 c f+4a b^2 c^2 e-3ab c^3 d+b^5 f-b^4 ce+b^3 c^2 d) x^2}{c(4ac-b^2)}-\frac{a(2a^2 c^2 f-4a b^2 c f+3ab c^2 e-2c^3 ad+b^4 f-b^3 c}{c(4ac-b^2)}}{c x^4+b x^2+a}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c*f*x^2+2*b*f-c*e)^2/c^4/f+1/2/c^3*((-(5*a^2*b*c^2*f-2*a^2*c^3*e-5*a*
b^3*c*f+4*a*b^2*c^2*e-3*a*b*c^3*d+b^5*f-b^4*c*e+b^3*c^2*d)/c/(4*a*c-b^2)*x^
2-a*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)
/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*a^2*c^2*f+14*a*b^2*c
*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2
*(11*a^2*b*c*f-6*a^2*c^2*e-3*a*b^3*f+2*a*b^2*c*e-a*b*c^2*d-1/2*(-8*a^2*c^2*
f+14*a*b^2*c*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)*b/c)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(306) = 612$.
time = 0.63, size = 2111, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b
```

```
^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d -
 (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c
+ 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^
4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3
)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^
3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*
c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*
c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a*b^4*c^2
 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3
)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f + (((b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e +
 (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*x^4 + ((b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*
e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*f)*x^2 + (a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*f)*log(c*x^4
 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^2), 1/4*(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3
 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d - (b^6*
c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c + 41*a
^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 + 2*(((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^
2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)
*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*c^3)
*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*c +
30*a^3*b*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) + 2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c
 - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^
2 - 8*a^4*c^3)*f + (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 -
32*a^3*c^4)*f)*x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c -
 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 3
2*a^3*b*c^3)*f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5
*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2
*c^2 - 32*a^4*c^3)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 +
16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.53, size = 424, normalized size = 1.32

$$-\frac{\left(b^{3} c^{2} d - 6 \, a b c^{3} d + 3 \, b^{5} f - 20 \, a b^{3} c f + 30 \, a^{2} b c^{2} f - 2 \, b^{4} c e + 12 \, a b^{2} c^{2} e - 12 \, a^{2} c^{3} e\right)\arctan\left(\frac{2 \, c x^{2} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{2 \, {\left(b^{2} c^{4} - 4 \, a c^{5}\right)} \sqrt{-b^{2} + 4 \, a c}} - \frac{b^{2} c^{3} d x^{4} - 4 \, a c^{4} d x^{4} + 3 \, b^{4} c f x^{4} - 14 \, a b^{2} c^{2} f x^{4} + 8 \, a^{2} c^{3} f x^{4} - 2 \, b^{3} c^{2} x^{4} e + 8 \, a b c^{3} x^{4} e - b^{3} c^{2} d x^{2} + 2 \, a b c^{3} d x^{2} + b^{5} f x^{2} - 4 \, a b^{3} c f x^{2} - 2 \, a^{2} b c^{2} f x^{2} + 4 \, a^{2} c^{3} x^{2} e - a b^{2} c^{2} d + a b^{4} f - 6 \, a^{2} b^{2} c f + 4 \, a^{3} c^{2} f + 2 \, a^{2} b c^{2} e}{4 \, {\left(b^{2} c^{4} - 4 \, a c^{5}\right)} {\left(c x^{4} + b x^{2} + a\right)}} + \frac{{\left(c^{2} d + 3 \, b^{2} f - 2 \, a c f - 2 \, b c e\right)} \log\left(c x^{4} + b x^{2} + a\right)}{4 \, c^{4}} + \frac{c^{2} f x^{4} - 4 \, b c f x^{2} + 2 \, c^{2} x^{2} e}{4 \, c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^2*d - 6*a*b*c^3*d + 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f - 2
*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 - 4*a
*c^4*d*x^4 + 3*b^4*c*f*x^4 - 14*a*b^2*c^2*f*x^4 + 8*a^2*c^3*f*x^4 - 2*b^3*c
^2*x^4*e + 8*a*b*c^3*x^4*e - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + b^5*f*x^2 -
4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 + 4*a^2*c^3*x^2*e - a*b^2*c^2*d + a*b^4
*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f + 2*a^2*b*c^2*e)/((b^2*c^4 - 4*a*c^5)*(c*x
^4 + b*x^2 + a)) + 1/4*(c^2*d + 3*b^2*f - 2*a*c*f - 2*b*c*e)*log(c*x^4 + b*
x^2 + a)/c^4 + 1/4*(c^2*f*x^4 - 4*b*c*f*x^2 + 2*c^2*x^2*e)/c^4
```

Mupad [B]
time = 1.33, size = 2500, normalized size = 7.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(e/(2*c^2) - (b*f)/c^3) - ((2*a^3*c^2*f - 2*a^2*c^3*d + a*b^4*f - a*b^3
*c*e + a*b^2*c^2*d + 3*a^2*b*c^2*e - 4*a^2*b^2*c*f)/(2*c*(4*a*c - b^2)) + (
x^2*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f
+ 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c*(4*a*c - b^2)))/(a*c^3 + c^4*x^4 + b
*c^3*x^2) - (log(a + b*x^2 + c*x^4)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d
+ 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^
2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*
c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*
a^2*b^2*c^6)) + (f*x^4)/(4*c^2) + (atan(((8*a*c^7*(4*a*c - b^2)^3 - 2*b^2*c
^6*(4*a*c - b^2)^3)*(((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2
*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4
*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f -
 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a
^3*b*c^4*e))/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6))*(3
```

*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f
+ 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) - (a*(3*b^5
*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12
*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256
*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4
*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e
 + 256*a^3*b*c^4*e))/(c^2*(4*a*c - b^2)^(3/2)*(256*a^3*c^7 - 4*b^6*c^4 + 48
*a*b^4*c^5 - 192*a^2*b^2*c^6)))/(a*(4*a*c - b^2)) - x^2*(((((24*a^2*c^7*e -
 6*b^3*c^6*d + 12*b^4*c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e
+ 96*a*b^3*c^5*f - 92*a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a
*b*c^9)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e
+ 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^
3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2
*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*
c^6)))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a
*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) -
((8*b^3*c^8 - 32*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e -
 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 1
28*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d -
 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f -
 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(16*c^4*(4*a*c - b^2)^
(3/2)*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2
*b^2*c^6)))/(a*(4*a*c - b^2)) + (b*((((24*a^2*c^7*e - 6*b^3*c^6*d + 12*b^4*
c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e + 96*a*b^3*c^5*f - 92*
a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a*b*c^9)*(6*b^8*f - 128
*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 1
92*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 2
4*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(4*a*c^7 - b^2*c^6)*(
256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))*(6*b^8*f - 128*
a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 19
2*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24
*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c
^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (9*b^7*f^2 + b^3*c^4*d^2 + 4*b^5*c^
2*e^2 - 20*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - 38*a^3*b*c^3*f^2 - 12*b^6*c*e
*f + 91*a^2*b^3*c^2*f^2 - 5*a*b*c^5*d^2 - 57*a*b^5*c*f^2 - 6*a^2*c^5*d*e -
4*b^4*c^3*d*e + 12*a^3*c^4*e*f + 6*b^5*c^2*d*f + 20*a*b^2*c^4*d*e - 34*a*b^
3*c^3*d*f + 29*a^2*b*c^4*d*f + 68*a*b^4*c^2*e*f - 76*a^2*b^2*c^3*e*f)/(4*a*
c^7 - b^2*c^6) + (((b^3*c^8)/2 - 2*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c
^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c
^2*f)^2)/(c^8*(4*a*c - b^2)^3*(4*a*c^7 - b^2*c^6))))/(2*a*(4*a*c - b^2)^(3/
2))) + (b*((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2*c^4*f)/c^6
 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*
c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^
2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e)
)/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6))*(6*b^8*f - 12

```
8*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d -
192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f -
24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6
*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (a*c^4*d^2 + 9*a*b^4*f^2 + 4*a^3*
c^2*f^2 + 4*a*b^2*c^2*e^2 - 12*a^2*b^2*c*f^2 - 4*a^2*c^3*d*f + 6*a*b^2*c^2*
d*f + 8*a^2*b*c^2*e*f - 4*a*b*c^3*d*e - 12*a*b^3*c*e*f)/c^6 + (a*(3*b^5*f -
 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b
^2*c^2*e + 30*a^2*b*c^2*f)^2)/(c^6*(4*a*c - b^2...
```

3.62 $\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=236

$$\frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)}+\frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(12a^2c^2f-b^3(ce$$

```
[Out] 1/2*(2*c^2*d+2*b^2*f-c*(6*a*f+b*e))*x^2/c^2/(-4*a*c+b^2)+1/2*x^4*(2*a*c*e-b
*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/2*(12*a^2*c^2*f-b^3*(-2*b*f+c*e)-2*a*c*(6*b^2*f-3*b*c*e+2*c^2*d))*arct
anh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/4*(-2*b*f+c*e)
*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A]
time = 0.29, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1677, 1658, 787, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac(6b^2f-3bce+2c^2d)-(b^3(ce-2bf))\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2(-c(6af+be)+2b^2f+2c^2d)}{2c^2\left(b^2-4ac\right)}+\frac{x^4(-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (x^4*(2
*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*
c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*(b^2 - 4*a*c)^(3/2)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+ex+fx^2)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \ldots$$

$$= \frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)} + \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-\ldots}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)} + \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-\ldots}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)} + \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-\ldots}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{(2c^2d+2b^2f-c(be+6af))\,x^2}{2c^2\,(b^2-4ac)} + \frac{x^4(2ace-b(cd+af)-(2c^2d-bce+b^2f-\ldots}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Mathematica [A]
time = 0.23, size = 236, normalized size = 1.00

$$\frac{2cfx^2-\frac{2(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2(12a^2c^2f+b^3(-ce+2bf)-2ac(2c^2d-3bce+6b^2f))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+(ce-2bf)\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e +
 f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^
2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*e)
+ 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^3)
```

Maple [A]
time = 0.10, size = 309, normalized size = 1.31

method	result

default	$\frac{f\,x^2}{2c^2}-\frac{-\frac{\left(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2c^3ad+b^4f-b^3ce+b^2c^2d\right)x^2}{c\left(4ac-b^2\right)}+\frac{a\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)}{c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(8abcf-4a\,c^2e-2b^3f+b^2ce\right)\ln\left(c\,x^4+\right.}{2c}}{2c^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/c^2-1/2/c^2*((-(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^4
*f-b^3*c*e+b^2*c^2*d)/c/(4*a*c-b^2)*x^2+a*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*
e-b*c^2*d)/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*f-4*a
*c^2*e-2*b^3*f+b^2*c*e)/c*ln(c*x^4+b*x^2+a)+2*(6*a^2*c*f-2*a*b^2*f+a*b*c*e-
2*a*c^2*d-1/2*(8*a*b*c*f-4*a*c^2*e-2*b^3*f+b^2*c*e)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(224) = 448$.
time = 0.46, size = 1455, normalized size = 6.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c
- 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^
3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^2*c^2
)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*e
```

```
- 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*
c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*
x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3
*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 +
16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*
a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 -
 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d +
 (b^4*c - 6*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^
3*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(a*b^3*c^
2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e - 2*(a*b^5 -
 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e -
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*x^2 + (a*b^4*c - 8*
a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Giac [**A**]
time = 5.88, size = 279, normalized size = 1.18

$$\frac{fx^2}{2\,c^2}-\frac{\left(4\,ac^3d-2\,b^4f+12\,ab^2cf-12\,a^2c^2f+b^3ce-6\,abc^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{2\,b^3fx^4-8\,abcfx^4-b^2cx^4e+4\,ac^2x^4e-2\,b^2cdx^2+4\,ac^2dx^2-4\,a^2cfx^2+b^3x^2e-2\,abcx^2e-2\,abcd-2\,a^2bf+ab^2e}{4\,\left(cx^4+bx^2+a\right)\left(b^2c^2-4\,ac^3\right)}-\frac{\left(2\,bf-ce\right)\log\left(cx^4+bx^2+a\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d - 2*b^4*f + 12*a*b^2*c*f - 12*a^2*c^2*f + b^
3*c*e - 6*a*b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4
```

```
*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^3*f*x^4 - 8*a*b*c*f*x^4 - b^2*c*x^4*
e + 4*a*c^2*x^4*e - 2*b^2*c*d*x^2 + 4*a*c^2*d*x^2 - 4*a^2*c*f*x^2 + b^3*x^2
*e - 2*a*b*c*x^2*e - 2*a*b*c*d - 2*a^2*b*f + a*b^2*e)/((c*x^4 + b*x^2 + a)*
(b^2*c^2 - 4*a*c^3)) - 1/4*(2*b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B]
time = 1.81, size = 2450, normalized size = 10.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c*(4*a*c - b^2)
) + (x^2*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2
*e - 4*a*b^2*c*f))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (f*
x^2)/(2*c^2) + (log(a + b*x^2 + c*x^4)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e
 - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 2
56*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c
^5)) - (atan(((8*a*c^5*(4*a*c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*((
(((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*c^6*d + 28*a*b*c^5*e - 5
6*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*b^7*f + 1
28*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*
c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^
6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(2*b^4*f + 12*a^2*c^2*f -
 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(8*c^3*(4*a*c - b^2)^(3
/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*
c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96
*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^
3*b*c^3*f))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 -
4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((4*b^
5*f^2 + b^3*c^2*e^2 + 12*a^2*b*c^2*f^2 + 2*a*c^4*d*e - 4*b^4*c*e*f - 5*a*b*
c^3*e^2 - 20*a*b^3*c*f^2 - 6*a^2*c^3*e*f + 20*a*b^2*c^2*e*f - 4*a*b*c^3*d*f
)/(4*a*c^5 - b^2*c^4) + (((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*
c^6*d + 28*a*b*c^5*e - 56*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 -
32*a*b*c^7)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a
^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^
5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(
4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f
- 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6
*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*b^4
*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)^2)/(c
^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4*a*c - b^2)^(3/2))) + ((((
8*a*c^4*e - 16*a*b*c^3*f)/c^4 - (8*a*c^2*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c
*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e -
```

256*a^3*b*c^3*f))/(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*a*c^4*e - 16*a*b*c^3*f)/c^4 - (8*a*c^2*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (4*a*b^2*f^2 + a*c^2*e^2 - 4*a*b*c*e*f)/c^4 + (a*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)^2)/(c^4*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*b^8*f^2 + 16*a^2*c^6*d^2 + 144*a^4*c^4*f^2 + b^6*c^2*e^2 - 12*a*b^4*c^3*e^2 - 4*b^7*c*e*f + 36*a^2*b^2*c^4*e^2 + 192*a^2*b^4*c^2*f^2 - 288*a^3*b^2*c^3*f^2 - 48*a*b^6*c*f^2 - 96*a^3*c^5*d*f + 8*a*b^3*c^4*d*e - 48*a^2*b*c^5*d*e - 16*a*b^4*c^3*d*f + 48*a*b^5*c^2*e*f + 144*a^3*b*c^4*e*f + 96*a^2*b^2*c^4*d*f - 168*a^2*b^3*c^3*e*f))*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(2*c^3*(4*a*c - b^2)^(3/2))

3.63 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=165

$$\frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{(4ac^2e+b^3f-2bc(cd+3af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2c^2\,(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*x^2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+
b^2)/(c*x^4+b*x^2+a)+1/2*(4*a*c^2*e+b^3*f-2*b*c*(3*a*f+c*d))*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*f*ln(c*x^4+b*x^2+a)/c^
2
```

Rubi [A]
time = 0.19, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1677, 1658, 648, 632, 212, 642}

$$\frac{x^2(-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2bc(3af+cd)+4ac^2e+b^3f)}{2c^2\,(b^2-4ac)^{3/2}}+\frac{f\log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(
2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e + b^3*f - 2*b*c*(c*d +
 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/
2)) + (f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex+fx^2)}{(a+bx+cx^2)^2}\,dx,x,x^2\right)$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{2ae-\frac{b(cd+af)}{c}-\left(b\right.}{a+bx+cx}\right.}{2\,(b^2-4}$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{f\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x\right.}{4c^2}$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{f\log\left(a+bx^2+cx^4\right)}{4c^2} +$$

$$= \frac{x^2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(4ac^2e+b^3f-2bc(cd}{2c^2\,(}$$

Mathematica [A]
time = 0.16, size = 175, normalized size = 1.06

$$\frac{\frac{2(-2a^2cf+b(c^2d-bce+b^2f)x^2+a(b^2f+2c^2(d+ex^2)-bc(e+3fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2(4ac^2e+b^3f-2bc(cd+3af))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*x
^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*c^
2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]
)/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.08, size = 228, normalized size = 1.38

method	result
default	$\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^2}{\left(4ac-b^2\right)c^2}+\frac{a\left(2acf-b^2f+bce-2c^2d\right)}{\left(4ac-b^2\right)c^2}}{2c\,x^4+2b\,x^2+2a}+\frac{\frac{\left(4acf-b^2f\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-abf+2ace-bcd-\frac{\left(4acf-b^2f\right)b}{2c}\right)\arctan}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*x^2+a*(2*a
*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/2/c/(4*a*c-b^2
)*(1/2*(4*a*c*f-b^2*f)/c*ln(c*x^4+b*x^2+a)+2*(-a*b*f+2*a*c*e-b*c*d-1/2*(4*a
*c*f-b^2*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(155) = 310$.
time = 0.41, size = 970, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (b
^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c
^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e
- (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*log((
2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*
e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 +
 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2
 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(2*a*b*c^2*d
- 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^
2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + 2*(a*b^4 - 6*a
^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*f*x^4 + (b^5
- 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
```

```
8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2
)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.72, size = 195, normalized size = 1.18

$$\frac{(2\,bc^2d - b^3f + 6\,abcf - 4\,ac^2e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2 - 4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{f\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{2\,ac^2d + ab^2f - 2\,a^2cf - abce + (bc^2d + b^3f - 3\,abcf - b^2ce + 2\,ac^2e)x^2}{2\,(cx^4+bx^2+a)(b^2-4\,ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c^2*d - b^3*f + 6*a*b*c*f - 4*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-
b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4 +
b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d + a*b^2*f - 2*a^2*c*f - a*b*c*e + (b*c^2*d
+ b^3*f - 3*a*b*c*f - b^2*c*e + 2*a*c^2*e)*x^2)/((c*x^4 + b*x^2 + a)*(b^2 -
 4*a*c)*c^2)
```

Mupad [**B**]
time = 2.72, size = 1651, normalized size = 10.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*(4*a*c - b^2)) + (x^2*(b^
3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^2*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6*f - 128*a^3*c^3*f + 96*a
^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 -
192*a^2*b^2*c^4)) - (atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^
2)^3)*((((8*a*f + (8*a*c^2*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24
*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(b
^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a
*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c
```

```
^2*e - 2*b*c^2*d - 6*a*b*c*f))/((4*a*c - b^2)^(3/2)*(256*a^3*c^5 - 4*b^6*c^
2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - x^2*(((((6*b^3*c^
2*f + 8*a*c^4*e - 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c
^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f
))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2
*b^2*c^4)))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^
2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2
*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(16*c^2
*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^
4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((b^3*f^2 - 5*a*b*c*f^2 +
 2*a*c^2*e*f - b*c^2*d*f)/(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2*f + 8*a*c^4*e
 - 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)
*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(4*a*c^3 -
 b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*b
^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 -
4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(
b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3
 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2))) + (b*(((8*a*f + (8*a*c^2*(2*b^6*f
 - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c
^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2
*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^
2*b^2*c^4)) + (a*f^2)/c^2 - (a*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f)^
2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6*f^2 + 16*a^2*c^
4*e^2 + 4*b^2*c^4*d^2 + 36*a^2*b^2*c^2*f^2 - 12*a*b^4*c*f^2 - 4*b^4*c^2*d*f
 + 24*a*b^2*c^3*d*f + 8*a*b^3*c^2*e*f - 48*a^2*b*c^3*e*f - 16*a*b*c^4*d*e))
*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(2*c^2*(4*a*c - b^2)^(3/2))
```

3.64 $\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=123

$$\frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{(2cd-be+2af)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1677, 1674, 12, 632, 212}

$$\frac{-(x^2(-2acf+b^2f-bce+2c^2d))-b(af+cd)+2ace}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(2*c*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*f)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{2cd-be+2af}{a+bx+cx^2}\,dx, x, x^2\right.}{2\,(b^2-4ac)}$$

$$= \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(2cd-be+2af)\text{Subst}\left(\int \right.}{2\,(b^2-4ac}$$

$$= \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(2cd-be+2af)\text{Subst}\left(\int \right.}{b^2-4}$$

$$= \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(2cd-be+2af)\tanh^{-1}\left(\right.}{(b^2-4ac)^{3/2}}$$

Mathematica [A]
time = 0.07, size = 130, normalized size = 1.06

$$\frac{abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2)}{2c\,(-b^2+4ac)\,(a+bx^2+cx^4)} - \frac{(-2cd+be-2af)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

[Out] (a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

Maple [A]
time = 0.05, size = 139, normalized size = 1.13

method	result
default	$\frac{-\frac{\left(2acf-b^2f+bce-2c^2d\right)x^2}{c\left(4ac-b^2\right)}+\frac{abf-2ace+bcd}{c\left(4ac-b^2\right)}}{2c\,x^4+2b\,x^2+2a}+\frac{(2fa-eb+2cd)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2acf-b^2f+bce-2c^2d\right)x^2}{2c\left(4ac-b^2\right)}+\frac{abf-2ace+bcd}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8a^2c-2a\,b^2\right)fa}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x\right.\right.}{2(-4ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^2+1/c*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(117) = 234.
time = 0.39, size = 650, normalized size = 5.28

$$\left[-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2+((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2d-abce+2\,a^2cf+(2\,bc^2d-b^2ce+2\,abcf)x^2)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^3c-4\,abc^2)d-2\,(ab^2c-4\,a^2c^2)e+(ab^3-4\,a^2bc)f}{2\,(ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2)},\ -\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2-2\,((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2d-abce+2\,a^2cf+(2\,bc^2d-b^2ce+2\,abcf)x^2)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^3c-4\,abc^2)d-2\,(ab^2c-4\,a^2c^2)e+(ab^3-4\,a^2bc)f}{2\,(ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*

```
b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c
))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e
 + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)
, -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
 + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d -
 a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3*c - 4*
a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.64, size = 140, normalized size = 1.14

$$-\frac{(2\,cd+2\,af-be)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2dx^2+b^2fx^2-2\,acfx^2-bcx^2e+bcd+abf-2\,ace}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d + 2*a*f - b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 + b^2*f*x^2 - 2*a*c*f*x^2 - b*c*
x^2*e + b*c*d + a*b*f - 2*a*c*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

Mupad [B]
time = 0.38, size = 342, normalized size = 2.78

$$\frac{\frac{a\,b\,f-2\,a\,c\,e+b\,c\,d}{2\,c\,(4\,a\,c-b^2)}+\frac{x^2\,(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c)}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}+\frac{\mathrm{atan}\left(\frac{{(4\,a\,c-b^2)}^4\,\left(x^2\,\left(\frac{\left(2\,c^3\,d+2\,a\,c^2\,f-b\,c^2\,e\right)\,(2\,a\,f-b\,e+2\,c\,d)}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)\,{(2\,a\,f-b\,e+2\,c\,d)}^2}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)-\frac{2\,c^2\,\left(b^3-4\,a\,b\,c\right)\,{(2\,a\,f-b\,e+2\,c\,d)}^2}{{\left(4\,a\,c-b^2\right)}^{11/2}}\right)}{8\,a^2\,c^2\,f^2-8\,a\,b\,c^2\,e\,f+16\,a\,c^3\,d\,f+2\,b^2\,c^2\,e^2-8\,b\,c^3\,d\,e+8\,c^4\,d^2}\right)\,(2\,a\,f-b\,e+2\,c\,d)}{{(4\,a\,c-b^2)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*b*f - 2*a*c*e + b*c*d)/(2*c*(4*a*c - b^2)) + (x^2*(2*c^2*d + b^2*f - 2*
a*c*f - b*c*e))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (atan(((4*a*c -
b^2)^4*(x^2*(((2*c^3*d + 2*a*c^2*f - b*c^2*e)*(2*a*f - b*e + 2*c*d))/(a*(4*
a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)*(2*a*f - b*e +
 2*c*d)^2)/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(b^3 - 4*a*b*c)*(2*a*f - b*
e + 2*c*d)^2)/(4*a*c - b^2)^(11/2)))/(8*c^4*d^2 + 8*a^2*c^2*f^2 + 2*b^2*c^2
*e^2 + 16*a*c^3*d*f - 8*b*c^3*d*e - 8*a*b*c^2*e*f))*(2*a*f - b*e + 2*c*d))/
(4*a*c - b^2)^(3/2)
```

3.65 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=166

$$\frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{(b^3d+4a^2ce-2ab(3cd+af))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}+$$

```
[Out] 1/2*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)+1/2*(b^3*d+4*a^2*c*e-2*a*b*(a*f+3*c*d))*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+d*ln(x)/a^2-1/4*d*ln(c*x^4+b*x^2
+a)/a^2
```

Rubi [A]
time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1677, 1660, 814, 648, 632, 212, 642}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce-2ab(af+3cd)+b^3d)}{2a^2(b^2-4ac)^{3/2}}-\frac{d\log(a+bx^2+cx^4)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(2*a*(b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d + a*f))
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (d
*Log[x])/a^2 - (d*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\frac{b^2}{a}-4c\right)d-\frac{(bcd-2ace}{a}}{x(a+bx+cx^2)}\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{(-b^2+4ac)d}{a^2x} + \frac{b^3d+}{}\right.\right.}{2}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b^3d+2a^2}{}\right.}{}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+c}\right.}{4a^2}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^2+\right.}{4a^2}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(b^3d+4a^2ce-2ab(3cd+a}{2a^2\left(b^2-\right.}$$

Mathematica [A]
time = 0.29, size = 268, normalized size = 1.61

$$-\frac{-\frac{2a\left(b^2d+b\left(-ae+cdx^2+afx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - 4d\log(x) + \frac{\left(b^3d+b^2\sqrt{b^2-4ac}\,d+4ac\left(-\sqrt{b^2-4ac}\,d+ae\right)-2ab(3cd+af)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^3d+b^2\sqrt{b^2-4ac}\,d-4ac\left(\sqrt{b^2-4ac}\,d+ae\right)+2ab(3cd+af)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x
^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*Sqr
t[b^2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) - 2*a*b*(3*c*d + a*
f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-(b^3*d)
+ b^2*Sqrt[b^2 - 4*a*c]*d - 4*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*(3*c*
d + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/a^2
```

Maple [A]
time = 0.07, size = 228, normalized size = 1.37

method	result

default	$\frac{\frac{-\frac{a(abf-2ace+bcd)x^2}{4ac-b^2}-\frac{a\left(2a^2f-abe-2acd+b^2d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-4a\,c^2d+b^2cd\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-a^2bf+2a^2ce-5abcd+b^3d-\frac{\left(-4a\,c^2d+b^2cd\right)b}{2c}\right)\text{arcta}}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{(abf-2ace+bcd)x^2}{2a\left(4ac-b^2\right)}-\frac{2a^2f-abe-2acd+b^2d}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{d\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^5c^3-48b^2c^2a^4+12c\,b^4a^3-b^6a^2\right)\_Z^2+\left(64a^3c^3d-48a^2b^2c^2d+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*((-a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^2-a*(2*a^2*f-a*b*e-2*a*c*d
+b^2*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a*c^2*d+b^2*c*d
)/c*ln(c*x^4+b*x^2+a)+2*(-a^2*b*f+2*a^2*c*e-5*a*b*c*d+b^3*d-1/2*(-4*a*c^2*d
+b^2*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))+d*
ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(156) = 312$.
time = 1.02, size = 1103, normalized size = 6.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3
 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f
+ (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*
c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
```

```
+ 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^
3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*
x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x)
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*((a*b^3*c -
 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^
2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*
c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^
3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*
a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Giac [**A**]
time = 3.87, size = 227, normalized size = 1.37

$$-\frac{\left(b^3d-6\,abcd-2\,a^2bf+4\,a^2ce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{d\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{d\log\left(x^2\right)}{2\,a^2}+\frac{b^2cdx^4-4\,ac^2dx^4+b^3dx^2-2\,abcdx^2+2\,a^2bfx^2-4\,a^2cx^2e+3\,ab^2d-8\,a^2cd+4\,a^3f-2\,a^2be}{4\,\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
[Out] -1/2*(b^3*d - 6*a*b*c*d - 2*a^2*b*f + 4*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*d*log(c*x^4 +
 b*x^2 + a)/a^2 + 1/2*d*log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4 + b
^3*d*x^2 - 2*a*b*c*d*x^2 + 2*a^2*b*f*x^2 - 4*a^2*c*x^2*e + 3*a*b^2*d - 8*a^
2*c*d + 4*a^3*f - 2*a^2*b*e)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

Mupad [**B**]
time = 11.85, size = 2500, normalized size = 15.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (d*log(x))/a^2 - ((b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2)) +
 (x^2*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
 (log((((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*
c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d
)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b
^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b
*c^2*(d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c -
 b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^3*d - a^
2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2))))/(4*a^2) + (c^2*(a^3*b^2
*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d*f - 36*a^
2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f))/(a^2*(4*a*
c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*d^2 + 4*a^2*b*c^2*e^2 - 20*
a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e - 20*a^2*b*c^2*d*f - 4*a^2*
b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) - (c^2*x^2*(a*b
*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) + (c^2*d*(a*b*f - 2*a*c*e +
b*c*d)^2)/(a^3*(4*a*c - b^2)^2))*(((d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*
e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d - a^2*(-(b^3*d - 2*a^2*
b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(2
0*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c
*f))/(a*(4*a*c - b^2)) + (b*c^2*(d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e -
 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^2 - (4*b*c^2*(b^3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2)
)))/(4*a^2) + (c^2*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^
2*d^2 - 4*a*b^4*d*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e -
 4*a^3*b*c*e*f))/(a^2*(4*a*c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*
d^2 + 4*a^2*b*c^2*e^2 - 20*a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e
- 20*a^2*b*c^2*d*f - 4*a^2*b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2
)))/(4*a^2) - (c^2*x^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) +
 (c^2*d*(a*b*f - 2*a*c*e + b*c*d)^2)/(a^3*(4*a*c - b^2)^2)))*(2*b^6*d - 128
*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3
- 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((x^2*((((b^3*c^5*d^3 - 8*a^3*c^5
*e^3 + a^3*b^3*c^2*f^3 - 6*a*b^2*c^5*d^2*e + 12*a^2*b*c^5*d*e^2 + 3*a*b^3*c
^4*d^2*f + 12*a^3*b*c^4*e^2*f + 3*a^2*b^3*c^3*d*f^2 - 6*a^3*b^2*c^3*e*f^2 -
 12*a^2*b^2*c^4*d*e*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2) - (((6*a*b^5*c^4*d^2 + 80*a^3*b*c^6*d^2 - 16*a^4*b*c^5*e^2 - 44*a^2*b^3*
c^5*d^2 + 4*a^3*b^3*c^4*e^2 + a^3*b^5*c^2*f^2 - 4*a^4*b^3*c^3*f^2 - 160*a^4
*c^6*d*e + 80*a^4*b*c^5*d*f - 14*a^2*b^4*c^4*d*e + 96*a^3*b^2*c^5*d*e + 7*a
^2*b^5*c^3*d*f - 48*a^3*b^3*c^4*d*f - 4*a^3*b^4*c^3*e*f + 16*a^4*b^2*c^4*e*
f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((640*a^6*c^6*
e - 2*a^2*b^7*c^3*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3
*e + 168*a^4*b^4*c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c
```

```
^3*f + 288*a^5*b^3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*
a^2*b^2*c^2*d - 24*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^
7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*
a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))
/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d -
 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*
c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*a^6*c^6*e - 2*a^2*b^7*c^3
*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*
c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^
3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 2
4*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*
b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b
^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((
2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b
*f + 4*a^2*c*e - 6*a*b*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*
c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3
*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3
 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b
*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2
*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64...
```

3.66 $\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{d}{2a^2x^2}-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(2b^4d-12ab^2cd-ab^3e+6a}{}$$

```
[Out] -1/2*d/a^2/x^2+1/2*(-b^3*d+a*b^2*e-2*a^2*c*e+a*b*(-a*f+3*c*d)-c*(b^2*d-a*b*
e-2*a*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(2*b^4*d-12*a*b
^2*c*d-a*b^3*e+6*a^2*b*c*e+4*a^2*c*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-a*e+2*b*d)*ln(x)/a^3+1/4*(-a*e+2*b*d
)*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A]
time = 0.48, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1677, 1660, 1642, 648, 632, 212, 642}

$$\frac{(2bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2bd-ae)}{a^3}-\frac{2a^2ce+cx^2(-abe-2a(cd-af)+b^2d)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{d}{2a^2x^2}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-af)-ab^3e-12ab^2cd+2b^4d\right)}{2a^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*d/(a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f) + c*(b^
2*d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b*c*e + 4*a^2*c*(3*c*d - a*
f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) -
 ((2*b*d - a*e)*Log[x])/a^3 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\Big(}{}$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\Big(}{}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A]
time = 0.42, size = 403, normalized size = 1.72

$$\frac{-\frac{2ad}{x^2}-\frac{2a\left(b^3d+b^2\left(-ae+cdx^2\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(-cdx^2+a\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+4(-2bd+ae)\log(x)+\frac{\left(2b^4d+b^3\left(2\sqrt{b^2-4ac}\,d-ae\right)+2abc\left(-4\sqrt{b^2-4ac}\,d+3ae\right)-ab^2\left(12cd+\sqrt{b^2-4ac}\,e\right)+4a^2c\left(3cd+\sqrt{b^2-4ac}\,e-af\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(-2b^4d+b^3\left(2\sqrt{b^2-4ac}\,d+ae\right)-2abc\left(4\sqrt{b^2-4ac}\,d+3ae\right)+ab^2\left(12cd-\sqrt{b^2-4ac}\,e\right)+4a^2c\left(-3cd+\sqrt{b^2-4ac}\,e+af\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d +
e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d
- a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqrt[b^
2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqrt[b^2
- 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(12*c*d
 - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f))*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Maple [A]
time = 0.09, size = 316, normalized size = 1.35

method	result
default	$\frac{\frac{\frac{ac\left(2a^2f-abe-2acd+b^2d\right)x^2}{4ac-b^2}+\frac{a\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-4a^2c^2e+a\,b^2ce+8ab\,c^2d-2b^3cd\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(2a^3cf-5a^2bce-\cdots\right.}{\cdots}}{\cdots}}{2a^3}$
risch	$\frac{\frac{c\left(2a^2f-abe-6acd+2b^2d\right)x^4}{2a^2\left(4ac-b^2\right)}+\frac{\left(a^2bf+2a^2ce-a\,b^2e-7abcd+2b^3d\right)x^2}{2\left(4ac-b^2\right)a^2}-\frac{d}{2a}}{x^2(c\,x^4+b\,x^2+a)}+\frac{\ln(x)e}{a^2}-\frac{2\ln(x)bd}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^6c^3-48a^5b^2c^2+\cdots\right.\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*((a*c*(2*a^2*f-a*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2)*x^2+a*(a^2*b*f+2*a^
2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(
1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c^2*d-2*b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(2
*a^3*c*f-5*a^2*b*c*e-6*a^2*c^2*d+a*b^3*e+10*a*b^2*c*d-2*b^4*d-1/2*(-4*a^2*c
^2*e+a*b^2*c*e+8*a*b*c^2*d-2*b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))))-1/2*d/a^2/x^2+(a*e-2*b*d)/a^3*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(222) = 444$.
time = 2.02, size = 1764, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b
*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c + 2
```

```
8*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^4*b
*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a*b^3
*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)
*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x
^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*
(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)
*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*
a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b
^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3
*b^3 - 4*a^4*b*c)*f)*x^2 - 2*((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2
*c^3)*d + (a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*
c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 -
 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d -
(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^
2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*
b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b
^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log
(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 5.06, size = 287, normalized size = 1.23

$$\frac{\left(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-4\,a^3cf-ab^3e+6\,a^2bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cdx^4-6\,ac^2dx^4+2\,a^2cfx^4-abcx^4e+2\,b^3dx^2-7\,abcdx^2+a^2bfx^2-ab^2x^2e+2\,a^2cx^2e+ab^2d-4\,a^2cd}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)}+\frac{(2\,bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^3}-\frac{(2\,bd-ae)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - 4*a^3*c*f - a*b^3*e + 6*a^2*b*
c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^
2 + 4*a*c)) - 1/2*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a*b*c*x^
4*e + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 + a^2*b*f*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2
*e + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) + 1
/4*(2*b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^3 - 1/2*(2*b*d - a*e)*log(x^2)/a^
3
```

Mupad [B]
time = 12.98, size = 2500, normalized size = 10.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(2*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 7*a*b*c*d))/(2*a^2*(4*a*c
- b^2)) - d/(2*a) + (c*x^4*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d))/(2*a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (log(x)*(a*e - 2*b*d))/a^3 + (log((
((((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*
a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((2*c^3*x^2*(2*b^4
*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e + 20*a^3*c*f + 4*a*b^2*c*d + 10*a
^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2*(2*b^4*d + 6*a^2*c^2*d - a*b^3*e
- 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (b*c^2*(a*
e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*
c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*
c*x^2))/a^3))/(4*a^3) + (c^3*(4*a^5*c*f^2 - 16*b^6*d^2 - 4*a^2*b^4*e^2 + 36
*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^5*d*e - 216*a^2*b^2*c^2*d^2 + 116*
a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*e*f - 24*a^4*c^2*d*f - 92*a^2*b^3*
c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*d*f - 36*a^4*b*c*e*f))/(a^4*(4*a*c
 - b^2)^2) - (2*c^4*x^2*(12*b^5*d^2 + 2*a^4*b*f^2 + 3*a^2*b^3*e^2 + 138*a^2
*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f - 82*a*b^3*c*d^2 - 10*a^3*b*c*e^2
+ 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3*b^2*e*f + 61*a^2*b^2*c*d*e - 52*a
^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*
c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b
^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b*d)*(2*b^2*d + 2*a^2*f - a*b*e - 6
*a*c*d)^2)/(a^6*(4*a*c - b^2)^2) + (c^5*x^2*(2*b^2*d + 2*a^2*f - a*b*e - 6*
a*c*d)^3)/(a^6*(4*a*c - b^2)^3))*(((((2*b*d - a*e + a^3*(-(2*b^4*d + 12*a^2
```

```
*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c -
b^2)^3))^(1/2))*((2*c^3*x^2*(2*b^4*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e
 + 20*a^3*c*f + 4*a*b^2*c*d + 10*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2
*(2*b^4*d + 6*a^2*c^2*d - a*b^3*e - 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e)
)/(a^2*(4*a*c - b^2)) - (b*c^2*(2*b*d - a*e + a^3*(-(2*b^4*d + 12*a^2*c^2*d
 - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3
))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3))/(4*a^3) - (c^3*(4*a^5*c*f^2
 - 16*b^6*d^2 - 4*a^2*b^4*e^2 + 36*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^
5*d*e - 216*a^2*b^2*c^2*d^2 + 116*a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*
e*f - 24*a^4*c^2*d*f - 92*a^2*b^3*c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*
d*f - 36*a^4*b*c*e*f))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2*(12*b^5*d^2 + 2*a
^4*b*f^2 + 3*a^2*b^3*e^2 + 138*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f
- 82*a*b^3*c*d^2 - 10*a^3*b*c*e^2 + 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3
*b^2*e*f + 61*a^2*b^2*c*d*e - 52*a^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(2*b*
d - a*e + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*
d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b
*d)*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d)^2)/(a^6*(4*a*c - b^2)^2) + (c^5*x
^2*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d)^3)/(a^6*(4*a*c - b^2)^3)))*(4*b^7*
d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a
*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 -
 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (atan((x^2*((((216*a^3*c^8*d^3 - 8*b^6*
c^5*d^3 - 8*a^6*c^5*f^3 + 72*a*b^4*c^6*d^3 - 216*a^4*c^7*d^2*f + 72*a^5*c^6
*d*f^2 - 216*a^2*b^2*c^7*d^3 + a^3*b^3*c^5*e^3 + 12*a*b^5*c^5*d^2*e + 108*a
^3*b*c^7*d^2*e + 12*a^5*b*c^5*e*f^2 - 72*a^2*b^3*c^6*d^2*e - 6*a^2*b^4*c^5*
d*e^2 + 18*a^3*b^2*c^6*d*e^2 - 24*a^2*b^4*c^5*d^2*f + 144*a^3*b^2*c^6*d^2*f
 - 24*a^4*b^2*c^5*d*f^2 - 6*a^4*b^2*c^5*e^2*f - 72*a^4*b*c^6*d*e*f + 24*a^3
*b^3*c^5*d*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + ((
(80*a^6*b*c^6*e^2 - 1104*a^5*b*c^7*d^2 - 16*a^7*b*c^5*f^2 + 24*a^2*b^7*c^4*
d^2 - 260*a^3*b^5*c^5*d^2 + 932*a^4*b^3*c^6*d^2 + 6*a^4*b^5*c^4*e^2 - 44*a^
5*b^3*c^5*e^2 + 4*a^6*b^3*c^4*f^2 + 480*a^6*c^7*d*e - 160*a^7*c^6*e*f + 416
*a^6*b*c^6*d*f - 24*a^3*b^6*c^4*d*e + 218*a^4*b^4*c^5*d*e - 608*a^5*b^2*c^6
*d*e + 28*a^4*b^5*c^4*d*f - 216*a^5*b^3*c^5*d*f - 14*a^5*b^4*c^4*e*f + 96*a
^6*b^2*c^5*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + ((
(1920*a^8*c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*
a^6*b^4*c^5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 1
92*a^7*b^3*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f
 - 320*a^8*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
- ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 -
 2688*a^9*b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d
 - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6
*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6
*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d
+ 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2
*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + ...
```

3.67 $\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=329

$$-\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce-ab(3cd$$

```
[Out] -1/4*d/a^2/x^4+1/2*(-a*e+2*b*d)/a^3/x^2+1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+2*a^
2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*
d))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(3*b^5*d-2*a*b^4*e+12*a^2*b^2
*c*e-12*a^3*c^2*e+6*a^2*b*c*(-a*f+5*c*d)-a*b^3*(-a*f+20*c*d))*arctanh((2*c*
x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(3*b^2*d-2*a*b*e-a*(-a*f+
2*c*d))*ln(x)/a^4-1/4*(3*b^2*d-2*a*b*e-a*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^
4
```

Rubi [A]
time = 0.78, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1677, 1660, 1642, 648, 632, 212, 642}

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-2abe-a(2cd-af)+3b^2d\right)}{4a^4}+\frac{\log(x)\left(-2abe-a(2cd-af)+3b^2d\right)}{a^4}+\frac{2bd-ae}{2a^3x^2}-\frac{d}{4a^2x^4}+\frac{cx^2(2a^2ce-ab^2e-ab(3cd-af)+b^3d)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-12a^3c^2e+12a^2b^2ce+6a^2bc(5cd-af)-2ab^4e-ab^3(20cd-af)+3b^5d\right)}{2a^4\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*d/(a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3*a^2*b*c
*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d - a*b^2*e + 2*a^2
*c*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d -
a*f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a
^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^
4 - ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce\ -}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce\ -}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+}{2a^3\left(b^2-4ac\right)\left(a+bx\right.}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+}{2a^3\left(b^2-4ac\right)\left(a+bx\right.}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+}{2a^3\left(b^2-4ac\right)\left(a+bx\right.}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+}{2a^3\left(b^2-4ac\right)\left(a+bx\right.}$$

Mathematica [A]
time = 0.78, size = 592, normalized size = 1.80

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e -
c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x
^2) + 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*Log[x] + ((3*b^5*d + b^4*(3*Sqrt[
b^2 - 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e - 3*a*f
) + a*b^3*(-20*c*d - 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqrt[b^2
- 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^2 - 4*a
*c]*d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2])/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-15*c*d + 4*
Sqrt[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e -
 a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d + 6*a*e) + a*S
```

```
qrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3
/2))/a^4
```

Maple [A]
time = 0.11, size = 466, normalized size = 1.42

method	result
default	$-\frac{\frac{ac\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^2}{4ac-b^2}-\frac{a\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-b^4d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4a^3c^2f-a^2b^2cf-8a^2b\,c^2e-8a^2c^3d+2a\,b^3 \ldots\right.}{2c}}{\ldots}$
risch	$\frac{-\frac{c\left(a^2bf+6a^2ce-2a\,b^2e-11abcd+3b^3d\right)x^6}{2a^3\left(4ac-b^2\right)}+\frac{\left(4a^3cf-2a^2b^2f-14a^2bce-8a^2c^2d+4a\,b^3e+25a\,b^2cd-6b^4d\right)x^4}{4a^3\left(4ac-b^2\right)}-\frac{(2ae-3bd)x^2}{4a^2}-\frac{d}{4a}}{x^4(c\,x^4+b\,x^2+a)}+\frac{\ln(x)f}{a^2}-\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4*((a*c*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^2-
a*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(
4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*a^3*c^2*f-a^2*b^2*c*f-8*a
^2*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)/c*ln(c*x^4+b*x
^2+a)+2*(5*a^3*b*c*f+6*e*a^3*c^2-a^2*b^3*f-10*a^2*b^2*c*e-19*a^2*b*c^2*d+2*
a*b^4*e+17*a*b^3*c*d-3*b^5*d-1/2*(4*a^3*c^2*f-a^2*b^2*c*f-8*a^2*b*c^2*e-8*a
^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))-1/4*d/a^2/x^4-1/2*(a*e-2*b*d)/a^3/x^2+(a
^2*f-2*a*b*e-2*a*c*d+3*b^2*d)/a^4*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(315) = 630$.
time = 4.31, size = 2567, normalized size = 7.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4), 1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x^2 + 2*(((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)

```
*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c
^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2
 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^
5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^
2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a
^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a
^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c
^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.45, size = 535, normalized size = 1.63

$$-\frac{\left(3 b^{5} d - 20 a b^{3} c d + 30 a^{2} b c^{2} d + a^{2} b^{3} f - 6 a^{3} b c f - 2 a b^{4} e + 12 a^{2} b^{2} c e - 12 a^{3} c^{2} e\right) \arctan\left(\frac{2 c x^{2}+b}{\sqrt{-b^{2}+4 a c}}\right)}{2 \left(a^{4} b^{2} - 4 a^{5} c\right) \sqrt{-b^{2}+4 a c}} + \frac{3 b^{4} c d x^{4} - 14 a b^{2} c^{2} d x^{4} + 8 a^{2} c^{3} d x^{4} + a^{2} b^{2} c f x^{4} - 4 a^{3} c^{2} f x^{4} - 2 a b^{3} c x^{4} e + 8 a^{2} b c^{2} x^{4} e + 3 b^{5} d x^{2} - 12 a b^{3} c d x^{2} + 2 a^{2} b c^{2} d x^{2} + a^{2} b^{3} f x^{2} - 2 a^{3} b c f x^{2} - 2 a b^{4} x^{2} e + 6 a^{2} b^{2} c x^{2} e + 4 a^{3} c^{2} x^{2} e + 5 a b^{4} d - 22 a^{2} b^{2} c d + 12 a^{3} c^{2} d + 3 a^{3} b^{2} f - 8 a^{4} c f - 4 a^{2} b^{3} e + 14 a^{3} b c e}{4 \left(a^{4} b^{2} - 4 a^{5} c\right) \left(c x^{4} + b x^{2} + a\right)} - \frac{\left(3 b^{2} d - 2 a c d + a^{2} f - 2 a b e\right) \log\left(c x^{4} + b x^{2} + a\right)}{4 a^{4}} + \frac{\left(3 b^{2} d - 2 a c d + a^{2} f - 2 a b e\right) \log\left(x^{2}\right)}{2 a^{4}} - \frac{9 b^{2} d x^{4} - 6 a c d x^{4} + 3 a^{2} f x^{4} - 6 a b x^{4} e - 4 a b d x^{2} + 2 a^{2} x^{2} e + a^{2} d}{4 a^{4} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d + a^2*b^3*f - 6*a^3*b*c*f - 2
*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*d*x^4 - 14*
a*b^2*c^2*d*x^4 + 8*a^2*c^3*d*x^4 + a^2*b^2*c*f*x^4 - 4*a^3*c^2*f*x^4 - 2*a
*b^3*c*x^4*e + 8*a^2*b*c^2*x^4*e + 3*b^5*d*x^2 - 12*a*b^3*c*d*x^2 + 2*a^2*b
*c^2*d*x^2 + a^2*b^3*f*x^2 - 2*a^3*b*c*f*x^2 - 2*a*b^4*x^2*e + 6*a^2*b^2*c*
x^2*e + 4*a^3*c^2*x^2*e + 5*a*b^4*d - 22*a^2*b^2*c*d + 12*a^3*c^2*d + 3*a^3
*b^2*f - 8*a^4*c*f - 4*a^2*b^3*e + 14*a^3*b*c*e)/((a^4*b^2 - 4*a^5*c)*(c*x^
4 + b*x^2 + a)) - 1/4*(3*b^2*d - 2*a*c*d + a^2*f - 2*a*b*e)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(3*b^2*d - 2*a*c*d + a^2*f - 2*a*b*e)*log(x^2)/a^4 - 1/4*
(9*b^2*d*x^4 - 6*a*c*d*x^4 + 3*a^2*f*x^4 - 6*a*b*x^4*e - 4*a*b*d*x^2 + 2*a^
2*x^2*e + a^2*d)/(a^4*x^4)
```

Mupad [**B**]
time = 21.02, size = 2500, normalized size = 7.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(x)*(3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 - (log((((((4*b*c^2*(3*
b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e - 17*a*b^3*c*d - 5*a^3*b*c*f +
19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)) - (b*c^2*(a*b + 3*b^2
*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e -
 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*
c - b^2)^3))^(1/2) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 + (2*c^3*x^2
*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a*b^3*c*d - 10*a^3*b*c
*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)))*(a^4*(-(3*b^5*d
+ a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^
2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) + 3*b^2*d + a^2*
f - 2*a*b*e - 2*a*c*d))/(4*a^4) + (c^3*(36*b^8*d^2 + 16*a^2*b^6*e^2 + 4*a^4
*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c*f^2 - 48*a*b^7
*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b^2*c^2*e^2 - 30
9*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^5*c*d*e + 324*a
^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^5*b*c^2*e*f - 8
32*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2)^2) + (c^4*x^2
*(54*b^7*d^2 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3
*b^3*c*e^2 + 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441*
a*b^5*c*d^2 - 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^
4*e*f - 120*a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b
*c^2*d*f + 122*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2)
)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6
*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2
) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/(4*a^4) - (c^4*(3*b^2*d + a^2*f -
 2*a*b*e - 2*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d
)^2)/(a^9*(4*a*c - b^2)^2) + (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^
2*c*e - 11*a*b*c*d)^3)/(a^9*(4*a*c - b^2)^3))*((((c^3*(36*b^8*d^2 + 16*a^2*
b^6*e^2 + 4*a^4*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c
*f^2 - 48*a*b^7*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b
^2*c^2*e^2 - 309*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^
5*c*d*e + 324*a^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^
5*b*c^2*e*f - 832*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2
)^2) - (((b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f
- 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d +
12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) - 3*b^2*d - a^2*f + 2*a*b*e
+ 2*a*c*d))/a^4 + (4*b*c^2*(3*b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e -
 17*a*b^3*c*d - 5*a^3*b*c*f + 19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c
 - b^2)) + (2*c^3*x^2*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a
*b^3*c*d - 10*a^3*b*c*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^
2)))*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d
- 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(
```

```
1/2) - 3*b^2*d - a^2*f + 2*a*b*e + 2*a*c*d))/(4*a^4) + (c^4*x^2*(54*b^7*d^2
 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3*b^3*c*e^2 +
 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441*a*b^5*c*d^2
- 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^4*e*f - 120*
a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b*c^2*d*f + 1
22*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2))*(a^4*(-(3*
b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f +
 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) - 3*b^2*d
- a^2*f + 2*a*b*e + 2*a*c*d))/(4*a^4) + (c^4*(3*b^2*d + a^2*f - 2*a*b*e - 2
*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d)^2)/(a^9*(4
*a*c - b^2)^2) - (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a
*b*c*d)^3)/(a^9*(4*a*c - b^2)^3)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f -
 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^
3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*
c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)) - (d/(4*a) + (x^2*(2*a*e - 3*b*d))/(4*a^2) + (x^4*(6*b^4*d + 8
*a^2*c^2*d + 2*a^2*b^2*f - 4*a*b^3*e - 4*a^3*c*f - 25*a*b^2*c*d + 14*a^2*b*
c*e))/(4*a^3*(4*a*c - b^2)) + (c*x^6*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2
*c*e - 11*a*b*c*d))/(2*a^3*(4*a*c - b^2)))/(a*x^4 + b*x^6 + c*x^8) + (atan(
(x^2*((((((1760*a^7*b*c^8*d^2 - 1104*a^8*b*c^7*e^2 + 80*a^9*b*c^6*f^2 + 54*
a^3*b^9*c^4*d^2 - 657*a^4*b^7*c^5*d^2 + 2775*a^5*b^5*c^6*d^2 - 4484*a^6*b^3
*c^7*d^2 + 24*a^5*b^7*c^4*e^2 - 260*a^6*b^5*c^5*e^2 + 932*a^7*b^3*c^6*e^2 +
 6*a^7*b^5*c^4*f^2 - 44*a^8*b^3*c^5*f^2 - 960*a^8*c^8*d*e + 480*a^9*c^7*e*f
 - 1040*a^8*b*c^7*d*f - 72*a^4*b^8*c^4*d*e + 828*a^5*b^6*c^5*d*e - 3232*a^6
*b^4*c^6*d*e + 4528*a^7*b^2*c^7*d*e + 36*a^5*b^7*c^4*d*f - 351*a^6*b^5*c^5*
d*f + 1088*a^7*b^3*c^6*d*f - 24*a^6*b^6*c^4*e*f...
```

3.68 $\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=550

$$\frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (-2*b*f+c*e)*x/c^3+1/3*f*x^3/c^2+1/2*x*(a*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*
a*f+c*d))+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))
*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*f+c*d)+
2*a*c^2*(-7*a*f+3*c*d)+(-3*b^4*c*e+19*a*b^2*c^2*e-20*a^2*c^3*e+5*b^5*f+b^3*
c*(-34*a*f+c*d)-4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*
a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*
f+c*d)+2*a*c^2*(-7*a*f+3*c*d)+(3*b^4*c*e-19*a*b^2*c^2*e+20*a^2*c^3*e-5*b^5*
f-b^3*c*(-34*a*f+c*d)+4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2
)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 9.87, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1682, 1690, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7af)-5b^4f+3b^3ce\right)}{2\sqrt{2}\,c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7af)-5b^4f+3b^3ce\right)}{2\sqrt{2}\,c^{7/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(a(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)+x^2(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce))}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x(ce-2bf)}{c^3}+\frac{fx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^2*e - b^3*
f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*
f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c
*d - 7*a*f) - (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*
(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b
^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e
 + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*
f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$

$$= \frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e}{2c^3(b^2-4ac)(a+bx}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e}{2c^3(b^2-4ac)(a+bx}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e}{2c^3(b^2-4ac)(a+bx}$$

Mathematica [**A**]
time = 1.30, size = 648, normalized size = 1.18

$$\frac{12\sqrt{c}(ce-2bf)x+4c^{3/2}fx^3-\frac{6\sqrt{c}\,x(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\left(-5b^5f+abc^2\left(8cd+13\sqrt{b^2-4ac}\,e-52af\right)-b^3c\left(cd+3\sqrt{b^2-4ac}\,e-34af\right)+b^4\left(3ce+5\sqrt{b^2-4ac}\,f\right)+b^2c\left(c\sqrt{b^2-4ac}\,d-19ace-24a\sqrt{b^2-4ac}\,f\right)+2ac^2\left(-3c\sqrt{b^2-4ac}\,d+10ace+7a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\left(5b^5f+b^3c\left(cd-3\sqrt{b^2-4ac}\,e-34af\right)+abc^2\left(-8cd+13\sqrt{b^2-4ac}\,e+52af\right)+b^4\left(-3ce+5\sqrt{b^2-4ac}\,f\right)+b^2c\left(c\sqrt{b^2-4ac}\,d+19ace-24a\sqrt{b^2-4ac}\,f\right)-2ac^2\left(3c\sqrt{b^2-4ac}\,d+10ace-7a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2*(c^2*d -
b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*
x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f + a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a*c]*
e - 52*a*f) - b^3*c*(c*d + 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*e + 5
*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*a*Sqrt
[b^2 - 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*f + b^
3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + a*b*c^2*(-8*c*d + 13*Sqrt[b^2
- 4*a*c]*e + 52*a*f) + b^4*(-3*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt
[b^2 - 4*a*c]*d + 19*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^2*(3*c*Sqrt[
b^2 - 4*a*c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(12*c^(7/2))
```

Maple [A]
time = 0.10, size = 682, normalized size = 1.24

method	result
risch	$\frac{f x^3}{3c^2} - \frac{2bfx}{c^3} + \frac{ex}{c^2} + \frac{\frac{\left(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2c^3ad+b^4f-b^3ce+b^2c^2d\right)x^3}{8ac-2b^2} - \frac{a\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c^3\left(c\,x^4+b\,x^2+a\right)} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_\ldots\right.}}{\ldots}$
default	$-\frac{-\frac{1}{3}c\,x^3f+2bfx-cex}{c^3} + \frac{\frac{\left(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2c^3ad+b^4f-b^3ce+b^2c^2d\right)x^3}{8ac-2b^2} - \frac{a\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a} + \frac{2c\left(-\frac{\left(-14a^2c^2\ldots\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*c*x^3*f+2*b*f*x-c*e*x)+1/c^3*((1/2*(2*a^2*c^2*f-4*a*b^2*c*f+3*
a*b*c^2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*c
*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c
-b^2)*c*(-1/8*(-14*a^2*c^2*f*(-4*a*c+b^2)^(1/2)+24*a*b^2*c*f*(-4*a*c+b^2)^(
1/2)-13*a*b*c^2*e*(-4*a*c+b^2)^(1/2)+6*c^3*a*d*(-4*a*c+b^2)^(1/2)-5*b^4*f*(
-4*a*c+b^2)^(1/2)+3*b^3*c*e*(-4*a*c+b^2)^(1/2)-b^2*c^2*d*(-4*a*c+b^2)^(1/2)
+52*a^2*b*c^2*f-20*a^2*c^3*e-34*a*b^3*c*f+19*a*b^2*c^2*e-8*a*b*c^3*d+5*b^5*
f-3*b^4*c*e+b^3*c^2*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-14*
a^2*c^2*f*(-4*a*c+b^2)^(1/2)+24*a*b^2*c*f*(-4*a*c+b^2)^(1/2)-13*a*b*c^2*e*(
-4*a*c+b^2)^(1/2)+6*c^3*a*d*(-4*a*c+b^2)^(1/2)-5*b^4*f*(-4*a*c+b^2)^(1/2)+3
*b^3*c*e*(-4*a*c+b^2)^(1/2)-b^2*c^2*d*(-4*a*c+b^2)^(1/2)-52*a^2*b*c^2*f+20*
a^2*c^3*e+34*a*b^3*c*f-19*a*b^2*c^2*e+8*a*b*c^3*d-5*b^5*f+3*b^4*c*e-b^3*c^2
*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3*c*e - 3*a*b*c^2*e - (b^2*c^2 - 2*a*c^3)*d - (b^4 - 4*a*b^2*c + 2*
a^2*c^2)*f)*x^3 - (a*b*c^2*d - a*b^2*c*e + 2*a^2*c^2*e + (a*b^3 - 3*a^2*b*c
```

```
)*f)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b
*c^4)*x^2) + 1/2*integrate((a*b*c^2*d - 3*a*b^2*c*e + 10*a^2*c^2*e - (3*b^3
*c*e - 13*a*b*c^2*e - (b^2*c^2 - 6*a*c^3)*d - (5*b^4 - 24*a*b^2*c + 14*a^2*
c^2)*f)*x^2 + (5*a*b^3 - 19*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 -
4*a*c^4) + 1/3*(c*f*x^3 - 3*(2*b*f - c*e)*x)/c^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 18909 vs. $2(506) = 1012$.
time = 75.86, size = 18909, normalized size = 34.38

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(b^2*c^2 - 4*a*c^3)*f*x^7 + 4*(3*(b^2*c^2 - 4*a*c^3)*e - 5*(b^3*c -
 4*a*b*c^2)*f)*x^5 - 2*(3*(b^2*c^2 - 2*a*c^3)*d - 3*(3*b^3*c - 11*a*b*c^2)*
e + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*f)*x^3 + 3*sqrt(1/2)*(a*b^2*c^3 - 4*
a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)*sqrt(-((b^5*
c^4 - 15*a*b^3*c^5 + 60*a^2*b*c^6)*d^2 - 2*(3*b^6*c^3 - 40*a*b^4*c^4 + 150*
a^2*b^2*c^5 - 120*a^3*c^6)*d*e + (9*b^7*c^2 - 105*a*b^5*c^3 + 385*a^2*b^3*c
^4 - 420*a^3*b*c^5)*e^2 + (25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a
^3*b^3*c^3 + 1260*a^4*b*c^4)*f^2 + 2*((5*b^7*c^2 - 69*a*b^5*c^3 + 285*a^2*b
^3*c^4 - 340*a^3*b*c^5)*d - (15*b^8*c - 182*a*b^6*c^2 + 735*a^2*b^4*c^3 - 1
050*a^3*b^2*c^4 + 280*a^4*c^5)*e)*f + (b^6*c^7 - 12*a*b^4*c^8 + 48*a^2*b^2*
c^9 - 64*a^3*c^10)*sqrt(((b^4*c^8 - 18*a*b^2*c^9 + 81*a^2*c^10)*d^4 - 4*(3*
b^5*c^7 - 49*a*b^3*c^8 + 198*a^2*b*c^9)*d^3*e + 6*(9*b^6*c^6 - 132*a*b^4*c^
7 + 484*a^2*b^2*c^8 - 75*a^3*c^9)*d^2*e^2 - 4*(27*b^7*c^5 - 351*a*b^5*c^6 +
 1197*a^2*b^3*c^7 - 550*a ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8957 vs. $2(517) = 1034$.
time = 7.93, size = 8957, normalized size = 16.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(b^2*c^2*d*x^3 - 2*a*c^3*d*x^3 + b^4*f*x^3 - 4*a*b^2*c*f*x^3 + 2*a^2*c
^2*f*x^3 - b^3*c*x^3*e + 3*a*b*c^2*x^3*e + a*b*c^2*d*x + a*b^3*f*x - 3*a^2*
b*c*f*x - a*b^2*c*x*e + 2*a^2*c^2*x*e)/((b^2*c^3 - 4*a*c^4)*(c*x^4 + b*x^2
+ a)) - 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c
^3 - 4*a*c^4)^2*d + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*c^4 - 112*a^3*
c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 44*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 110*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 48*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 48*(
b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 - 4*a*c^4)^2*f
- (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2
*c^3 - 4*a*c^4)^2*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^8 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 + 16*a^2*b^3*
c^8 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 - 32*a^3*b*c^9 +
2*(b^2 - 4*a*c)*a*b^3*c^7 - 8*(b^2 - 4*a*c)*a^2*b*c^8)*d*abs(b^2*c^3 - 4*a*
c^4) - 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^6*c^5 - 10*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^3*c^6 + 78*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*
c^6 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 118*a^2*b^5*c^6
```

```
- 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^7 - 464*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c^8 + 608*a^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2
- 4*a*c)*a^2*b^3*c^6 + 152*(b^2 - 4*a*c)*a^3*b*c^7)*f*abs(b^2*c^3 - 4*a*c^4
) + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 +
 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 - 160
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^8 - 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*c^9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*b^2
*c^7 + 80*(b^2 - 4*a*c)*a^3*c^8)*abs(b^2*c^3 - 4*a*c^4)*e - (2*b^8*c^10 - 3
2*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^2*c^13 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(2)*...
```

Mupad [B]
time = 4.10, size = 2500, normalized size = 4.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x*(e/c^2 - (2*b*f)/c^3) + ((x^3*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*
d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2
*e + a*b^3*f + a*b*c^2*d - a*b^2*c*e - 3*a^2*b*c*f))/(2*(4*a*c - b^2)))/(a*
c^3 + c^4*x^4 + b*c^3*x^2) - atan(((((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d -
 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2
*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16
*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a
^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x
*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2
)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7
*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^
```

4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 614

```
4*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^
2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*
a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 7
18*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4
*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c
^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b
^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f
 - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^
13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2...
```

3.69 $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=436

$$\frac{fx}{c^2}+\frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2ce-6ac^2e-3b^3f+bc(c\right.}{}$$

```
[Out] f*x/c^2+1/2*x*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)-(b^2*c*e-2*a*c^2*e-b^3*f-b*
c*(-3*a*f+c*d))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6*a*c^2*e-3*b^3*f+b*c*(13*a*
f+c*d)+(-b^3*c*e+8*a*b*c^2*e+3*b^4*f-4*a*c^2*(-5*a*f+c*d)-b^2*c*(19*a*f+c*d
))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6
*a*c^2*e-3*b^3*f+b*c*(13*a*f+c*d)+(b^3*c*e-8*a*b*c^2*e-3*b^4*f+4*a*c^2*(-5*
a*f+c*d)+b^2*c*(19*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 3.60, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1682, 1690, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{fx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*
e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) - (b^3*c*e - 8*a
*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^
2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a
*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \int \frac{a^2(\cdots}{\cdots}$$

$$= \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \int\left(-\cdots\right.$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \cdots$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \cdots$$

$$= \frac{fx}{c^2} + \frac{x(a(2c^2d-bce+b^2f-2acf)-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2)}{2c^2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \cdots$$

Mathematica [A]
time = 0.96, size = 511, normalized size = 1.17

$$\frac{4\sqrt{c}fx + \frac{2\sqrt{c}x(-2a^2cf+b(c^2d-bce+b^2f)x^2+a(b^2f+2c^2(d+ex^2)-bc(e+3fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4f+2ac^2\left(2cd+3\sqrt{b^2-4ac}\,e-10af\right)+b^2c\left(cd-\sqrt{b^2-4ac}\,e+19af\right)+b^3\left(ce+3\sqrt{b^2-4ac}\,f\right)-bc\left(c\sqrt{b^2-4ac}\,d+8ace+13a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(3b^4f+2ac^2\left(-2cd+3\sqrt{b^2-4ac}\,e+10af\right)-b^2c\left(cd+\sqrt{b^2-4ac}\,e+19af\right)+b^3\left(-ce+3\sqrt{b^2-4ac}\,f\right)-bc\left(c\sqrt{b^2-4ac}\,d-8ace+13a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 +
 a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - 10*a*f) + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*Sqrt
[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*c^2*(-2
*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e +
 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*
d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
4*c^(5/2))
```

Maple [A]
time = 0.07, size = 522, normalized size = 1.20

method	result
risch	$\frac{fx}{c^2}+\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{8ac-2b^2}+\frac{a\left(2acf-b^2f+bce-2c^2d\right)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(13abcf-6a\,c^2e-3b^3f+b^2ce\right.}{4ac-b^2}\right.}{}}{4c^2}$
default	$\frac{fx}{c^2}-\frac{-\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{2\left(4ac-b^2\right)}-\frac{a\left(2acf-b^2f+bce-2c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(13\sqrt{-4ac+b^2}\,abcf-6a\,c^2e\sqrt{-4ac+b^2}-3b\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] f*x/c^2-1/c^2*((-1/2*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x^3-1/2*a*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(13*(-4*a*c+b^2)^(1/2)*a*b*c*f-6*a*c^2*e*(-4*a*c+b^2)^(1/2)-3*b^3*f*(-4*a*c+b^2)^(1/2)+b^2*c*e*(-4*a*c+b^2)^(1/2)+b*c^2*d*(-4*a*c+b^2)^(1/2)+20*a^2*c^2*f-19*a*b^2*c*f+8*a*b*c^2*e-4*c^3*a*d+3*b^4*f-b^3*c*e-b^2*c^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(13*(-4*a*c+b^2)^(1/2)*a*b*c*f-6*a*c^2*e*(-4*a*c+b^2)^(1/2)-3*b^3*f*(-4*a*c+b^2)^(1/2)+b^2*c*e*(-4*a*c+b^2)^(1/2)+b*c^2*d*(-4*a*c+b^2)^(1/2)-20*a^2*c^2*f+19*a*b^2*c*f-8*a*b*c^2*e+4*c^3*a*d-3*b^4*f+b^3*c*e+b^2*c^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/2*((b*c^2*d - b^2*c*e + 2*a*c^2*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d - a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^2*d - a*b*c*e - (b*c^2*d + b^2*c*e - 6*a*c^2*e - (3*b^3 - 13*a*b*c)*f)*x^2 + (3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12597 vs. 2(394) = 788.
time = 22.84, size = 12597, normalized size = 28.89

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*f*x^5 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (3*b^3
- 11*a*b*c)*f)*x^3 + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)
*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^
4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*
c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*
((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 15
0*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e
^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*
e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4
*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)
*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 +
 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^
4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a
^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)
*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*
a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)
)*log(((3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^
4*c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*
d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1
971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6
*c + 3213*a^2*b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1
323*a^2*b^5*c + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2
- 117*a*b^4*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^
5*c^2 + 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c
^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^
5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3
 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^
4)*e^3)*f)*x + 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 +
3*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*
c^6 + 16*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^
3*b*c^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3
 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4
*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^
2*b^5*c^3 - 2536*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*
```

```
a*b^4*c^5 - 32*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c
^5 + 16*a^3*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a
^3*b^2*c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*
c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^
3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 -
 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^
2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*
b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*
b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 5
50*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9
*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 +
484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(
3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*
e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^
2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 +
60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)
*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4
*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a
^2*b^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c
^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*
a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
+ 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125
*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)
*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51
*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c
^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4
*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*
c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7496 vs. 2(411) = 822.
time = 7.06, size = 7496, normalized size = 17.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/c^2 + 1/2*(b*c^2*d*x^3 + b^3*f*x^3 - 3*a*b*c*f*x^3 - b^2*c*x^3*e + 2*a*
c^2*x^3*e + 2*a*c^2*d*x + a*b^2*f*x - 2*a^2*c*f*x - a*b*c*x*e)/((c*x^4 + b*
x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*
c^3)^2*d - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(
b^2*c^2 - 4*a*c^3)^2*f + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^
4)*(b^2*c^2 - 4*a*c^3)^2*e - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 16*a^2*b
^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^8 + 32*a^3*c^8 - 2
*(b^2 - 4*a*c)*a*b^2*c^6 + 8*(b^2 - 4*a*c)*a^2*c^7)*d*abs(-b^2*c^2 + 4*a*c^
3) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5
+ 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 16
0*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^
2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*f*abs(-b^2*c^2 + 4*a*c^3) + 2*(sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
```

```
*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5
+ 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2
- 4*a*c)*a^2*b*c^6)*abs(-b^2*c^2 + 4*a*c^3)*e - (2*b^7*c^8 - 8*a*b^5*c^9 -
32*a^2*b^3*c^10 + 128*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^7*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^6*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c^8 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^9 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*d + (6*b^9
*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4...
```

Mupad [B]
time = 2.65, size = 2500, normalized size = 5.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (f*x)/c^2 - atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d +
 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c
^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7
*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^
2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*
d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 38
40*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a
```

^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^
(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*
c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^
4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2
)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*
b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*
a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8
*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6
*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c
^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b
^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) -
9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 -
512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4
*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5
*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
- b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*
e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e
- 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 19
2*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^
7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e
*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f +
6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1
280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8
*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6
*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^
4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^
3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*
d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c
^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((
768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c
^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2
+ 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*
c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2
*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c
^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*
f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^

```
6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5
*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c -
 b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b
^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*
d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 2
2400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2
)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 +
b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^
4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*f - 2048*a^4*c^8*d
 - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3
*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f...
```

3.70 $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=362

$$-\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\,\sqrt{c}\,\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2}}}$$

```
[Out] -1/2*x*(b*c*d-2*a*c*e+a*b*f+(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b
^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(2*c*d-b*e+6*a*f-b^2*f/c+(b^2*c*e+4*a*c^2*e+b^3*f-4*b*c*(2*a*f+c*d))/c/
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-b*e+6*
a*f-b^2*f/c+(-b^2*c*e-4*a*c^2*e-b^3*f+4*b*c*(2*a*f+c*d))/c/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.74, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1682, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af-\frac{b^2f}{c}-be+2cd\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)
)/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c
 + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]
))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt
[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b
^2*f)/c - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 -
 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{-\frac{a(bcd-2ace+abf)}{c}+a\left(2cd \cdots\right.}{a+bx^2+c \cdots}}{2a\,(b^2- \cdots}$$

$$= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(2cd-be+6af-\frac{b^2 \cdots}{c}\right.}{\cdots}$$

$$= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(2cd-be+6af-\frac{b^2 \cdots}{c}\right.}{2 \cdots}$$

Mathematica [A]
time = 0.68, size = 414, normalized size = 1.14

$$\frac{-\frac{2\sqrt{c}\,x(abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-b^3f+bc\left(4cd+\sqrt{b^2-4ac}\,e+8af\right)+b^2\left(-ce+\sqrt{b^2-4ac}\,f\right)-2c\left(c\sqrt{b^2-4ac}\,d+2ace+3a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^3f+bc\left(-4cd+\sqrt{b^2-4ac}\,e-8af\right)+b^2\left(ce+\sqrt{b^2-4ac}\,f\right)-2c\left(c\sqrt{b^2-4ac}\,d-2ace+3a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(
e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*f) + b*c
*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f)
 - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 - 4*a*c
]*e - 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d
 - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
c^(3/2))
```

Maple [A]
time = 0.06, size = 415, normalized size = 1.15

method	result
risch	$\frac{-\frac{\left(2acf-b^2f+bce-2c^2d\right)x^3}{2c\left(4ac-b^2\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6acf-b^2f-bce+2c^2d\right)\_R^2}{4ac-b^2}-\frac{abf-2ace+bcd}{4ac-b^2}\right)\ln\left(x-\_\right.}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{\left(2acf-b^2f+bce-2c^2d\right)x^3}{2c\left(4ac-b^2\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{-\frac{\left(6acf\sqrt{-4ac+b^2}-b^2f\sqrt{-4ac+b^2}-bce\sqrt{-4ac+b^2}+2c^2d\sqrt{-4\right.}{4c\sqrt{-4ac+b^2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^3+1/2/c*(a*b*f-2*a*c*e+
b*c*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(-1/8*(6*a*c*f*(-4*a*c+
b^2)^(1/2)-b^2*f*(-4*a*c+b^2)^(1/2)-b*c*e*(-4*a*c+b^2)^(1/2)+2*c^2*d*(-4*a*
c+b^2)^(1/2)-8*a*b*c*f+4*a*c^2*e+b^3*f+b^2*c*e-4*b*c^2*d)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*a*c*f*(-4*a*c+b^2)^(1/2)-b^2*f*(-4*a*c+b^2
)^(1/2)-b*c*e*(-4*a*c+b^2)^(1/2)+2*c^2*d*(-4*a*c+b^2)^(1/2)+8*a*b*c*f-4*a*c
^2*e-b^3*f-b^2*c*e+4*b*c^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d + a*b*f - 2*a*c*e)*x
)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
 - 1/2*integrate(-(b*c*d + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6*a*c)*f)*x^2
- 2*a*c*e)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8951 vs. $2(320) = 640$.
time = 12.98, size = 8951, normalized size = 24.73

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a
*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-((b^3*c^3
+ 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b
*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 2
8*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 -
 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*
e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b
^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*
d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 +
 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*
b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a
^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 -
 (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b
^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 + ((
a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*c + 324*a^4
*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5
*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^
2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2
*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*
a^3*b*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a
^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*
c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 - 144*a^5*b*c^3)*f^3 -
((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a
^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f - ((a*b
^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a^2*b^7*c^4
- 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^2*b^8*c^3 - 24*a^3
*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^
```

```
4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f
^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 -
2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3
*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c
^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*s
qrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3
*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((
3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f
 + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4
 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^
4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2
*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*
c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^
3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a
*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) - sqrt(1/2)*((b^
2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(
-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2
 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*
b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (
a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2
*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 -
4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12
*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)
*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^
3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6
*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a
*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3
 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^
2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*
c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d
^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*
b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c
^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^
3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 1
6*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^
4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c +...
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6208 vs. $2(328) = 656$.
time = 6.07, size = 6208, normalized size = 17.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c^2*d*x^3 + b^2*f*x^3 - 2*a*c*f*x^3 - b*c*x^3*e + b*c*d*x + a*b*f*x
 - 2*a*c*x*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*(2*(2*b^2*c^4
- 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^4 - 2*(b^2 - 4*a*c)*c^4)
*(b^2*c - 4*a*c^2)^2*d - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^
2*c - 4*a*c^2)^2*f - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*e - 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 2
*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^5 + 16*a*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^
5)*d*abs(b^2*c - 4*a*c^2) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^
2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*
c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(b^2*c -
4*a*c^2) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^
```

```
2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b
^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*abs(b^2*c - 4*a*c^2)*e - 4*(2*b^6*c^6 - 1
6*a*b^4*c^7 + 32*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7
 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7)*d + (2*b^8*c^4 - 32
*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c
^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f + (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^
3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5
- 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 -
32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 +
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^...
```

Mupad [B]
time = 6.54, size = 2500, normalized size = 6.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)) + (x*(a*b*f
- 2*a*c*e + b*c*d))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((204
8*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a
```

^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 10
24*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^
6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c^7*d^2
 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*
e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*
a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*
c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a
*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c
 - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c
^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f -
192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10
*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c
^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^
7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6
 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7
*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*
c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^
2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*
c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 +
 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*
b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f +
12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*
a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b
^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*
f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*
b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^
12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^
4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e
^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^
5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*
d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f
 + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*
d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c
^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c
f^2(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 +
96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b
^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 1
2*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^
7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f
 - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b
^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^1

```
2*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4
*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*1i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 76
8*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*
f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e
 + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3)) + (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a
*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2
*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2
*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96
*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c
^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2
- 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f
+ 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c
^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f -
3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b
^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*
b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 7...
```

3.71 $\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=346

$$\frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-}}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-}}}$$

```
[Out] 1/2*x*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*(b*c*d-2*a*c*e+a*b*f+(4*a*b*c*e+b^2*(-a*f+c*d)-4*a*c*(a*f+3*c*d))/(-4*a*c+
b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*e+a*b*
f+(-4*a*b*c*e-b^2*(-a*f+c*d)+4*a*c*(a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a/(-4*a
*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.26, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1692, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e + a*b*f + (4*a*b*c*e
 + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e +
b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d-abe+2a(3cd+af)+(-bcd+2 \cdots}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right) \cdots}$$

$$= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf-\frac{4abce+ \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf+\frac{4abce+ \cdots}{\cdots}\right.}{2\sqrt{2}\,a\sqrt{\cdots}}$$

Mathematica [A]
time = 0.66, size = 382, normalized size = 1.10

$$\frac{\frac{2x\left(b^2d+b\left(-ae+cdx^2+afx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2(cd-af)-2ac\left(6cd+\sqrt{b^2-4ac}\,e+2af\right)+b\left(c\sqrt{b^2-4ac}\,d+4ace+a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2(-cd+af)+2ac\left(6cd-\sqrt{b^2-4ac}\,e+2af\right)+b\left(c\sqrt{b^2-4ac}\,d-4ace+a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))))
/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*(6
*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e +
```

```
a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*
(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sq
rt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.06, size = 387, normalized size = 1.12

method	result
risch	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a(4ac-b^2)}-\frac{(2a^2f-abe-2acd+b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{(abf-2ace+bcd)\_R^2}{4ac-b^2}+\frac{2a^2f-abe+6acd-b^2d}{4ac-b^2}\right)\ln(\cdots}{2c\_R^3+\_Rb}}{4a}$
default	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a(4ac-b^2)}-\frac{(2a^2f-abe-2acd+b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(-\frac{\left(-\sqrt{-4ac+b^2}\,abf+2ace\sqrt{-4ac+b^2}-bcd\sqrt{-4ac+b^2}+4a^2\cdots\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\cdots}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b^
2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(-1/8*(-(-4*a*c+b^2
)^(1/2)*a*b*f+2*a*c*e*(-4*a*c+b^2)^(1/2)-b*c*d*(-4*a*c+b^2)^(1/2)+4*a^2*c*f
+a*b^2*f-4*a*b*c*e+12*a*c^2*d-b^2*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))+1/8*(-(-4*a*c+b^2)^(1/2)*a*b*f+2*a*c*e*(-4*a*c+b^2)^(1/2)-b*c*d*(-4*
a*c+b^2)^(1/2)-4*a^2*c*f-a*b^2*f+4*a*b*c*e-12*a*c^2*d+b^2*c*d)/(-4*a*c+b^2)
^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d + a*b*f - 2*a*c*e)*x^3 + (2*a^2*f - a*b*e + (b^2 - 2*a*c)*d)*x)
/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

```
+ 1/2*integrate(-(2*a^2*f - (b*c*d + a*b*f - 2*a*c*e)*x^2 - a*b*e - (b^2 -
6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8991 vs. $2(304) = 608$.
time = 11.41, size = 8991, normalized size = 25.99

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e + a*b*f)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^
2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2
*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c -
 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b
^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4
 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4
*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*
(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*
b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^
2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4
*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a
^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4
*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*
a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a
^4*b^2*c - 48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c -
6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b
^6*c - 15*a*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a
^3*b*c^3)*d^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^
4*b*c^2)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3
 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^
3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b
^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3
 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c +
160*a^5*b^2*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e
)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d
^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*
b^2*c^2 + 16*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c
^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*
a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c
^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^
3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*
a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e +
```

```
 a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^
4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c
^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b
*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*
a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^
2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*
a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*
d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
- 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d
*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e
+ a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b
^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^
5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*
b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12
*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c
^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48
*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c
*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e
 + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a
^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4
 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a
^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2
 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c -
48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c
^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a
*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d
^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^
3)*f)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*
b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 -
 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6356 vs. $2(310) = 620$.

time = 6.99, size = 6356, normalized size = 18.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 + a*b*f*x^3 - 2*a*c*x^3*e + b^2*d*x - 2*a*c*d*x + 2*a^2*f*x
- a*b*x*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^3 - 8*a
*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c
^3)*(a*b^2 - 4*a^2*c)^2*d + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2
*f - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^
6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^3 + 28*a^2*b^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 2
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 192*a^4*c^5 + 2*(b^2 -
4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)
*d*abs(a*b^2 - 4*a^2*c) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 16*a^
4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^4
+ 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*f*abs(a*b^2 - 4*a^
2*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^4*c^2 - 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 -
```

```
4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4*a*c)*a^3*b*c^3)*abs(a*b^2 - 4*a^2*c)*e + (2
*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d - (2
*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 3
2*(b^2 - 4*a*c)*a^5*b*c^4)*f + 4*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b
^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*
c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2
- 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 ...
```

Mupad [B]
time = 6.55, size = 2500, normalized size = 7.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4
*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^
4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 1
6*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2)) - (x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c
- b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^
```

2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*
a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*
c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 -
3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f
- 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*
d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 12
8*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^
6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^
8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f
^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(
-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^
4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 +
96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^
3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^
6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*
a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*
c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f
- 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2
+ 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)
))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2
- 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^
2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f +
4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
- b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) -
a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b
^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2
+ 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a
^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*
e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b
^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d
*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384
*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^
6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c
^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b
^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^
3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 102
4*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) +

(x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^...

3.72 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=399

$$-\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c(b^2d-abe-2a(cd-af))\,x^2\right)}{2a^2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\sqrt{c}\,\Bigl(3b^2d-abe-2a(5cd\ldots}{\ldots}$$

```
[Out] -d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e))+c*(b^2*d-a*b*e-2*a*
(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+(
3*b^3*d-a*b^2*e+12*a^2*c*e-4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+(-3*b^
3*d+a*b^2*e-12*a^2*c*e+4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b
^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.40, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1683, 1678, 1180, 211}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}\,a^2\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{2\sqrt{2}\,a^2\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2(-abe-2a(cd-af)+b^2d)\right)}{2a^2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{d}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b
^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) + (3*b^3*d - a*b^2*e +
 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3
*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a
*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int}{}$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int}{}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A]
time = 0.82, size = 444, normalized size = 1.11

$$\frac{-\frac{4d}{x}-\frac{2x\left(b^3d+b^2\left(-ae+cdx^2\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(-cdx^2+a\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3d+b^2\left(-3\sqrt{b^2-4ac}\,d+ae\right)+ab\left(16cd+\sqrt{b^2-4ac}\,e+4af\right)-2a\left(-5c\sqrt{b^2-4ac}\,d+6ace+a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3d-b^2\left(3\sqrt{b^2-4ac}\,d+ae\right)+ab\left(-16cd+\sqrt{b^2-4ac}\,e-4af\right)+2a\left(5c\sqrt{b^2-4ac}\,d+6ace-a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x^
2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b
*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6
*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d + Sqrt
[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [A]
time = 0.11, size = 438, normalized size = 1.10

method	result

default	$\frac{\frac{c\left(2a^{2}f-abe-2acd+b^{2}d\right)x^{3}}{8ac-2b^{2}}+\frac{\left(a^{2}bf+2a^{2}ce-a\,b^{2}e-3abcd+b^{3}d\right)x}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+ 2c\left(-\frac{\left(2a^{2}f\sqrt{-4ac+b^{2}}-abe\sqrt{-4ac+b^{2}}-10\sqrt{-4ac+\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*((1/2*c*(2*a^2*f-a*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2)*x^3+1/2*(a^2*b*f+2*
a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^
2)*c*(-1/8*(2*a^2*f*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+
b^2)^(1/2)*a*c*d+3*(-4*a*c+b^2)^(1/2)*b^2*d-4*a^2*b*f+12*a^2*c*e-a*b^2*e-16
*a*b*c*d+3*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(2*a^2*f*(-4
*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*a*c*d+3*(-4*
a*c+b^2)^(1/2)*b^2*d+4*a^2*b*f-12*a^2*c*e+a*b^2*e+16*a*b*c*d-3*b^3*d)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-d/a^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a^2*c*f - a*b*c*e + (3*b^2*c - 10*a*c^2)*d)*x^4 + (a^2*b*f - a*b^2
*e + 2*a^2*c*e + (3*b^3 - 11*a*b*c)*d)*x^2 + 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b
^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
- 1/2*integrate(-(a^2*b*f + a*b^2*e - 6*a^2*c*e - (2*a^2*c*f - a*b*c*e + (3
*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d)/(c*x^4 + b*x^2 + a), x)/(
a^2*b^2 - 4*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13111 vs. 2(357) = 714.
time = 26.38, size = 13111, normalized size = 32.86

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - 2*(a^2*b*f + (3
*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 + sqrt(1/2)*((a^2*b^2*c - 4*a
^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*
b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*
a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c +
60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3
*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*
a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*
b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5
- 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^
2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*
b^2*c + 25*a^6*c^2)*d^2 - ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7182 vs. 2(366) = 732.
time = 8.21, size = 7182, normalized size = 18.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a*b*c*x^4*e + 3*b^3*
d*x^2 - 11*a*b*c*d*x^2 + a^2*b*f*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2*e + 2*a*b^
2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) + 1/16*((6*b^4
*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
```

```
*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b
^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(
2*a^2*b^2*c^2 - 8*a^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2
- 2*(b^2 - 4*a*c)*a^2*c^2)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a*b^3*c^2 - 8*a^2*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b
*c^2)*(a^2*b^2 - 4*a^3*c)^2*e - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*
c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^
3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a
^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*d*ab
s(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5 -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c - 2*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*a^4*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^6*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 16*a^5*b^3*c
^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 32*a^6*b*c^3 + 2
*(b^2 - 4*a*c)*a^4*b^3*c - 8*(b^2 - 4*a*c)*a^5*b*c^2)*f*abs(a^2*b^2 - 4*a^3
*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^4
 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2*c^2 +
 48*(b^2 - 4*a*c)*a^5*c^3)*abs(a^2*b^2 - 4*a^3*c)*e + (6*a^4*b^8*c^2 - 80*a
^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(
b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*d - 4*(2*a^6*b^6*
c^2 - 16*a^7*b^4*c^3 + 32*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^8*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^6*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c...
```

Mupad [B]
time = 6.86, size = 2500, normalized size = 6.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(3*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*c
 - b^2)) - d/a + (c*x^4*(3*b^2*d + 2*a^2*f - a*b*e - 10*a*c*d))/(2*a^2*(4*a
*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan(((x*(204800*a^12*c^9*d^2 - 73728*a
^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*
d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*
d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 +
 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 +
160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 1638
4*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^1
3*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^
5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3
*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f
 + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 92
16*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^
11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 +
 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e -
2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 4480
0*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840
*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*
d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*
```

f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*
c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*
c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*
b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*
a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d
e(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c
+ 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5
)))^(1/2)*(x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^
9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6
*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/
2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^
7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b
^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f
^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^
6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*
b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36
*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2
*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
 + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*
b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e
*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^
3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256
*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*
c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*a^15*c^8*e + 192
*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*
b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^
2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e -
 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*
a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b
^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f))*((27*a^3*b^9*c*e^2 -
a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5
*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*
d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2
+ 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2
d^2(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2
- 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b
^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*
c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^
3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*...

3.73 $\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=575

$$-\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab(3cd-a\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^2
*(b*e+4*c*d)/a+b^2*f-2*a*c*f)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))*
x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d+b^3*(-3*a*e+5*d*(-4*a*c+b^2)^(1/2))-
a*b^2*(29*c*d-a*f+3*e*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f+5*e*(-4*a*c
+b^2)^(1/2))-a*b*(-16*a*c*e+19*c*d*(-4*a*c+b^2)^(1/2)-a*f*(-4*a*c+b^2)^(1/2
)))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d-b^3*(3*a*e
+5*d*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f-5*e*(-4*a*c+b^2)^(1/2))-a*b^
2*(29*c*d-a*f-3*e*(-4*a*c+b^2)^(1/2))+a*b*(16*a*c*e+19*c*d*(-4*a*c+b^2)^(1/
2)-a*f*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A]
time = 6.51, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1683, 1678, 1180, 211}

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(3e\sqrt{b^2-4ac}-af+29cd\right)-ab\left(19cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}-16ace\right)+b^3\left(5d\sqrt{b^2-4ac}-3ae\right)+5b^4d\right)}{2\sqrt{2}\,a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(2a^2c\left(-5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(-3e\sqrt{b^2-4ac}-af+29cd\right)+ab\left(19cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+16ace\right)-b^3\left(5d\sqrt{b^2-4ac}+3ae\right)+5b^4d\right)}{2\sqrt{2}\,a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/3*d/(a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d +
 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(b^3*d - a*b^2*e +
2*a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14
*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]
*e - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*
f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3
*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3
*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e
- 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^
2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x
```

```
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b^3d-ab^2e+2a^2ce-ab\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2-\right.}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2-\right.}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2-\right.}$$

Mathematica [A]
time = 1.12, size = 548, normalized size = 0.95

$$\frac{-\frac{4ad}{x^3}+\frac{24bd-12ae}{x}+\frac{6x\left(b^4d+b^3(-ae+cdx^2)+abc(3ae-3cdx^2+afx^2)+2a^2c(-af+c(d+ex^2))+ab^2(af-c(4d+ex^2))\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4d+b^3\left(5\sqrt{b^2-4ac}\,d-3ae\right)+2a^2c\left(14cd+5\sqrt{b^2-4ac}\,e-6af\right)+ab^2\left(-29cd-3\sqrt{b^2-4ac}\,e+af\right)+ab\left(-19c\sqrt{b^2-4ac}\,d+16ace+a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-5b^4d+b^3\left(5\sqrt{b^2-4ac}\,d+3ae\right)-ab^2\left(-29cd+3\sqrt{b^2-4ac}\,e+af\right)+2a^2c\left(-14cd+5\sqrt{b^2-4ac}\,e+6af\right)+ab\left(-19c\sqrt{b^2-4ac}\,d-16ace+a\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2)) +
a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sq
rt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*
c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]
*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4*d + b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a
*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-19*c*Sqrt[b
^2 - 4*a*c]*d - 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]]))/(12*a^3)
```

Maple [A]
time = 0.13, size = 570, normalized size = 0.99

method	result
default	$\frac{-\frac{c\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-b^4d\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+2c\left(-\frac{\left(-a^2bf\sqrt{-4ac+b^2}-10a^2ce\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((-1/2*c*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^3+
1/2*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)
/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(-a^2*b*f*(-4*a*c+b^2
)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*(-4*a
*c+b^2)^(1/2)*a*b*c*d-5*(-4*a*c+b^2)^(1/2)*b^3*d+12*a^3*c*f-a^2*b^2*f-16*a^
2*b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*b^4*d)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))+1/8*(-a^2*b*f*(-4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)
^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*(-4*a*c+b^2)^(1/2)*a*b*c*d-5*(-4*a*c
+b^2)^(1/2)*b^3*d-12*a^3*c*f+a^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-
29*a*b^2*c*d+5*b^4*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/3*d/a^2/x^3
-(a*e-2*b*d)/a^3/x
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^2*b*c*f - 3*a*b^2*c*e + 10*a^2*c^2*e + (5*b^3*c - 19*a*b*c^2)*d)*
x^6 - (9*a*b^3*e - 33*a^2*b*c*e - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*
(a^2*b^2 - 2*a^3*c)*f)*x^4 - 2*(3*a^2*b^2*e - 12*a^3*c*e - 5*(a*b^3 - 4*a^2
*b*c)*d)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(-(3*a*b^3*
e - 13*a^2*b*c*e - (a^2*b*c*f - 3*a*b^2*c*e + 10*a^2*c^2*e + (5*b^3*c - 19*
a*b*c^2)*d)*x^2 - (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*d - (a^2*b^2 - 6*a^3*c)
*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19333 vs. $2(510) = 1020$.
time = 84.86, size = 19333, normalized size = 33.62

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)
*x^6 + 2*((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^3 - 11*a^2*b*c)*e
 + 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 4*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 -
 4*a^3*c)*e)*x^2 + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*
a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-((25*b^9 - 315*a*b^7*c + 1386
*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4)*d^2 - 2*(15*a*b^8 - 182*a
^2*b^6*c + 735*a^3*b^4*c^2 - 1050*a^4*b^2*c^3 + 280*a^5*c^4)*d*e + (9*a^2*b
^7 - 105*a^3*b^5*c + 385*a^4*b^3*c^2 - 420*a^5*b*c^3)*e^2 + (a^4*b^5 - 15*a
^5*b^3*c + 60*a^6*b*c^2)*f^2 + 2*((5*a^2*b^7 - 69*a^3*b^5*c + 285*a^4*b^3*c
^2 - 340*a^5*b*c^3)*d - (3*a^3*b^6 - 40*a^4*b^4*c + 150*a^5*b^2*c^2 - 120*a
^6*c^3)*e)*f + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt
(((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686
*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)*d^4 - 4*(375*a*b^11 - 4775
*a^2*b^9*c + 21195*a^3*b^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8660 vs. $2(523) = 1046$.
time = 8.75, size = 8660, normalized size = 15.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 + a^2*b*c*f*x^3 - a*b^2*c*x^3*e + 2*a^2*
c^2*x^3*e + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x + a^2*b^2*f*x - 2*a^3*c
```

*f*x - a*b^3*x*e + 3*a^2*b*c*x*e)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a))
 + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*
(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 -
4*a^4*c)^2*f - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*
c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 22
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4
*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*d*abs
(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6 -
14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 28*a^6*b^
4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b^2 - 4*a*
c)*a^6*b^2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) - 2*(3*

```
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^6*c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*
c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3
*c^3 - 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c
^4 + 416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c
^2 + 104*(b^2 - 4*a*c)*a^6*b*c^3)*abs(a^3*b^2 - 4*a^4*c)*e + (10*a^6*b^9*c^
2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 -
 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*...
```

Mupad [B]
time = 7.37, size = 2500, normalized size = 4.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 +
 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 -
488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d
^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*
e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^
7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f
^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^
2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f -
480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e +
 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*
d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d
*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^
8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e
*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 +
9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^
```

2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d
*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2
 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c
^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*
e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f
^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*
f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^1
2*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 2
46*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^
6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f +
 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b
^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e
*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f -
 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 9
17504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 +
 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*
(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^
3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^
6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 3024
0*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^
(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 150
4*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d
*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*
(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*
a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e +
 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e -
161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4
*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^
3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*
c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7
*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2
*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^1

```
0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b
^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^
3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 157286
4*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b
^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b
^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^...
```

3.74 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}+\frac{414+415x^2}{2(2+3x^2+x^4)}+2\log(1+x^2)+392\log(2+x^2)$$

```
[Out] -293/2*x^2+49/2*x^4-9/2*x^6+5/8*x^8+1/2*(415*x^2+414)/(x^4+3*x^2+2)+2*ln(x^
2+1)+392*ln(x^2+2)
```

Rubi [A]

time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1677, 1674, 1671, 646, 31}

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\log(x^2+1)+392\log(x^2+2)+\frac{415x^2+414}{2(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^2)/(2*(2 +
 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right)$$

$$= \frac{414+415x^2}{2(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{-206-105x+53x^2-27x^3+12x^4-5x^5}{2+3x+x^2}\,d$$

$$= \frac{414+415x^2}{2(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \left(293-98x+27x^2-5x^3-\frac{4(198+197x)}{2+3x+x^2}\right.\right.$$

$$= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2\text{Subst}\left(\int \frac{198+197x}{2+3x+x^2}\right.$$

$$= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x,\right.$$

$$= -\frac{293x^2}{2} + \frac{49x^4}{2} - \frac{9x^6}{2} + \frac{5x^8}{8} + \frac{414+415x^2}{2(2+3x^2+x^4)} + 2\log(1+x^2) + 392\log$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.91

$$\frac{1}{8}\left(-1172x^2+196x^4-36x^6+5x^8+\frac{4(414+415x^2)}{2+3x^2+x^4}+16\log(1+x^2)+3136\log(2+x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 + 3*x^2 + x^
4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

Maple [A]
time = 0.04, size = 56, normalized size = 0.82

method	result	size
default	$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}-\frac{1}{2(x^2+1)}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)+\frac{208}{x^2+2}$	56
norman	$\frac{1086x^2-82x^6+\frac{49}{4}x^8-\frac{21}{8}x^{10}+\frac{5}{8}x^{12}+988}{x^4+3x^2+2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)$	58
risch	$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{\frac{415x^2}{2}+207}{x^4+3x^2+2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2-1/2/(x^2+1)+2*ln(x^2+1)+392*ln(x^2+2)+20
8/(x^2+2)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.85

$$\frac{5}{8}\,x^8-\frac{9}{2}\,x^6+\frac{49}{2}\,x^4-\frac{293}{2}\,x^2+\frac{415\,x^2+414}{2\,\left(x^4+3\,x^2+2\right)}+392\,\log\left(x^2+2\right)+2\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^2
 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Fricas [A]
time = 0.37, size = 82, normalized size = 1.21

$$\frac{5\,x^{12}-21\,x^{10}+98\,x^8-656\,x^6-3124\,x^4-684\,x^2+3136\,\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)+16\,\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)+1656}{8\,\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4 +
 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4 +
 3*x^2 + 2)
```

Sympy [A]
time = 0.06, size = 61, normalized size = 0.90

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{415x^2+414}{2x^4+6x^2+4}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 + 6
*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

Giac [A]
time = 4.83, size = 63, normalized size = 0.93

$$\frac{5}{8}\,x^{8}-\frac{9}{2}\,x^{6}+\frac{49}{2}\,x^{4}-\frac{293}{2}\,x^{2}-\frac{394\,x^{4}+767\,x^{2}+374}{2\,(x^{4}+3\,x^{2}+2)}+392\,\log\left(x^{2}+2\right)+2\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(x
^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Mupad [B]
time = 0.06, size = 57, normalized size = 0.84

$$2\,\ln\left(x^{2}+1\right)+392\,\ln\left(x^{2}+2\right)+\frac{\frac{415\,x^{2}}{2}+207}{x^{4}+3\,x^{2}+2}-\frac{293\,x^{2}}{2}+\frac{49\,x^{4}}{2}-\frac{9\,x^{6}}{2}+\frac{5\,x^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 2*log(x^2 + 1) + 392*log(x^2 + 2) + ((415*x^2)/2 + 207)/(3*x^2 + x^4 + 2) -
 (293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8
```

3.75 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=61

$$49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2(2+3x^2+x^4)}-\frac{5}{2}\log(1+x^2)-144\log(2+x^2)$$

```
[Out] 49*x^2-27/4*x^4+5/6*x^6+1/2*(-207*x^2-206)/(x^4+3*x^2+2)-5/2*ln(x^2+1)-144*
ln(x^2+2)
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1677, 1674, 1671, 646, 31}

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5}{2}\log(x^2+1)-144\log(x^2+2)-\frac{207x^2+206}{2(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (
5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx,x,x^2\right)\\
&= -\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{102+53x-27x^2+12x^3-5x^4}{2+3x+x^2}\,dx,x,x^2\right)\\
&= -\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-98+27x-5x^2+\frac{298+293x}{2+3x+x^2}\right)\,dx,x,\right.\\
&= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{298+293x}{2+3x+x^2}\,dx,x,x^2\right)\\
&= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-14\\
&= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{5}{2}\log\left(1+x^2\right)-144\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 1.00

$$49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}+\frac{-206-207x^2}{2\left(2+3x^2+x^4\right)}-\frac{5}{2}\log\left(1+x^2\right)-144\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) -
(5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Maple [A]
time = 0.03, size = 51, normalized size = 0.84

method	result	size
default	$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{1}{2x^2+2}-\frac{5\ln(x^2+1)}{2}-144\ln\left(x^2+2\right)-\frac{104}{x^2+2}$	51
norman	$\frac{-406x^2+\frac{365}{12}x^6-\frac{17}{4}x^8+\frac{5}{6}x^{10}-370}{x^4+3x^2+2}-\frac{5\ln(x^2+1)}{2}-144\ln\left(x^2+2\right)$	53
risch	$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-\frac{207x^2}{2}-103}{x^4+3x^2+2}-\frac{5\ln(x^2+1)}{2}-144\ln\left(x^2+2\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/6*x^6-27/4*x^4+49*x^2+1/2/(x^2+1)-5/2*ln(x^2+1)-144*ln(x^2+2)-104/(x^2+2)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.87

$$\frac{5}{6}\,x^6-\frac{27}{4}\,x^4+49\,x^2-\frac{207\,x^2+206}{2\,(x^4+3\,x^2+2)}-144\,\log\left(x^2+2\right)-\frac{5}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 + 2) - 144*l
og(x^2 + 2) - 5/2*log(x^2 + 1)
```

Fricas [A]
time = 0.37, size = 77, normalized size = 1.26

$$\frac{10\,x^{10}-51\,x^8+365\,x^6+1602\,x^4-66\,x^2-1728\,(x^4+3\,x^2+2)\log\left(x^2+2\right)-30\,(x^4+3\,x^2+2)\log\left(x^2+1\right)-1236}{12\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 +
2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2 +
2)
```

Sympy [A]
time = 0.06, size = 56, normalized size = 0.92

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-207x^2-206}{2x^4+6x^2+4}-\frac{5\log\left(x^2+1\right)}{2}-144\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**6/6 - 27*x**4/4 + 49*x**2 + (-207*x**2 - 206)/(2*x**4 + 6*x**2 + 4) -
5*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

Giac [A]
time = 5.20, size = 58, normalized size = 0.95

$$\frac{5}{6}\,x^{6} - \frac{27}{4}\,x^{4} + 49\,x^{2} + \frac{293\,x^{4} + 465\,x^{2} + 174}{4\,(x^{4} + 3\,x^{2} + 2)} - 144\,\log\left(x^{2} + 2\right) - \frac{5}{2}\,\log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4 + 3*x^2 +
2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 53, normalized size = 0.87

$$49\,x^2-144\,\ln\left(x^2+2\right)-\frac{\frac{207\,x^2}{2}+103}{x^4+3\,x^2+2}-\frac{5\,\ln\left(x^2+1\right)}{2}-\frac{27\,x^4}{4}+\frac{5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 49*x^2 - 144*log(x^2 + 2) - ((207*x^2)/2 + 103)/(3*x^2 + x^4 + 2) - (5*log(
x^2 + 1))/2 - (27*x^4)/4 + (5*x^6)/6
```

3.76 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2(2+3x^2+x^4)}+3\log(1+x^2)+46\log(2+x^2)$$

```
[Out] -27/2*x^2+5/4*x^4+1/2*(103*x^2+102)/(x^4+3*x^2+2)+3*ln(x^2+1)+46*ln(x^2+2)
```

Rubi [A]
time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1677, 1674, 1671, 646, 31}

$$\frac{5x^4}{4}-\frac{27x^2}{2}+3\log(x^2+1)+46\log(x^2+2)+\frac{103x^2+102}{2(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right)\\ &= \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-50-27x+12x^2-5x^3}{2+3x+x^2}\,dx, x, x^2\right)\\ &= \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(27-5x-\frac{2(52+49x)}{2+3x+x^2}\right)\,dx, x, x^2\right)\\ &= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+\text{Subst}\left(\int \frac{52+49x}{2+3x+x^2}\,dx, x, x^2\right)\\ &= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+3\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)+46\text{Subst}\left(\right.\\ &= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+3\log\left(1+x^2\right)+46\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.00

$$-\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+3\log\left(1+x^2\right)+46\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Maple [A]
time = 0.04, size = 46, normalized size = 0.85

method	result	size
default	$\frac{5x^4}{4} - \frac{27x^2}{2} - \frac{1}{2(x^2+1)} + 3\ln(x^2+1) + 46\ln(x^2+2) + \frac{52}{x^2+2}$	46
norman	$\frac{\frac{277}{2}x^2 - \frac{39}{4}x^6 + \frac{5}{4}x^8 + 127}{x^4+3x^2+2} + 3\ln(x^2+1) + 46\ln(x^2+2)$	48
risch	$\frac{5x^4}{4} - \frac{27x^2}{2} + \frac{729}{20} + \frac{\frac{103x^2}{2}+51}{x^4+3x^2+2} + 3\ln(x^2+1) + 46\ln(x^2+2)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x^4-27/2*x^2-1/2/(x^2+1)+3*ln(x^2+1)+46*ln(x^2+2)+52/(x^2+2)
```

Maxima [A]
time = 0.26, size = 48, normalized size = 0.89

$$\frac{5}{4}x^4 - \frac{27}{2}x^2 + \frac{103\,x^2 + 102}{2\,(x^4 + 3\,x^2 + 2)} + 46\,\log\left(x^2 + 2\right) + 3\,\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 + 2
) + 3*log(x^2 + 1)
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.33

$$\frac{5\,x^8 - 39\,x^6 - 152\,x^4 + 98\,x^2 + 184\,(x^4 + 3\,x^2 + 2)\log\left(x^2 + 2\right) + 12\,(x^4 + 3\,x^2 + 2)\log\left(x^2 + 1\right) + 204}{4\,(x^4 + 3\,x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2)
 + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.06, size = 48, normalized size = 0.89

$$\frac{5x^4}{4} - \frac{27x^2}{2} + \frac{103x^2 + 102}{2x^4 + 6x^2 + 4} + 3\log\left(x^2 + 1\right) + 46\log\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2
+ 1) + 46*log(x**2 + 2)
```

Giac [A]
time = 3.83, size = 53, normalized size = 0.98

$$\frac{5}{4}\,x^4-\frac{27}{2}\,x^2-\frac{49\,x^4+44\,x^2-4}{2\,(x^4+3\,x^2+2)}+46\,\log\left(x^2+2\right)+3\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(x
^2 + 2) + 3*log(x^2 + 1)
```

Mupad [B]
time = 0.90, size = 47, normalized size = 0.87

$$3\,\ln\left(x^2+1\right)+46\,\ln\left(x^2+2\right)+\frac{\frac{103\,x^2}{2}+51}{x^4+3\,x^2+2}-\frac{27\,x^2}{2}+\frac{5\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 3*log(x^2 + 1) + 46*log(x^2 + 2) + ((103*x^2)/2 + 51)/(3*x^2 + x^4 + 2) - (
27*x^2)/2 + (5*x^4)/4
```

3.77 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=49

$$\frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{7}{2}\log\left(1+x^2\right)-10\log\left(2+x^2\right)$$

```
[Out] 5/2*x^2+1/2*(-51*x^2-50)/(x^4+3*x^2+2)-7/2*ln(x^2+1)-10*ln(x^2+2)
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1677, 1674, 1671, 646, 31}

$$\frac{5x^2}{2}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*L
og[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(4+x+3x^2+5x^3)}{(2+3x+x^2)^2}\,dx, x, x^2\right)\\ &= -\frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{24+12x-5x^2}{2+3x+x^2}\,dx, x, x^2\right)\\ &= -\frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \left(-5+\frac{34+27x}{2+3x+x^2}\right)dx, x, x^2\right)\\ &= \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{1}{2}\text{Subst}\left(\int \frac{34+27x}{2+3x+x^2}\,dx, x, x^2\right)\\ &= \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{7}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - 10\text{Subst}\left(\int \frac{1}{2+x}\right.\\ &= \frac{5x^2}{2} - \frac{50+51x^2}{2(2+3x^2+x^4)} - \frac{7}{2}\log(1+x^2) - 10\log(2+x^2)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$\frac{5x^2}{2}+\frac{-50-51x^2}{2(2+3x^2+x^4)}-\frac{7}{2}\log(1+x^2)-10\log(2+x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*
Log[2 + x^2]
```

Maple [A]
time = 0.03, size = 41, normalized size = 0.84

method	result	size
default	$\frac{5x^2}{2}+\frac{1}{2x^2+2}-\frac{7\ln(x^2+1)}{2}-10\ln\left(x^2+2\right)-\frac{26}{x^2+2}$	41
norman	$\frac{-43x^2+\frac{5}{2}x^6-40}{x^4+3x^2+2}-\frac{7\ln(x^2+1)}{2}-10\ln\left(x^2+2\right)$	43
risch	$\frac{5x^2}{2}+\frac{-\frac{51x^2}{2}-25}{x^4+3x^2+2}-\frac{7\ln(x^2+1)}{2}-10\ln\left(x^2+2\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*x^2+1/2/(x^2+1)-7/2*ln(x^2+1)-10*ln(x^2+2)-26/(x^2+2)
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.88

$$\frac{5}{2}\,x^2-\frac{51\,x^2+50}{2\,(x^4+3\,x^2+2)}-10\,\log\left(x^2+2\right)-\frac{7}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) - 7/2*log(x
^2 + 1)
```

Fricas [A]
time = 0.37, size = 67, normalized size = 1.37

$$\frac{5\,x^6+15\,x^4-41\,x^2-20\,(x^4+3\,x^2+2)\log\left(x^2+2\right)-7\,(x^4+3\,x^2+2)\log\left(x^2+1\right)-50}{2\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4 +
 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.06, size = 44, normalized size = 0.90

$$\frac{5x^2}{2}+\frac{-51x^2-50}{2x^4+6x^2+4}-\frac{7\log\left(x^2+1\right)}{2}-10\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**2/2 + (-51*x**2 - 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)/2 - 10*l
og(x**2 + 2)
```

Giac [A]
time = 2.65, size = 45, normalized size = 0.92

$$\frac{5}{2}x^2 - \frac{51x^2 + 50}{2(x^2+2)(x^2+1)} - 10\log\left(x^2+2\right) - \frac{7}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*log(x^2 + 2) - 7/2*l
og(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.88

$$\frac{5x^2}{2} - 10\ln\left(x^2+2\right) - \frac{\frac{51x^2}{2} + 25}{x^4 + 3x^2 + 2} - \frac{7\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (5*x^2)/2 - 10*log(x^2 + 2) - ((51*x^2)/2 + 25)/(3*x^2 + x^4 + 2) - (7*log(
x^2 + 1))/2
```

3.78 $\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=42

$$\frac{24+25x^2}{2\left(2+3x^2+x^4\right)}+4\log\left(1+x^2\right)-\frac{3}{2}\log\left(2+x^2\right)$$

[Out] 1/2*(25*x^2+24)/(x^4+3*x^2+2)+4*ln(x^2+1)-3/2*ln(x^2+2)

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1677, 1674, 646, 31}

$$4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)+\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]

[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{(2+3x+x^2)^2}\,dx, x, x^2\right)\\ &= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-13-5x}{2+3x+x^2}\,dx, x, x^2\right)\\ &= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right) + 4\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)\\ &= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)} + 4\log\left(1+x^2\right) - \frac{3}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{24+25x^2}{2\left(2+3x^2+x^4\right)} + 4\log\left(1+x^2\right) - \frac{3}{2}\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.86

method	result	size
default	$-\frac{1}{2(x^2+1)} + 4\ln\left(x^2+1\right) - \frac{3\ln\left(x^2+2\right)}{2} + \frac{13}{x^2+2}$	36
norman	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2} + 4\ln\left(x^2+1\right) - \frac{3\ln\left(x^2+2\right)}{2}$	38
risch	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2} + 4\ln\left(x^2+1\right) - \frac{3\ln\left(x^2+2\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2+1)+4*ln(x^2+1)-3/2*ln(x^2+2)+13/(x^2+2)
```

Maxima [A]
time = 0.30, size = 38, normalized size = 0.90

$$\frac{25\,x^2+24}{2\,(x^4+3\,x^2+2)}-\frac{3}{2}\,\log\left(x^2+2\right)+4\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] 1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.36

$$\frac{25\,x^2-3\,(x^4+3\,x^2+2)\log\left(x^2+2\right)+8\,(x^4+3\,x^2+2)\log\left(x^2+1\right)+24}{2\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x^
2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.06, size = 36, normalized size = 0.86

$$\frac{25x^2+24}{2x^4+6x^2+4}+4\log\left(x^2+1\right)-\frac{3\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
[Out] (25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2
```

Giac [A]
time = 3.01, size = 40, normalized size = 0.95

$$\frac{25\,x^2+24}{2\,(x^2+2)(x^2+1)}-\frac{3}{2}\,\log\left(x^2+2\right)+4\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.88

$$4\,\ln\left(x^2+1\right)-\frac{3\,\ln\left(x^2+2\right)}{2}+\frac{\frac{25\,x^2}{2}+12}{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
[Out] 4*log(x^2 + 1) - (3*log(x^2 + 2))/2 + ((25*x^2)/2 + 12)/(3*x^2 + x^4 + 2)
```

3.79 $\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)$$

```
[Out] 1/2*(-12*x^2-11)/(x^4+3*x^2+2)+ln(x)-9/2*ln(x^2+1)+4*ln(x^2+2)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1677, 1660, 814}

$$-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/2*(11 + 12*x^2)/(2 + 3*x^2 + x^4) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{-2+7x}{x\left(2+3x+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{9}{1+x}-\frac{8}{2+x}\right)\,dx,x,x^2\right)\\
&= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$\frac{-11-12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
[Out] (-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Maple [A]
time = 0.03, size = 38, normalized size = 0.86

method	result	size
default	$\frac{1}{2x^2+2}-\frac{9\ln\left(x^2+1\right)}{2}+\ln\left(x\right)+4\ln\left(x^2+2\right)-\frac{13}{2\left(x^2+2\right)}$	38
norman	$\frac{-6x^2-\frac{11}{2}}{x^4+3x^2+2}-\frac{9\ln\left(x^2+1\right)}{2}+4\ln\left(x^2+2\right)+\ln\left(x\right)$	40
risch	$\frac{-6x^2-\frac{11}{2}}{x^4+3x^2+2}-\frac{9\ln\left(x^2+1\right)}{2}+4\ln\left(x^2+2\right)+\ln\left(x\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/(x^2+1)-9/2*ln(x^2+1)+ln(x)+4*ln(x^2+2)-13/2/(x^2+2)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 1.00

$$-\frac{12\,x^2+11}{2\,(x^4+3\,x^2+2)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) +
1/2*log(x^2)
```

Fricas [A]
time = 0.36, size = 71, normalized size = 1.61

$$-\frac{12\,x^2-8\,(x^4+3\,x^2+2)\log\left(x^2+2\right)+9\,(x^4+3\,x^2+2)\log\left(x^2+1\right)-2\,(x^4+3\,x^2+2)\log\left(x\right)+11}{2\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(x
^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 0.93

$$\frac{-12x^2-11}{2x^4+6x^2+4}+\log\left(x\right)-\frac{9\log\left(x^2+1\right)}{2}+4\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

```
[Out] (-12*x**2 - 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log(
x**2 + 2)
```

Giac [A]
time = 4.17, size = 47, normalized size = 1.07

$$\frac{x^4-21\,x^2-20}{4\,(x^4+3\,x^2+2)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 +
1) + 1/2*log(x^2)
```

Mupad [B]
time = 0.04, size = 40, normalized size = 0.91

$$4\,\ln\left(x^2+2\right)-\frac{9\,\ln\left(x^2+1\right)}{2}+\ln\left(x\right)-\frac{6\,x^2+\frac{11}{2}}{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(3*x^2 + x^4 + 2)^2),x)
[Out] 4*log(x^2 + 2) - (9*log(x^2 + 1))/2 + log(x) - (6*x^2 + 11/2)/(3*x^2 + x^4
+ 2)
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} \, dx$

Optimal. Leaf size=55

$$-\frac{1}{2x^2}+\frac{9+11x^2}{4(2+3x^2+x^4)}-\frac{11\log(x)}{4}+5\log(1+x^2)-\frac{29}{8}\log(2+x^2)$$

```
[Out] -1/2/x^2+1/4*(11*x^2+9)/(x^4+3*x^2+2)-11/4*ln(x)+5*ln(x^2+1)-29/8*ln(x^2+2)
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1677, 1660, 1642}

$$-\frac{1}{2x^2}+5\log(x^2+1)-\frac{29}{8}\log(x^2+2)+\frac{11x^2+9}{4(x^4+3x^2+2)}-\frac{11\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/2*1/x^2 + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 + 5*Log[1 +
 x^2] - (29*Log[2 + x^2])/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{11x^2}{2}}{x^2\left(2+3x+x^2\right)}\,dx,x,x^2\right)\\
&= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{11}{4x}-\frac{10}{1+x}+\frac{29}{4(2+x)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{9+11x^2}{4\left(2+3x^2+x^4\right)}-\frac{11\log(x)}{4}+5\log\left(1+x^2\right)-\frac{29}{8}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.91

$$\frac{1}{8}\left(-\frac{4}{x^2}+\frac{18+22x^2}{2+3x^2+x^4}-22\log(x)+40\log\left(1+x^2\right)-29\log\left(2+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] - 2
9*Log[2 + x^2])/8
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.82

method	result	size
default	$-\frac{1}{2(x^2+1)}+5\ln\left(x^2+1\right)-\frac{1}{2x^2}-\frac{11\ln(x)}{4}-\frac{29\ln(x^2+2)}{8}+\frac{13}{4(x^2+2)}$	45
norman	$\frac{-1+\frac{3}{4}x^2+\frac{9}{4}x^4}{x^2(x^4+3x^2+2)}-\frac{11\ln(x)}{4}+5\ln\left(x^2+1\right)-\frac{29\ln(x^2+2)}{8}$	50
risch	$\frac{-1+\frac{3}{4}x^2+\frac{9}{4}x^4}{x^2(x^4+3x^2+2)}-\frac{11\ln(x)}{4}+5\ln\left(x^2+1\right)-\frac{29\ln(x^2+2)}{8}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2+1)+5*ln(x^2+1)-1/2/x^2-11/4*ln(x)-29/8*ln(x^2+2)+13/4/(x^2+2)
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.96

$$\frac{9\,x^4+3\,x^2-4}{4\,(x^6+3\,x^4+2\,x^2)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

Fricas [A]
time = 0.37, size = 92, normalized size = 1.67

$$\frac{18\,x^4+6\,x^2-29\,(x^6+3\,x^4+2\,x^2)\log\left(x^2+2\right)+40\,(x^6+3\,x^4+2\,x^2)\log\left(x^2+1\right)-22\,(x^6+3\,x^4+2\,x^2)\log\left(x\right)-8}{8\,(x^6+3\,x^4+2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*x
^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*x^
4 + 2*x^2)
```

Sympy [A]
time = 0.08, size = 51, normalized size = 0.93

$$\frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2}-\frac{11\log\left(x\right)}{4}+5\log\left(x^2+1\right)-\frac{29\log\left(x^2+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

```
[Out] (9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 + 5*log(x**
2 + 1) - 29*log(x**2 + 2)/8
```

Giac [A]
time = 5.43, size = 53, normalized size = 0.96

$$\frac{9\,x^4+3\,x^2-4}{4\,(x^6+3\,x^4+2\,x^2)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x^2 + 1) - 11/8*log(x^2)

Mupad [B]
time = 0.04, size = 50, normalized size = 0.91

$$5\ln\left(x^2+1\right)-\frac{29\ln\left(x^2+2\right)}{8}-\frac{11\ln\left(x\right)}{4}+\frac{\frac{9x^4}{4}+\frac{3x^2}{4}-1}{x^6+3x^4+2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(3*x^2 + x^4 + 2)^2),x)

[Out] 5*log(x^2 + 1) - (29*log(x^2 + 2))/8 - (11*log(x))/4 + ((3*x^2)/4 + (9*x^4)/4 - 1)/(2*x^2 + 3*x^4 + x^6)

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=64

$$-\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4}-\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)$$

```
[Out] -1/4/x^4+11/8/x^2+1/8*(-9*x^2-5)/(x^4+3*x^2+2)+23/4*ln(x)-11/2*ln(x^2+1)+21
/8*ln(x^2+2)
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1677, 1660, 1642}

$$-\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{23\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/4*1/x^4 + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23*Log[x])/4
 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{17x^2}{4}+\frac{9x^3}{4}}{x^3\left(2+3x+x^2\right)}\,dx, x, x^2\right)\\
&= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^3}+\frac{11}{4x^2}-\frac{23}{4x}+\frac{11}{1+x}-\frac{21}{4(2+x)}\right)dx, x\right.\\
&= -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4}-\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right.
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.88

$$\frac{1}{8}\left(-\frac{2}{x^4}+\frac{11}{x^2}-\frac{5+9x^2}{2+3x^2+x^4}+46\log(x)-44\log\left(1+x^2\right)+21\log\left(2+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 + x
^2] + 21*Log[2 + x^2])/8
```

Maple [A]
time = 0.03, size = 50, normalized size = 0.78

method	result	size
default	$\frac{1}{2x^2+2}-\frac{11\ln(x^2+1)}{2}-\frac{1}{4x^4}+\frac{11}{8x^2}+\frac{23\ln(x)}{4}+\frac{21\ln(x^2+2)}{8}-\frac{13}{8(x^2+2)}$	50
norman	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{21\ln(x^2+2)}{8}$	55
risch	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23\ln(x)}{4}-\frac{11\ln(x^2+1)}{2}+\frac{21\ln(x^2+2)}{8}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(x^2+1)-11/2*ln(x^2+1)-1/4/x^4+11/8/x^2+23/4*ln(x)+21/8*ln(x^2+2)-13/8/
(x^2+2)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.88

$$\frac{x^6+13\,x^4+8\,x^2-2}{4\,(x^8+3\,x^6+2\,x^4)}+\frac{21}{8}\,\log\left(x^2+2\right)-\frac{11}{2}\,\log\left(x^2+1\right)+\frac{23}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x^2 + 2) -
11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Fricas [A]
time = 0.37, size = 97, normalized size = 1.52

$$\frac{2\,x^6+26\,x^4+16\,x^2+21\,(x^8+3\,x^6+2\,x^4)\log\left(x^2+2\right)-44\,(x^8+3\,x^6+2\,x^4)\log\left(x^2+1\right)+46\,(x^8+3\,x^6+2\,x^4)\log\left(x\right)-4}{8\,(x^8+3\,x^6+2\,x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*(
x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/(x
^8 + 3*x^6 + 2*x^4)
```

Sympy [A]
time = 0.08, size = 56, normalized size = 0.88

$$\frac{23\log\left(x\right)}{4}-\frac{11\log\left(x^2+1\right)}{2}+\frac{21\log\left(x^2+2\right)}{8}+\frac{x^6+13x^4+8x^2-2}{4x^8+12x^6+8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

```
[Out] 23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 13*x**4 + 8
*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

Giac [A]
time = 6.02, size = 66, normalized size = 1.03

$$\frac{23\,x^4+51\,x^2+36}{16\,(x^4+3\,x^2+2)}-\frac{69\,x^4-22\,x^2+4}{16\,x^4}+\frac{21}{8}\,\log\left(x^2+2\right)-\frac{11}{2}\,\log\left(x^2+1\right)+\frac{23}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)/
x^4 + 21/8*log(x^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Mupad [B]
time = 0.92, size = 55, normalized size = 0.86

$$\frac{21\ln\left(x^2+2\right)}{8}-\frac{11\ln\left(x^2+1\right)}{2}+\frac{23\ln\left(x\right)}{4}+\frac{\frac{x^6}{4}+\frac{13\,x^4}{4}+2\,x^2-\frac{1}{2}}{x^8+3\,x^6+2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (21*log(x^2 + 2))/8 - (11*log(x^2 + 1))/2 + (23*log(x))/4 + (2*x^2 + (13*x^
4)/4 + x^6/4 - 1/2)/(2*x^4 + 3*x^6 + x^8)
```

3.82 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=70

$$-293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2\,(2+3x^2+x^4)}+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -293*x+98/3*x^3-27/5*x^5+5/7*x^7-1/2*x*(207*x^2+206)/(x^4+3*x^2+2)+9/2*arct
an(x)+340*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$\frac{9\text{ArcTan}(x)}{2}+340\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)+\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)\,x}{2\,(x^4+3x^2+2)}-293x$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2))/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= -\frac{x(206+207x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{4}\int \frac{-412-6x^2+212x^4-108x^6+48x^8-20x^{10}}{2+3x^2+x^4}\,dx \\
&= -\frac{x(206+207x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{4}\int \Bigg(1172-392x^2+108x^4-20x^6-\frac{2(1378+1369x}{2+3x^2+x^4} \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2\,(2+3x^2+x^4)}+\frac{1}{2}\int \frac{1378+1369x^2}{2+3x^2+x^4}\,dx \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2\,(2+3x^2+x^4)}+\frac{9}{2}\int \frac{1}{1+x^2}\,dx+680\int \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2\,(2+3x^2+x^4)}+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\,\text{ta}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.01

$$-293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}+\frac{-206x-207x^3}{2\,(2+3x^2+x^4)}+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 + 3
*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A]
time = 0.03, size = 56, normalized size = 0.80

method	result	size

default	$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{x}{2x^2+2}+\frac{9\arctan(x)}{2}-\frac{104x}{x^2+2}+340\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	56
risch	$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{-\frac{207}{2}x^3-103x}{x^4+3x^2+2}+\frac{9\arctan(x)}{2}+340\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/7*x^7-27/5*x^5+98/3*x^3-293*x+1/2*x/(x^2+1)+9/2*arctan(x)-104*x/(x^2+2)+3
40*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 58, normalized size = 0.83

$$\frac{5}{7}\,x^7-\frac{27}{5}\,x^5+\frac{98}{3}\,x^3+340\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-293\,x-\frac{207\,x^3+206\,x}{2\,(x^4+3\,x^2+2)}+\frac{9}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Fricas [A]
time = 0.37, size = 79, normalized size = 1.13

$$\frac{150\,x^{11}-684\,x^9+3758\,x^7-43218\,x^5-192605\,x^3+71400\,\sqrt{2}\,(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+945\,(x^4+3\,x^2+2)\arctan\left(x\right)-144690\,x}{210\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt(
2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan(x
) - 144690*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.08, size = 68, normalized size = 0.97

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{-207x^3-206x}{2x^4+6x^2+4}+\frac{9\,\mathrm{atan}\,(x)}{2}+340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x + (-207*x**3 - 206*x)/(2*x**4 + 6*
x**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A]
time = 5.88, size = 58, normalized size = 0.83

$$\frac{5}{7}\,x^{7}-\frac{27}{5}\,x^{5}+\frac{98}{3}\,x^{3}+340\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-293\,x-\frac{207\,x^{3}+206\,x}{2\,(x^{4}+3\,x^{2}+2)}+\frac{9}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Mupad [B]
time = 0.95, size = 58, normalized size = 0.83

$$\frac{9\,\mathrm{atan}(x)}{2}-293\,x+340\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)-\frac{\frac{207\,x^{3}}{2}+103\,x}{x^{4}+3\,x^{2}+2}+\frac{98\,x^{3}}{3}-\frac{27\,x^{5}}{5}+\frac{5\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (9*atan(x))/2 - 293*x + 340*2^(1/2)*atan((2^(1/2)*x)/2) - (103*x + (207*x^3
)/2)/(3*x^2 + x^4 + 2) + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7
```

3.83 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=57

$$98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)}-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 98*x-9*x^3+x^5+1/2*x*(103*x^2+102)/(x^4+3*x^2+2)-11/2*arctan(x)-118*arctan(
1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$-\frac{11\text{ArcTan}(x)}{2}-118\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)+x^5-9x^3+\frac{(103x^2+102)\,x}{2(x^4+3x^2+2)}+98x$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan
[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{x(102+103x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{4}\int \frac{204+6x^2-108x^4+48x^6-20x^8}{2+3x^2+x^4}\,dx \\
&= \frac{x(102+103x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{4}\int \left(-392+108x^2-20x^4+\frac{2(494+483x^2)}{2+3x^2+x^4}\right)\,dx \\
&= 98x-9x^3+x^5+\frac{x(102+103x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{2}\int \frac{494+483x^2}{2+3x^2+x^4}\,dx \\
&= 98x-9x^3+x^5+\frac{x(102+103x^2)}{2\,(2+3x^2+x^4)}-\frac{11}{2}\int \frac{1}{1+x^2}\,dx-236\int \frac{1}{2+x^2}\,dx \\
&= 98x-9x^3+x^5+\frac{x(102+103x^2)}{2\,(2+3x^2+x^4)}-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 1.02

$$98x-9x^3+x^5+\frac{102x+103x^3}{2\,(2+3x^2+x^4)}-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[x
])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A]
time = 0.04, size = 49, normalized size = 0.86

method	result	size

default	$x^5-9x^3+98x-\frac{x}{2(x^2+1)}-\frac{11\arctan(x)}{2}+\frac{52x}{x^2+2}-118\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	49
risch	$x^5-9x^3+98x+\frac{\frac{103}{2}x^3+51x}{x^4+3x^2+2}-\frac{11\arctan(x)}{2}-118\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-9*x^3+98*x-1/2*x/(x^2+1)-11/2*arctan(x)+52*x/(x^2+2)-118*arctan(1/2*2^(
1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.48, size = 51, normalized size = 0.89

$$x^5-9\,x^3-118\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+98\,x+\frac{103\,x^3+102\,x}{2\,(x^4+3\,x^2+2)}-\frac{11}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Fricas [A]
time = 0.37, size = 74, normalized size = 1.30

$$\frac{2\,x^9-12\,x^7+146\,x^5+655\,x^3-236\,\sqrt{2}\,(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-11\,(x^4+3\,x^2+2)\arctan\left(x\right)+494\,x}{2\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*arc
tan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2 +
 2)
```

Sympy [A]
time = 0.08, size = 54, normalized size = 0.95

$$x^5-9x^3+98x+\frac{103x^3+102x}{2x^4+6x^2+4}-\frac{11\,\mathrm{atan}\,(x)}{2}-118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*atan(x
)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A]
time = 5.19, size = 51, normalized size = 0.89

$$x^5 - 9\,x^3 - 118\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) + 98\,x + \frac{103\,x^3 + 102\,x}{2\,(x^4 + 3\,x^2 + 2)} - \frac{11}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Mupad [B]
time = 0.05, size = 50, normalized size = 0.88

$$98\,x - \frac{11\,\mathrm{atan}(x)}{2} - 118\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{103\,x^3}{2} + 51\,x}{x^4 + 3\,x^2 + 2} - 9\,x^3 + x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 98*x - (11*atan(x))/2 - 118*2^(1/2)*atan((2^(1/2)*x)/2) + (51*x + (103*x^3)
/2)/(3*x^2 + x^4 + 2) - 9*x^3 + x^5
```

3.84 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=56

$$-27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2(2+3x^2+x^4)}+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -27*x+5/3*x^3-1/2*x*(51*x^2+50)/(x^4+3*x^2+2)+13/2*arctan(x)+33*arctan(1/2*
x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1682, 1690, 1180, 209}

$$\frac{13\text{ArcTan}(x)}{2}+33\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)+\frac{5x^3}{3}-\frac{(51x^2+50)\,x}{2(x^4+3x^2+2)}-27x$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= -\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{4}\int \frac{-100-6x^2+48x^4-20x^6}{2+3x^2+x^4}\,dx \\
&= -\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}-\frac{1}{4}\int \left(108-20x^2-\frac{2(158+145x^2)}{2+3x^2+x^4}\right)dx \\
&= -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}+\frac{1}{2}\int \frac{158+145x^2}{2+3x^2+x^4}\,dx \\
&= -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}+\frac{13}{2}\int \frac{1}{1+x^2}\,dx+66\int \frac{1}{2+x^2}\,dx \\
&= -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2\,(2+3x^2+x^4)}+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.02

$$-27x+\frac{5x^3}{3}+\frac{-50x-51x^3}{2\,(2+3x^2+x^4)}+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x])
/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A]
time = 0.03, size = 46, normalized size = 0.82

method	result	size

default	$\frac{5x^3}{3}-27x+\frac{x}{2x^2+2}+\frac{13\arctan(x)}{2}-\frac{26x}{x^2+2}+33\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	46
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{51}{2}x^3-25x}{x^4+3x^2+2}+33\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}+\frac{13\arctan(x)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/3*x^3-27*x+1/2*x/(x^2+1)+13/2*arctan(x)-26*x/(x^2+2)+33*arctan(1/2*2^(1/2
)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 48, normalized size = 0.86

$$\frac{5}{3}x^3+33\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-27x-\frac{51x^3+50x}{2(x^4+3x^2+2)}+\frac{13}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Fricas [A]
time = 0.35, size = 69, normalized size = 1.23

$$\frac{10x^7-132x^5-619x^3+198\sqrt{2}(x^4+3x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+39(x^4+3x^2+2)\arctan(x)-474x}{6(x^4+3x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*
sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.08, size = 54, normalized size = 0.96

$$\frac{5x^3}{3}-27x+\frac{-51x^3-50x}{2x^4+6x^2+4}+\frac{13\operatorname{atan}(x)}{2}+33\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**3/3 - 27*x + (-51*x**3 - 50*x)/(2*x**4 + 6*x**2 + 4) + 13*atan(x)/2 +
33*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A]
time = 5.47, size = 48, normalized size = 0.86

$$\frac{5}{3}\,x^3 + 33\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - 27\,x - \frac{51\,x^3 + 50\,x}{2\,(x^4 + 3\,x^2 + 2)} + \frac{13}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Mupad [B]
time = 0.92, size = 48, normalized size = 0.86

$$\frac{13\,\mathrm{atan}(x)}{2} - 27\,x + 33\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{51\,x^3}{2} + 25\,x}{x^4 + 3\,x^2 + 2} + \frac{5\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (13*atan(x))/2 - 27*x + 33*2^(1/2)*atan((2^(1/2)*x)/2) - (25*x + (51*x^3)/2
)/(3*x^2 + x^4 + 2) + (5*x^3)/3
```

3.85 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=49

$$5x+\frac{x(24+25x^2)}{2(2+3x^2+x^4)}-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 5*x+1/2*x*(25*x^2+24)/(x^4+3*x^2+2)-15/2*arctan(x)-7/2*arctan(1/2*x*2^(1/2)
)*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1682, 1690, 1180, 209}

$$-\frac{15\text{ArcTan}(x)}{2}-\frac{7\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{(25x^2+24)\,x}{2(x^4+3x^2+2)}+5x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTa
n[x/Sqrt[2]])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
```

```
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx &= \frac{x(24+25x^2)}{2(2+3x^2+x^4)}-\frac{1}{4}\int \frac{48-2x^2-20x^4}{2+3x^2+x^4}\,dx\\
&= \frac{x(24+25x^2)}{2(2+3x^2+x^4)}-\frac{1}{4}\int \left(-20+\frac{2(44+29x^2)}{2+3x^2+x^4}\right)\,dx\\
&= 5x+\frac{x(24+25x^2)}{2(2+3x^2+x^4)}-\frac{1}{2}\int \frac{44+29x^2}{2+3x^2+x^4}\,dx\\
&= 5x+\frac{x(24+25x^2)}{2(2+3x^2+x^4)}-7\int \frac{1}{2+x^2}\,dx-\frac{15}{2}\int \frac{1}{1+x^2}\,dx\\
&= 5x+\frac{x(24+25x^2)}{2(2+3x^2+x^4)}-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 1.02

$$5x+\frac{24x+25x^3}{2(2+3x^2+x^4)}-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan[
x/Sqrt[2]])/Sqrt[2]
```

Maple [A]
time = 0.03, size = 41, normalized size = 0.84

method	result	size
default	$5x-\frac{x}{2(x^{2}+1)}-\frac{15\arctan(x)}{2}+\frac{13x}{x^{2}+2}-\frac{7\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$	41
risch	$5x+\frac{\frac{25}{2}x^{3}+12x}{x^{4}+3x^{2}+2}-\frac{15\arctan(x)}{2}-\frac{7\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x-1/2*x/(x^2+1)-15/2*arctan(x)+13*x/(x^2+2)-7/2*arctan(1/2*2^(1/2)*x)*2^(
1/2)
```

Maxima [A]
time = 0.48, size = 43, normalized size = 0.88

$$-\frac{7}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+5\,x+\frac{25\,x^{3}+24\,x}{2\,(x^{4}+3\,x^{2}+2)}-\frac{15}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

Fricas [A]
time = 0.37, size = 64, normalized size = 1.31

$$\frac{10\,x^{5}+55\,x^{3}-7\,\sqrt{2}\,(x^{4}+3\,x^{2}+2)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-15\,(x^{4}+3\,x^{2}+2)\arctan\left(x\right)+44\,x}{2\,(x^{4}+3\,x^{2}+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) -
15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.08, size = 48, normalized size = 0.98

$$5x+\frac{25x^{3}+24x}{2x^{4}+6x^{2}+4}-\frac{15\,\mathrm{atan}\,(x)}{2}-\frac{7\sqrt{2}\,\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*ata
n(sqrt(2)*x/2)/2
```

Giac [A]
time = 4.62, size = 43, normalized size = 0.88

$$-\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5x+\frac{25x^3+24x}{2(x^4+3x^2+2)}-\frac{15}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

Mupad [B]
time = 0.07, size = 42, normalized size = 0.86

$$5x-\frac{15\,\mathrm{atan}(x)}{2}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\frac{25x^3}{2}+12x}{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 5*x - (15*atan(x))/2 - (7*2^(1/2)*atan((2^(1/2)*x)/2))/2 + (12*x + (25*x^3)
/2)/(3*x^2 + x^4 + 2)
```

3.86 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{x(11+12x^2)}{2\left(2+3x^2+x^4\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -1/2*x*(12*x^2+11)/(x^4+3*x^2+2)+17/2*arctan(x)-19/4*arctan(1/2*x*2^(1/2))*
2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1692, 1180, 209}

$$\frac{17\text{ArcTan}(x)}{2}-\frac{19\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{x(12x^2+11)}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -1/2*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + (17*ArcTan[x])/2 - (19*ArcTan[x/
Sqrt[2]])/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
```

```
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx &= -\frac{x(11+12x^2)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-30+4x^2}{2+3x^2+x^4}\,dx\\ &= -\frac{x(11+12x^2)}{2\left(2+3x^2+x^4\right)}+\frac{17}{2}\int \frac{1}{1+x^2}\,dx-\frac{19}{2}\int \frac{1}{2+x^2}\,dx\\ &= -\frac{x(11+12x^2)}{2\left(2+3x^2+x^4\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2x(11+12x^2)}{2+3x^2+x^4}+34\tan^{-1}(x)-19\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
[Out] ((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan[
x/Sqrt[2]])/4
```

Maple [A]
time = 0.03, size = 38, normalized size = 0.79

method	result	size
default	$\frac{x}{2x^2+2}+\frac{17\arctan(x)}{2}-\frac{13x}{2(x^2+2)}-\frac{19\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{4}$	38
risch	$\frac{-6x^3-\frac{11}{2}x}{x^4+3x^2+2}-\frac{19\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{4}+\frac{17\arctan(x)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(x^2+1)+17/2*arctan(x)-13/2*x/(x^2+2)-19/4*arctan(1/2*2^(1/2)*x)*2^(1
/2)
```

Maxima [A]
time = 0.49, size = 40, normalized size = 0.83

$$-\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)}+\frac{17}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

Fricas [A]
time = 0.37, size = 59, normalized size = 1.23

$$-\frac{24\,x^3+19\,\sqrt{2}\,(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-34\,(x^4+3\,x^2+2)\arctan(x)+22\,x}{4\,(x^4+3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^4
 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 0.96

$$\frac{-12x^3-11x}{2x^4+6x^2+4}+\frac{17\,\mathrm{atan}(x)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] (-12*x**3 - 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(sq
rt(2)*x/2)/4
```

Giac [A]
time = 3.70, size = 40, normalized size = 0.83

$$-\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)}+\frac{17}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

Mupad [B]
time = 0.07, size = 40, normalized size = 0.83

$$\frac{17\,\mathrm{atan}(x)}{2}-\frac{19\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4}-\frac{6\,x^3+\frac{11\,x}{2}}{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (17*atan(x))/2 - (19*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((11*x)/2 + 6*x^3)/(3
*x^2 + x^4 + 2)
```

3.87 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} \, dx$

Optimal. Leaf size=53

$$-\frac{1}{x}+\frac{x(9+11x^2)}{4(2+3x^2+x^4)}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/x+1/4*x*(11*x^2+9)/(x^4+3*x^2+2)-19/2*arctan(x)+45/8*arctan(1/2*x*2^(1/2
))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1683, 1678, 209}

$$-\frac{19\text{ArcTan}(x)}{2}+\frac{45\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{x(11x^2+9)}{4(x^4+3x^2+2)}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])/2 + (45*A
rcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
```

```
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx &= \frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-8+19x^2-11x^4}{x^2\left(2+3x^2+x^4\right)}\,dx \\
&= \frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-\frac{4}{x^2}+\frac{38}{1+x^2}-\frac{45}{2+x^2}\right)dx \\
&= -\frac{1}{x}+\frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)}-\frac{19}{2}\int \frac{1}{1+x^2}\,dx+\frac{45}{4}\int \frac{1}{2+x^2}\,dx \\
&= -\frac{1}{x}+\frac{x(9+11x^2)}{4\left(2+3x^2+x^4\right)}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{8}{x}+\frac{2x(9+11x^2)}{2+3x^2+x^4}-76\tan^{-1}(x)+45\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
[Out] (-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/8
```

Maple [A]
time = 0.03, size = 43, normalized size = 0.81

method	result	size
default	$-\frac{x}{2(x^2+1)}-\frac{19\arctan(x)}{2}-\frac{1}{x}+\frac{13x}{4(x^2+2)}+\frac{45\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{8}$	43
risch	$\frac{\frac{7}{4}x^4-\frac{3}{4}x^2-2}{x(x^4+3x^2+2)}+\frac{45\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{8}-\frac{19\arctan(x)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(x^2+1)-19/2*arctan(x)-1/x+13/4*x/(x^2+2)+45/8*arctan(1/2*2^(1/2)*x)
*2^(1/2)
```

Maxima [A]
time = 0.50, size = 45, normalized size = 0.85

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{7\,x^4-3\,x^2-8}{4\,(x^5+3\,x^3+2\,x)}-\frac{19}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

Fricas [A]
time = 0.36, size = 68, normalized size = 1.28

$$\frac{14\,x^4+45\,\sqrt{2}\,(x^5+3\,x^3+2\,x)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-6\,x^2-76\,(x^5+3\,x^3+2\,x)\arctan\left(x\right)-16}{8\,(x^5+3\,x^3+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2
- 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)
```

Sympy [A]
time = 0.09, size = 49, normalized size = 0.92

$$\frac{7x^4-3x^2-8}{4x^5+12x^3+8x}-\frac{19\,\mathrm{atan}\left(x\right)}{2}+\frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

```
[Out] (7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 45*sqrt(2)*
atan(sqrt(2)*x/2)/8
```

Giac [A]
time = 3.30, size = 45, normalized size = 0.85

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{7\,x^4-3\,x^2-8}{4\,(x^5+3\,x^3+2\,x)}-\frac{19}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

Mupad [B]
time = 0.07, size = 45, normalized size = 0.85

$$\frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{19\,\mathrm{atan}(x)}{2}-\frac{-\frac{7\,x^4}{4}+\frac{3\,x^2}{4}+2}{x^5+3\,x^3+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (45*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (19*atan(x))/2 - ((3*x^2)/4 - (7*x^4)/
4 + 2)/(2*x + 3*x^3 + x^5)
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8(2+3x^2+x^4)}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] -1/3/x^3+11/4/x-1/8*x*(9*x^2+5)/(x^4+3*x^2+2)+21/2*arctan(x)-71/16*arctan(1
/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1683, 1678, 209}

$$\frac{21\text{ArcTan}(x)}{2}-\frac{71\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{1}{3x^3}-\frac{x(9x^2+5)}{8(x^4+3x^2+2)}+\frac{11}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/3*1/x^3 + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (21*ArcTan[
x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
```

```
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx &= -\frac{x(5+9x^2)}{8\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-8+10x^2-\frac{39x^4}{2}+\frac{9x^6}{2}}{x^4\left(2+3x^2+x^4\right)}\,dx \\
&= -\frac{x(5+9x^2)}{8\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-\frac{4}{x^4}+\frac{11}{x^2}-\frac{42}{1+x^2}+\frac{71}{2\left(2+x^2\right)}\right)dx \\
&= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8\left(2+3x^2+x^4\right)}-\frac{71}{8}\int \frac{1}{2+x^2}\,dx+\frac{21}{2}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8\left(2+3x^2+x^4\right)}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.90

$$\frac{1}{48}\left(-\frac{16}{x^3}+\frac{132}{x}-\frac{6x(5+9x^2)}{2+3x^2+x^4}+504\tan^{-1}(x)-213\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
[Out] (-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 21
3*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

Maple [A]
time = 0.04, size = 48, normalized size = 0.77

method	result	size
default	$\frac{x}{2x^2+2}+\frac{21\arctan(x)}{2}-\frac{1}{3x^3}+\frac{11}{4x}-\frac{13x}{8(x^2+2)}-\frac{71\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{16}$	48
risch	$\frac{\frac{13}{8}x^6+\frac{175}{24}x^4+\frac{9}{2}x^2-\frac{2}{3}}{x^3(x^4+3x^2+2)}-\frac{71\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{16}+\frac{21\arctan(x)}{2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(x^2+1)+21/2*arctan(x)-1/3/x^3+11/4/x-13/8*x/(x^2+2)-71/16*arctan(1/2
*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 52, normalized size = 0.84

$$-\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{39\,x^6+175\,x^4+108\,x^2-16}{24\,(x^7+3\,x^5+2\,x^3)}+\frac{21}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 - 1
6)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

Fricas [A]
time = 0.36, size = 79, normalized size = 1.27

$$\frac{78\,x^6+350\,x^4-213\,\sqrt{2}\,(x^7+3\,x^5+2\,x^3)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+216\,x^2+504\,(x^7+3\,x^5+2\,x^3)\arctan\left(x\right)-32}{48\,(x^7+3\,x^5+2\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt(
2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5 +
2*x^3)
```

Sympy [A]
time = 0.10, size = 56, normalized size = 0.90

$$\frac{21\,\mathrm{atan}\,(x)}{2}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16}+\frac{39x^6+175x^4+108x^2-16}{24x^7+72x^5+48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

```
[Out] 21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x**4 + 108*
x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

Giac [A]
time = 3.81, size = 52, normalized size = 0.84

$$-\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{9\,x^3+5\,x}{8\,(x^4+3\,x^2+2)}+\frac{33\,x^2-4}{12\,x^3}+\frac{21}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2)
+ 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

Mupad [B]
time = 0.92, size = 51, normalized size = 0.82

$$\frac{21\,\mathrm{atan}(x)}{2}-\frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16}+\frac{\frac{13\,x^6}{8}+\frac{175\,x^4}{24}+\frac{9\,x^2}{2}-\frac{2}{3}}{x^7+3\,x^5+2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (21*atan(x))/2 - (71*2^(1/2)*atan((2^(1/2)*x)/2))/16 + ((9*x^2)/2 + (175*x^
4)/24 + (13*x^6)/8 - 2/3)/(2*x^3 + 3*x^5 + x^7)
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16(2+3x^2+x^4)}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/5/x^5+11/12/x^3-23/4/x-1/16*x*(-5*x^2+3)/(x^4+3*x^2+2)-23/2*arctan(x)+97
/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$-\frac{23\text{ArcTan}(x)}{2}+\frac{97\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{x(3-5x^2)}{16(x^4+3x^2+2)}-\frac{23}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/5*1/x^5 + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*x^2 + x^4)
) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
```

```
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x]])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx &= -\frac{x(3-5x^2)}{16\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{39x^6}{4}-\frac{5x^8}{4}}{x^6\left(2+3x^2+x^4\right)}\,dx\\ &= -\frac{x(3-5x^2)}{16\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-\frac{4}{x^6}+\frac{11}{x^4}-\frac{23}{x^2}+\frac{46}{1+x^2}-\frac{97}{4\left(2+x^2\right)}\right)dx\\ &= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16\left(2+3x^2+x^4\right)}+\frac{97}{16}\int \frac{1}{2+x^2}\,dx-\frac{23}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16\left(2+3x^2+x^4\right)}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.88

$$\frac{1}{480}\left(-\frac{96}{x^5}+\frac{440}{x^3}-\frac{2760}{x}+\frac{30x(-3+5x^2)}{2+3x^2+x^4}-5520\tan^{-1}(x)+1455\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520*
ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

Maple [A]
time = 0.04, size = 53, normalized size = 0.77

method	result	size
default	$-\frac{x}{2(x^2+1)}-\frac{23\arctan(x)}{2}-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}+\frac{13x}{16(x^2+2)}+\frac{97\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{32}$	53
risch	$\frac{-\frac{87}{16}x^8-\frac{793}{48}x^6-\frac{179}{20}x^4+\frac{37}{30}x^2-\frac{2}{5}}{x^5(x^4+3x^2+2)}+\frac{97\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{32}-\frac{23\arctan(x)}{2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(x^2+1)-23/2*arctan(x)-1/5/x^5+11/12/x^3-23/4/x+13/16*x/(x^2+2)+97/3
2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 57, normalized size = 0.83

$$\frac{97}{32}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{1305\,x^{8}+3965\,x^{6}+2148\,x^{4}-296\,x^{2}+96}{240\,(x^{9}+3\,x^{7}+2\,x^{5})}-\frac{23}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6 + 2148*x^4
 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

Fricas [A]
time = 0.38, size = 84, normalized size = 1.22

$$-\frac{2610\,x^{8}+7930\,x^{6}+4296\,x^{4}-1455\,\sqrt{2}\,(x^{9}+3\,x^{7}+2\,x^{5})\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-592\,x^{2}+5520\,(x^{9}+3\,x^{7}+2\,x^{5})\arctan\left(x\right)+192}{480\,(x^{9}+3\,x^{7}+2\,x^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5)
*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) + 1
92)/(x^9 + 3*x^7 + 2*x^5)
```

Sympy [A]
time = 0.10, size = 61, normalized size = 0.88

$$-\frac{23\operatorname{atan}\left(x\right)}{2}+\frac{97\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}+\frac{-1305x^{8}-3965x^{6}-2148x^{4}+296x^{2}-96}{240x^{9}+720x^{7}+480x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

```
[Out] -23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 + (-1305*x**8 - 3965*x**6 -
 2148*x**4 + 296*x**2 - 96)/(240*x**9 + 720*x**7 + 480*x**5)
```

Giac [A]
time = 5.11, size = 57, normalized size = 0.83

$$\frac{97}{32}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{5\,x^{3}-3\,x}{16\,(x^{4}+3\,x^{2}+2)}-\frac{345\,x^{4}-55\,x^{2}+12}{60\,x^{5}}-\frac{23}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3*x^2 + 2)
- 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

Mupad [B]
time = 0.92, size = 57, normalized size = 0.83

$$\frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{23\,\mathrm{atan}(x)}{2}-\frac{\frac{87\,x^8}{16}+\frac{793\,x^6}{48}+\frac{179\,x^4}{20}-\frac{37\,x^2}{30}+\frac{2}{5}}{x^9+3\,x^7+2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (97*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (23*atan(x))/2 - ((179*x^4)/20 - (37*
x^2)/30 + (793*x^6)/48 + (87*x^8)/16 + 2/5)/(2*x^5 + 3*x^7 + x^9)
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+1/32*x*(3*x^2+19)/(x^4+3*x^2+2)+25/2*
arctan(x)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1683, 1678, 209}

$$\frac{25\text{ArcTan}(x)}{2}-\frac{123\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{x(3x^2+19)}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*x^2))/(32*
(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx &= \frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{21x^6}{2}-\frac{39x^8}{8}-\frac{3x^{10}}{8}}{x^8\left(2+3x^2+x^4\right)}\,dx\\ &= \frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-\frac{4}{x^8}+\frac{11}{x^6}-\frac{23}{x^4}+\frac{137}{4x^2}-\frac{50}{1+x^2}+\frac{123}{8\left(2+x^2\right)}\right)dx\\ &= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}-\frac{123}{32}\int \frac{1}{2+x^2}\,dx+\frac{25}{2}\int \frac{1}{1\cdots}\\ &= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32\left(2+3x^2+x^4\right)}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\cdots\right)}{32\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 1.01

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{19x+3x^3}{32\left(2+3x^2+x^4\right)}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
[Out] -1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(2
 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2])
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.76

method	result	size
default	$\frac{x}{2x^2+2}+\frac{25\arctan(x)}{2}-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}-\frac{13x}{32(x^2+2)}-\frac{123\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{64}$	58
risch	$\frac{\frac{277}{32}x^{10}+\frac{2339}{96}x^8+\frac{477}{40}x^6-\frac{977}{420}x^4+\frac{47}{70}x^2-\frac{2}{7}}{x^7(x^4+3x^2+2)}-\frac{123\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{64}+\frac{25\arctan(x)}{2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(x^2+1)+25/2*arctan(x)-1/7/x^7+11/20/x^5-23/12/x^3+137/16/x-13/32*x/(
x^2+2)-123/64*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 62, normalized size = 0.82

$$-\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{29085\,x^{10}+81865\,x^{8}+40068\,x^{6}-7816\,x^{4}+2256\,x^{2}-960}{3360\,(x^{11}+3\,x^{9}+2\,x^{7})}+\frac{25}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 81865*x^8 + 40
068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*x^7) + 25/2*arctan(x
)
```

Fricas [A]
time = 0.38, size = 89, normalized size = 1.17

$$\frac{58170\,x^{10}+163730\,x^{8}+80136\,x^{6}-15632\,x^{4}-12915\,\sqrt{2}\,(x^{11}+3\,x^{9}+2\,x^{7})\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+4512\,x^{2}+84000\,(x^{11}+3\,x^{9}+2\,x^{7})\arctan\left(x\right)-1920}{6720\,(x^{11}+3\,x^{9}+2\,x^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x^
11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^9
+ 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)
```

Sympy [A]
time = 0.11, size = 66, normalized size = 0.87

$$\frac{25\,\mathrm{atan}\left(x\right)}{2}-\frac{123\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}+\frac{29085x^{10}+81865x^{8}+40068x^{6}-7816x^{4}+2256x^{2}-960}{3360x^{11}+10080x^{9}+6720x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

```
[Out] 25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 + 81865*x**8
 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**11 + 10080*x**9 + 672
0*x**7)
```

Giac [A]
time = 4.76, size = 62, normalized size = 0.82

$$-\frac{123}{64}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{3\,x^{3}+19\,x}{32\,\left(x^{4}+3\,x^{2}+2\right)}+\frac{14385\,x^{6}-3220\,x^{4}+924\,x^{2}-240}{1680\,x^{7}}+\frac{25}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 +
2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*arctan(x)
```

Mupad [B]
time = 0.07, size = 61, normalized size = 0.80

$$\frac{25\,\mathrm{atan}(x)}{2}-\frac{123\,\sqrt{2}\,\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}+\frac{\frac{277\,x^{10}}{32}+\frac{2339\,x^{8}}{96}+\frac{477\,x^{6}}{40}-\frac{977\,x^{4}}{420}+\frac{47\,x^{2}}{70}-\frac{2}{7}}{x^{11}+3\,x^{9}+2\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^8*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (25*atan(x))/2 - (123*2^(1/2)*atan((2^(1/2)*x)/2))/64 + ((47*x^2)/70 - (977
*x^4)/420 + (477*x^6)/40 + (2339*x^8)/96 + (277*x^10)/32 - 2/7)/(2*x^7 + 3*
x^9 + x^11)
```

3.91 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=81

$$214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 214*x-14*x^3+x^5+1/4*x*(415*x^2+414)/(x^4+3*x^2+2)^2+1/8*x*(1669*x^2+824)/(
x^4+3*x^2+2)+477/8*arctan(x)-351*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1682, 1692, 1690, 1180, 209}

$$\frac{477\text{ArcTan}(x)}{8}-351\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)+x^5-14x^3+\frac{(1669x^2+824)\,x}{8(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4(x^4+3x^2+2)^2}+214x$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(82
4 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcT
an[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
```

```
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(414+415x^2)}{4(2+3x^2+x^4)^2} - \frac{1}{8}\int \frac{828-2478x^2-840x^4+424x^6-216x^8+96x^{10}}{(2+3x^2+x^4)^2}$$

$$= \frac{x(414+415x^2)}{4(2+3x^2+x^4)^2} + \frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{1}{32}\int \frac{-4952-2700x^2+3136x^4}{2+3x^2+x^4}$$

$$= \frac{x(414+415x^2)}{4(2+3x^2+x^4)^2} + \frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{1}{32}\int \Big(6848-1344x^2+160x^4-$$

$$= 214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2} + \frac{x(824+1669x^2)}{8(2+3x^2+x^4)} - \frac{9}{8}\int \frac{518+57}{2+3x^2}$$

$$= 214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2} + \frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{477}{8}\int \frac{1}{1+x^2}$$

$$= 214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2} + \frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{477}{8}\tan^{-1}(x)$$

Mathematica [A]

time = 0.04, size = 71, normalized size = 0.88

$$\frac{x(9324+26736x^2+26775x^4+10581x^6+1144x^8-64x^{10}+8x^{12})}{8\,(2+3x^2+x^4)^2}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12)
)/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2
]]
```

Maple [A]
time = 0.03, size = 64, normalized size = 0.79

method	result	size
risch	$x^5-14x^3+214x+\frac{\frac{1669}{8}x^7+\frac{5831}{8}x^5+830x^3+\frac{619}{2}x}{(x^4+3x^2+2)^2}-351\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}+\frac{477\arctan(x)}{8}$	61
default	$x^5-14x^3+214x+\frac{-\frac{11}{8}x^3-\frac{13}{8}x}{(x^2+1)^2}+\frac{477\arctan(x)}{8}-\frac{16(-\frac{105}{8}x^3-\frac{79}{4}x)}{(x^2+2)^2}-351\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-14*x^3+214*x+(-11/8*x^3-13/8*x)/(x^2+1)^2+477/8*arctan(x)-16*(-105/8*x^
3-79/4*x)/(x^2+2)^2-351*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.51, size = 71, normalized size = 0.88

$$x^5-14\,x^3-351\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+214\,x+\frac{1669\,x^7+5831\,x^5+6640\,x^3+2476\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{477}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8*a
rctan(x)
```

Fricas [A]
time = 0.36, size = 114, normalized size = 1.41

$$\frac{8\,x^{13}-64\,x^{11}+1144\,x^9+10581\,x^7+26775\,x^5+26736\,x^3-2808\,\sqrt{2}\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+477\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(x\right)+9324\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 2808
*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*(x
^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*x^4
 + 12*x^2 + 4)
```

Sympy [A]
time = 0.10, size = 75, normalized size = 0.93

$$x^5 - 14x^3 + 214x + \frac{1669x^7 + 5831x^5 + 6640x^3 + 2476x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{477\operatorname{atan}(x)}{8} - 351\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 2476*x)/(8*x*
*8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8 - 351*sqrt(2)*atan(
sqrt(2)*x/2)
```

Giac [A]
time = 4.45, size = 61, normalized size = 0.75

$$x^5 - 14\,x^3 - 351\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{477}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)
```

Mupad [B]
time = 0.06, size = 70, normalized size = 0.86

$$214\,x + \frac{477\,\mathrm{atan}(x)}{8} - 351\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{1669\,x^7}{8} + \frac{5831\,x^5}{8} + 830\,x^3 + \frac{619\,x}{2}}{x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4} - 14\,x^3 + x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] 214*x + (477*atan(x))/8 - 351*2^(1/2)*atan((2^(1/2)*x)/2) + ((619*x)/2 + 83
0*x^3 + (5831*x^5)/8 + (1669*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4) -
14*x^3 + x^5
```

3.92 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=80

$$-42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(24-409x^2)}{8\left(2+3x^2+x^4\right)}-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -42*x+5/3*x^3-1/4*x*(207*x^2+206)/(x^4+3*x^2+2)^2+1/8*x*(-409*x^2+24)/(x^4+
3*x^2+2)-449/8*arctan(x)+219/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1682, 1692, 1690, 1180, 209}

$$-\frac{449\text{ArcTan}(x)}{8}+\frac{219\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{5x^3}{3}+\frac{(24-409x^2)\,x}{8\left(x^4+3x^2+2\right)}-\frac{(207x^2+206)\,x}{4\left(x^4+3x^2+2\right)^2}-42x$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(24 -
409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]
])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
```

```
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2} - \frac{1}{8}\int \frac{-412+1230x^2+424x^4-216x^6+96x^8-40x}{(2+3x^2+x^4)^2}$$

$$= -\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2} + \frac{x(24-409x^2)}{8(2+3x^2+x^4)} + \frac{1}{32}\int \frac{728+1500x^2-864x^4+1}{2+3x^2+x^4}$$

$$= -\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2} + \frac{x(24-409x^2)}{8(2+3x^2+x^4)} + \frac{1}{32}\int \left(-1344+160x^2+\frac{4(854}{2+3}\right.$$

$$= -42x + \frac{5x^3}{3} - \frac{x(206+207x^2)}{4(2+3x^2+x^4)^2} + \frac{x(24-409x^2)}{8(2+3x^2+x^4)} + \frac{1}{8}\int \frac{854+1303x^2}{2+3x^2+x^4}$$

$$= -42x + \frac{5x^3}{3} - \frac{x(206+207x^2)}{4(2+3x^2+x^4)^2} + \frac{x(24-409x^2)}{8(2+3x^2+x^4)} - \frac{449}{8}\int \frac{1}{1+x^2}\,dx +$$

$$= -42x + \frac{5x^3}{3} - \frac{x(206+207x^2)}{4(2+3x^2+x^4)^2} + \frac{x(24-409x^2)}{8(2+3x^2+x^4)} - \frac{449}{8}\tan^{-1}(x) + \frac{219}{}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.82

$$\frac{x(-5124-15416x^2-16233x^4-6755x^6-768x^8+40x^{10})}{24(2+3x^2+x^4)^2} - \frac{449}{8}\tan^{-1}(x) + \frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2 +
 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A]
time = 0.03, size = 62, normalized size = 0.78

method	result	size
risch	$\frac{5x^3}{3} - 42x + \frac{-\frac{409}{8}x^7 - \frac{1203}{8}x^5 - 145x^3 - \frac{91}{2}x}{(x^4+3x^2+2)^2} + \frac{219\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2} - \frac{449\arctan(x)}{8}$	58
default	$\frac{5x^3}{3} - 42x - \frac{-\frac{15}{8}x^3 - \frac{17}{8}x}{(x^2+1)^2} - \frac{449\arctan(x)}{8} + \frac{-53x^3-54x}{(x^2+2)^2} + \frac{219\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/3*x^3-42*x-(-15/8*x^3-17/8*x)/(x^2+1)^2-449/8*arctan(x)+16*(-53/16*x^3-27
/8*x)/(x^2+2)^2+219/2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 68, normalized size = 0.85

$$\frac{5}{3}\,x^{3}+\frac{219}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-42\,x-\frac{409\,x^{7}+1203\,x^{5}+1160\,x^{3}+364\,x}{8\,\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)}-\frac{449}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arctan(
x)
```

Fricas [A]
time = 0.38, size = 109, normalized size = 1.36

$$\frac{40\,x^{11}-768\,x^{9}-6755\,x^{7}-16233\,x^{5}-15416\,x^{3}+2628\,\sqrt{2}\,\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-1347\,\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)\arctan\left(x\right)-5124\,x}{24\,\left(x^{8}+6\,x^{6}+13\,x^{4}+12\,x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*(
x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*x^
6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2
 + 4)
```

Sympy [A]
time = 0.10, size = 76, normalized size = 0.95

$$\frac{5x^{3}}{3}-42x+\frac{-409x^{7}-1203x^{5}-1160x^{3}-364x}{8x^{8}+48x^{6}+104x^{4}+96x^{2}+32}-\frac{449\,\mathrm{atan}\,(x)}{8}+\frac{219\sqrt{2}\,\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x**3/3 - 42*x + (-409*x**7 - 1203*x**5 - 1160*x**3 - 364*x)/(8*x**8 + 48*
x**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2)*
x/2)/2
```

Giac [A]
time = 4.48, size = 58, normalized size = 0.72

$$\frac{5}{3}\,x^{3}+\frac{219}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-42\,x-\frac{409\,x^{7}+1203\,x^{5}+1160\,x^{3}+364\,x}{8\,\left(x^{4}+3\,x^{2}+2\right)^{2}}-\frac{449}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)
```

Mupad [B]
time = 0.05, size = 68, normalized size = 0.85

$$\frac{219\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{449\,\mathrm{atan}(x)}{8}-42\,x-\frac{\frac{409\,x^7}{8}+\frac{1203\,x^5}{8}+145\,x^3+\frac{91\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}+\frac{5\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (219*2^(1/2)*atan((2^(1/2)*x)/2))/2 - (449*atan(x))/8 - 42*x - ((91*x)/2 +
145*x^3 + (1203*x^5)/8 + (409*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4) +
 (5*x^3)/3
```

3.93 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=75

$$5x+\frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 5*x+1/4*x*(103*x^2+102)/(x^4+3*x^2+2)^2-1/8*x*(15*x^2+244)/(x^4+3*x^2+2)+41
3/8*arctan(x)-191/4*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.161$, Rules used = {1682, 1692, 1690, 1180, 209}

$$\frac{413\text{ArcTan}(x)}{8}-\frac{191\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{(15x^2+244)\,x}{8(x^4+3x^2+2)}+\frac{(103x^2+102)\,x}{4(x^4+3x^2+2)^2}+5x$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*x^2))/(8*(
2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
```

```
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx &= \frac{x(102+103x^2)}{4\,(2+3x^2+x^4)^2}-\frac{1}{8}\int \frac{204-606x^2-216x^4+96x^6-40x^8}{(2+3x^2+x^4)^2}\,dx\\
&= \frac{x(102+103x^2)}{4\,(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8\,(2+3x^2+x^4)}+\frac{1}{32}\int \frac{568-924x^2+160x^4}{2+3x^2+x^4}\,dx\\
&= \frac{x(102+103x^2)}{4\,(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8\,(2+3x^2+x^4)}+\frac{1}{32}\int \left(160+\frac{4(62-351x^2)}{2+3x^2+x^4}\right)dx\\
&= 5x+\frac{x(102+103x^2)}{4\,(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8\,(2+3x^2+x^4)}+\frac{1}{8}\int \frac{62-351x^2}{2+3x^2+x^4}\,dx\\
&= 5x+\frac{x(102+103x^2)}{4\,(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8\,(2+3x^2+x^4)}+\frac{413}{8}\int \frac{1}{1+x^2}\,dx-\frac{191}{2}\int \frac{}{2}\\
&= 5x+\frac{x(102+103x^2)}{4\,(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8\,(2+3x^2+x^4)}+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\Big(}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.80

$$\frac{1}{8}\left(\frac{x(-124-76x^2+231x^4+225x^6+40x^8)}{(2+3x^2+x^4)^2}+413\tan^{-1}(x)-382\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 413
*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A]
time = 0.04, size = 56, normalized size = 0.75

method	result	size
risch	$5x+\frac{-\frac{15}{8}x^7-\frac{289}{8}x^5-\frac{139}{2}x^3-\frac{71}{2}x}{(x^4+3x^2+2)^2}+\frac{413\arctan(x)}{8}-\frac{191\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{4}$	53
default	$5x+\frac{-\frac{19}{8}x^3-\frac{21}{8}x}{(x^2+1)^2}+\frac{413\arctan(x)}{8}-\frac{16(-\frac{1}{32}x^3+\frac{25}{16}x)}{(x^2+2)^2}-\frac{191\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{4}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x+(-19/8*x^3-21/8*x)/(x^2+1)^2+413/8*arctan(x)-16*(-1/32*x^3+25/16*x)/(x^
2+2)^2-191/4*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 63, normalized size = 0.84

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+5\,x-\frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{413}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)
```

Fricas [A]
time = 0.38, size = 104, normalized size = 1.39

$$\frac{40\,x^9+225\,x^7+231\,x^5-76\,x^3-382\,\sqrt{2}\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+413\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(x\right)-124\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x^
4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^2
+ 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A]
time = 0.10, size = 70, normalized size = 0.93

$$5x + \frac{-15x^7 - 289x^5 - 556x^3 - 284x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{413\,\mathrm{atan}\,(x)}{8} - \frac{191\sqrt{2}\;\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x + (-15*x**7 - 289*x**5 - 556*x**3 - 284*x)/(8*x**8 + 48*x**6 + 104*x**4
 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

Giac [A]
time = 2.88, size = 53, normalized size = 0.71

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right) + 5\,x - \frac{15\,x^7 + 289\,x^5 + 556\,x^3 + 284\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{413}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

Mupad [B]
time = 0.93, size = 63, normalized size = 0.84

$$5\,x + \frac{413\,\mathrm{atan}(x)}{8} - \frac{191\,\sqrt{2}\;\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} - \frac{\frac{15\,x^7}{8} + \frac{289\,x^5}{8} + \frac{139\,x^3}{2} + \frac{71\,x}{2}}{x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] 5*x + (413*atan(x))/8 - (191*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((71*x)/2 + (
139*x^3)/2 + (289*x^5)/8 + (15*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.94 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8(2+3x^2+x^4)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/4*x*(51*x^2+50)/(x^4+3*x^2+2)^2+1/8*x*(125*x^2+254)/(x^4+3*x^2+2)-369/8*
arctan(x)+267/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1682, 1692, 1180, 209}

$$-\frac{369\text{ArcTan}(x)}{8}+\frac{267\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}+\frac{x(125x^2+254)}{8(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -1/4*(x*(50 + 51*x^2))/(2 + 3*x^2 + x^4)^2 + (x*(254 + 125*x^2))/(8*(2 + 3*
x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
```

```
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx &= -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}-\frac{1}{8}\int \frac{-100+294x^2+96x^4-40x^6}{(2+3x^2+x^4)^2}\,dx\\ &= -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int \frac{-816+660x^2}{2+3x^2+x^4}\,dx\\ &= -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8(2+3x^2+x^4)}-\frac{369}{8}\int \frac{1}{1+x^2}\,dx+\frac{267}{4}\int \frac{1}{2+x\ldots}\\ &= -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2}+\frac{x(254+125x^2)}{8(2+3x^2+x^4)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\ldots\right.}{4\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.76

$$\frac{1}{8}\left(\frac{x(408+910x^2+629x^4+125x^6)}{(2+3x^2+x^4)^2}-369\tan^{-1}(x)+267\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[x
] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Maple [A]
time = 0.03, size = 54, normalized size = 0.75

method	result	size
risch	$\frac{\frac{125}{8}x^7+\frac{629}{8}x^5+\frac{455}{4}x^3+51x}{(x^4+3x^2+2)^2}-\frac{369\arctan(x)}{8}+\frac{267\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{8}$	50
default	$-\frac{-\frac{23}{8}x^3-\frac{25}{8}x}{(x^2+1)^2}-\frac{369\arctan(x)}{8}+\frac{\frac{51}{4}x^3+\frac{77}{2}x}{(x^2+2)^2}+\frac{267\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{8}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-23/8*x^3-25/8*x)/(x^2+1)^2-369/8*arctan(x)+2*(51/8*x^3+77/4*x)/(x^2+2)^2
+267/8*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 60, normalized size = 0.83

$$\frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}-\frac{369}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)
```

Fricas [A]
time = 0.37, size = 99, normalized size = 1.38

$$\frac{125\,x^7+629\,x^5+910\,x^3+267\,\sqrt{2}\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-369\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(x\right)+408\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x
^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc
tan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A]
time = 0.10, size = 65, normalized size = 0.90

$$\frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32}-\frac{369\,\mathrm{atan}\,(x)}{8}+\frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*
x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Giac [A]
time = 3.10, size = 50, normalized size = 0.69

$$\frac{267}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^4+3\,x^2+2)^2}-\frac{369}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

Mupad [B]
time = 0.93, size = 59, normalized size = 0.82

$$\frac{267\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{369\,\mathrm{atan}(x)}{8}+\frac{\frac{125\,x^7}{8}+\frac{629\,x^5}{8}+\frac{455\,x^3}{4}+51\,x}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (267*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (369*atan(x))/8 + (51*x + (455*x^3)/4
 + (629*x^5)/8 + (125*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.95 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=72

$$\frac{x(24+25x^2)}{4\,(2+3x^2+x^4)^2}-\frac{x(211+130x^2)}{8\,(2+3x^2+x^4)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/4*x*(25*x^2+24)/(x^4+3*x^2+2)^2-1/8*x*(130*x^2+211)/(x^4+3*x^2+2)+317/8*a
rctan(x)-447/16*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1682, 1692, 1180, 209}

$$\frac{317\text{ArcTan}(x)}{8}-\frac{447\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{x(25x^2+24)}{4\,(x^4+3x^2+2)^2}-\frac{x(130x^2+211)}{8\,(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(8*(2 + 3*x
^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
```

```
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx &= \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2}-\frac{1}{8}\int \frac{48-154x^2-40x^4}{(2+3x^2+x^4)^2}\,dx\\ &= \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2}-\frac{x(211+130x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int \frac{748-520x^2}{2+3x^2+x^4}\,dx\\ &= \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2}-\frac{x(211+130x^2)}{8(2+3x^2+x^4)}+\frac{317}{8}\int \frac{1}{1+x^2}\,dx-\frac{447}{8}\int \frac{1}{2+x^2}\\ &= \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2}-\frac{x(211+130x^2)}{8(2+3x^2+x^4)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.78

$$\frac{1}{16}\left(-\frac{2x(374+843x^2+601x^4+130x^6)}{(2+3x^2+x^4)^2}+634\tan^{-1}(x)-447\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

[Out] ((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcTan[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16

Maple [A]
time = 0.03, size = 53, normalized size = 0.74

method	result	size
risch	$\frac{-\frac{65}{4}x^7-\frac{601}{8}x^5-\frac{843}{8}x^3-\frac{187}{4}x}{(x^4+3x^2+2)^2}+\frac{317\arctan(x)}{8}-\frac{447\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{16}$	50
default	$\frac{-\frac{27}{8}x^3-\frac{29}{8}x}{(x^2+1)^2}+\frac{317\arctan(x)}{8}-\frac{\frac{103}{8}x^3+\frac{129}{4}x}{(x^2+2)^2}-\frac{447\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{16}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)

[Out] (-27/8*x^3-29/8*x)/(x^2+1)^2+317/8*arctan(x)-(103/8*x^3+129/4*x)/(x^2+2)^2-447/16*arctan(1/2*2^(1/2)*x)*2^(1/2)

Maxima [A]
time = 0.50, size = 60, normalized size = 0.83

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}+\frac{317}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 + 374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)

Fricas [A]
time = 0.36, size = 99, normalized size = 1.38

$$-\frac{260\,x^7+1202\,x^5+1686\,x^3+447\,\sqrt{2}\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-634\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan(x)+748\,x}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] -1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)

Sympy [A]
time = 0.10, size = 66, normalized size = 0.92

$$\frac{-130x^7 - 601x^5 - 843x^3 - 374x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} + \frac{317\,\mathrm{atan}\,(x)}{8} - \frac{447\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (-130*x**7 - 601*x**5 - 843*x**3 - 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96
*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16
```

Giac [A]
time = 4.32, size = 50, normalized size = 0.69

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right) - \frac{130\,x^7 + 601\,x^5 + 843\,x^3 + 374\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{317}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

Mupad [B]
time = 0.07, size = 60, normalized size = 0.83

$$\frac{317\,\mathrm{atan}(x)}{8} - \frac{447\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} - \frac{\frac{65\,x^7}{4} + \frac{601\,x^5}{8} + \frac{843\,x^3}{8} + \frac{187\,x}{4}}{x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (317*atan(x))/8 - (447*2^(1/2)*atan((2^(1/2)*x)/2))/16 - ((187*x)/4 + (843*
x^3)/8 + (601*x^5)/8 + (65*x^7)/4)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.96 $\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}+\frac{x(335+217x^2)}{16(2+3x^2+x^4)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x*(12*x^2+11)/(x^4+3*x^2+2)^2+1/16*x*(217*x^2+335)/(x^4+3*x^2+2)-257/8
*arctan(x)+731/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1692, 1192, 1180, 209}

$$-\frac{257\text{ArcTan}(x)}{8}+\frac{731\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}+\frac{x(217x^2+335)}{16(x^4+3x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -1/4*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4)^2 + (x*(335 + 217*x^2))/(16*(2 + 3
*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
```

```
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx &= -\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}-\frac{1}{8}\int \frac{-38+80x^2}{(2+3x^2+x^4)^2}\,dx\\ &= -\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}+\frac{x(335+217x^2)}{16(2+3x^2+x^4)}+\frac{1}{32}\int \frac{-594+434x^2}{2+3x^2+x^4}\,dx\\ &= -\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}+\frac{x(335+217x^2)}{16(2+3x^2+x^4)}-\frac{257}{8}\int \frac{1}{1+x^2}\,dx+\frac{731}{16}\int \frac{1}{2+x^2}\,d\\ &= -\frac{x(11+12x^2)}{4(2+3x^2+x^4)^2}+\frac{x(335+217x^2)}{16(2+3x^2+x^4)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.78

$$\frac{1}{32}\left(\frac{2x(626+1391x^2+986x^4+217x^6)}{(2+3x^2+x^4)^2}-1028\tan^{-1}(x)+731\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*ArcT
an[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

Maple [A]
time = 0.03, size = 53, normalized size = 0.74

method	result	size
risch	$\frac{\frac{217}{16}x^7+\frac{493}{8}x^5+\frac{1391}{16}x^3+\frac{313}{8}x}{(x^4+3x^2+2)^2}+\frac{731\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{32}-\frac{257\arctan(x)}{8}$	50
default	$-\frac{-\frac{31}{8}x^3-\frac{33}{8}x}{(x^2+1)^2}-\frac{257\arctan(x)}{8}+\frac{\frac{155}{16}x^3+\frac{181}{8}x}{(x^2+2)^2}+\frac{731\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{32}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-31/8*x^3-33/8*x)/(x^2+1)^2-257/8*arctan(x)+(155/16*x^3+181/8*x)/(x^2+2)^
2+731/32*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 60, normalized size = 0.83

$$\frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}-\frac{257}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)
```

Fricas [A]
time = 0.37, size = 99, normalized size = 1.38

$$\frac{434\,x^7+1972\,x^5+2782\,x^3+731\,\sqrt{2}\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-1028\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan(x)+1252\,x}{32\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 1
2*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A]
time = 0.10, size = 65, normalized size = 0.90

$$\frac{217x^7+986x^5+1391x^3+626x}{16x^8+96x^6+208x^4+192x^2+64}-\frac{257\operatorname{atan}(x)}{8}+\frac{731\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 + 1
92*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32
```

Giac [A]
time = 4.73, size = 50, normalized size = 0.69

$$\frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^4+3\,x^2+2)^2}-\frac{257}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

Mupad [B]
time = 0.07, size = 59, normalized size = 0.82

$$\frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{257\,\mathrm{atan}(x)}{8}+\frac{\frac{217\,x^7}{16}+\frac{493\,x^5}{8}+\frac{1391\,x^3}{16}+\frac{313\,x}{8}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (731*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (257*atan(x))/8 + ((313*x)/8 + (1391
*x^3)/16 + (493*x^5)/8 + (217*x^7)/16)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3} dx$

Optimal. Leaf size=79

$$-\frac{1}{2x}+\frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/2/x+1/8*x*(11*x^2+9)/(x^4+3*x^2+2)^2-1/32*x*(347*x^2+547)/(x^4+3*x^2+2)+
189/8*arctan(x)-1119/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1683, 1678, 209}

$$\frac{189\text{ArcTan}(x)}{8}-\frac{1119\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}+\frac{x(11x^2+9)}{8\left(x^4+3x^2+2\right)^2}-\frac{x(347x^2+547)}{32\left(x^4+3x^2+2\right)}-\frac{1}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/2*1/x + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 347*x^2))/(
32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTan[x/Sqrt[2]])/(32*Sq
rt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
```

```
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx &= \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-16+29x^2-55x^4}{x^2\left(2+3x^2+x^4\right)^2}\,dx\\ &= \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{32+441x^2-347x^4}{x^2\left(2+3x^2+x^4\right)}\,dx\\ &= \frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(\frac{16}{x^2}+\frac{756}{1+x^2}-\frac{1119}{2+x^2}\right)dx\\ &= -\frac{1}{2x}+\frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)}+\frac{189}{8}\int \frac{1}{1+x^2}\,dx-\frac{1119}{32}\int \frac{1}{2}\\ &= -\frac{1}{2x}+\frac{x(9+11x^2)}{8\left(2+3x^2+x^4\right)^2}-\frac{x(547+347x^2)}{32\left(2+3x^2+x^4\right)}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\right.}{32\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.80

$$\frac{1}{64}\left(-\frac{2(64+1250x^2+2499x^4+1684x^6+363x^8)}{x\left(2+3x^2+x^4\right)^2}+1512\tan^{-1}(x)-1119\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)^
2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.73

method	result	size
risch	$\frac{-\frac{363}{32}x^8-\frac{421}{8}x^6-\frac{2499}{32}x^4-\frac{625}{16}x^2-2}{x(x^4+3x^2+2)^2}+\frac{189\arctan(x)}{8}-\frac{1119\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{64}$	56

default	$\frac{-\frac{35}{8}x^3-\frac{37}{8}x}{(x^2+1)^2}+\frac{189\arctan(x)}{8}-\frac{1}{2x}-\frac{\frac{207}{16}x^3+\frac{233}{8}x}{2(x^2+2)^2}-\frac{1119\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{64}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-35/8*x^3-37/8*x)/(x^2+1)^2+189/8*arctan(x)-1/2/x-1/2*(207/16*x^3+233/8*x)
/(x^2+2)^2-1119/64*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 65, normalized size = 0.82

$$-\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{363\,x^8+1684\,x^6+2499\,x^4+1250\,x^2+64}{32\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)}+\frac{189}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x^
4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x)
```

Fricas [A]
time = 0.37, size = 108, normalized size = 1.37

$$-\frac{726\,x^8+3368\,x^6+4998\,x^4+1119\,\sqrt{2}\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+2500\,x^2-1512\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)\arctan\left(x\right)+128}{64\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5 +
 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*x^
5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

Sympy [A]
time = 0.11, size = 71, normalized size = 0.90

$$\frac{-363x^8-1684x^6-2499x^4-1250x^2-64}{32x^9+192x^7+416x^5+384x^3+128x}+\frac{189\operatorname{atan}\left(x\right)}{8}-\frac{1119\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

```
[Out] (-363*x**8 - 1684*x**6 - 2499*x**4 - 1250*x**2 - 64)/(32*x**9 + 192*x**7 +
416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*x/
2)/64
```

Giac [A]
time = 5.46, size = 55, normalized size = 0.70

$$-\frac{1119}{64}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{347\,x^{7}+1588\,x^{5}+2291\,x^{3}+1058\,x}{32\,\left(x^{4}+3\,x^{2}+2\right)^{2}}-\frac{1}{2\,x}+\frac{189}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x^
3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)
```

Mupad [B]
time = 0.92, size = 65, normalized size = 0.82

$$\frac{189\,\mathrm{atan}(x)}{8}-\frac{1119\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}-\frac{\frac{363\,x^8}{32}+\frac{421\,x^6}{8}+\frac{2499\,x^4}{32}+\frac{625\,x^2}{16}+2}{x^9+6\,x^7+13\,x^5+12\,x^3+4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] (189*atan(x))/8 - (1119*2^(1/2)*atan((2^(1/2)*x)/2))/64 - ((625*x^2)/16 + (
2499*x^4)/32 + (421*x^6)/8 + (363*x^8)/32 + 2)/(4*x + 12*x^3 + 13*x^5 + 6*x
^7 + x^9)
```

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{6x^3}+\frac{17}{8x}-\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

```
[Out] -1/6/x^3+17/8/x-1/16*x*(9*x^2+5)/(x^4+3*x^2+2)^2+1/64*x*(571*x^2+951)/(x^4+
3*x^2+2)-113/8*arctan(x)+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1683, 1678, 209}

$$-\frac{113\text{ArcTan}(x)}{8}+\frac{1611\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}-\frac{1}{6x^3}-\frac{x(9x^2+5)}{16\left(x^4+3x^2+2\right)^2}+\frac{x(571x^2+951)}{64\left(x^4+3x^2+2\right)}+\frac{17}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/6*1/x^3 + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2) + (x*(951
+ 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8 + (1611*ArcTan[x/Sqr
t[2]])/(64*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
```

```
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = -\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+20x^2-\frac{73x^4}{2}+\frac{45x^6}{2}}{x^4\left(2+3x^2+x^4\right)^2}\,dx$$

$$= -\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2} + \frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32-88x^2-\frac{573x^4}{2}+\frac{571x^6}{2}}{x^4\left(2+3x^2+x^4\right)}\,dx$$

$$= -\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2} + \frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^4} - \frac{68}{x^2} - \frac{452}{1+x^2} + \frac{16}{2(2+}\right.$$

$$= -\frac{1}{6x^3} + \frac{17}{8x} - \frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2} + \frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)} - \frac{113}{8}\int \frac{1}{1+x^2}\,dx + \frac{16}{}$$

$$= -\frac{1}{6x^3} + \frac{17}{8x} - \frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2} + \frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)} - \frac{113}{8}\tan^{-1}(x) + \frac{1611}{}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.91

$$\frac{1}{384}\left(-\frac{64}{x^3} + \frac{816}{x} - \frac{24x(5+9x^2)}{\left(2+3x^2+x^4\right)^2} + \frac{6x(951+571x^2)}{2+3x^2+x^4} - 5424\tan^{-1}(x) + 4833\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] (-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 571
*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2]*ArcTan[x/Sqrt[2]])
/384
```

Maple [A]
time = 0.04, size = 64, normalized size = 0.74

method	result	size

risch	$\frac{\frac{707}{64}x^{10}+\frac{1301}{24}x^8+\frac{5663}{64}x^6+\frac{5063}{96}x^4+\frac{13}{2}x^2-\frac{2}{3}}{x^3(x^4+3x^2+2)^2}-\frac{113\arctan(x)}{8}+\frac{1611\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{128}$	61
default	$-\frac{-\frac{39}{8}x^3-\frac{41}{8}x}{(x^2+1)^2}-\frac{113\arctan(x)}{8}-\frac{1}{6x^3}+\frac{17}{8x}+\frac{\frac{259}{8}x^3+\frac{285}{4}x}{8(x^2+2)^2}+\frac{1611\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{128}$	64

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)

[Out] -(-39/8*x^3-41/8*x)/(x^2+1)^2-113/8*arctan(x)-1/6/x^3+17/8/x+1/8*(259/8*x^3 +285/4*x)/(x^2+2)^2+1611/128*arctan(1/2*2^(1/2)*x)*2^(1/2)

Maxima [A]
time = 0.49, size = 72, normalized size = 0.84

$$\frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{2121\,x^{10}+10408\,x^8+16989\,x^6+10126\,x^4+1248\,x^2-128}{192\,(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)}-\frac{113}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")

[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 16989*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3) - 113/8*arctan(x)

Fricas [A]
time = 0.36, size = 119, normalized size = 1.38

$$\frac{4242\,x^{10}+20816\,x^8+33978\,x^6+20252\,x^4+4833\,\sqrt{2}\,(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+2496\,x^2-5424\,(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)\arctan(x)-256}{384\,(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")

[Out] 1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)

Sympy [A]
time = 0.12, size = 76, normalized size = 0.88

$$-\frac{113\,\operatorname{atan}(x)}{8}+\frac{1611\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{128}+\frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192x^{11}+1152x^9+2496x^7+2304x^5+768x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

```
[Out] -113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*x
**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 + 2
496*x**7 + 2304*x**5 + 768*x**3)
```

Giac [A]
time = 5.32, size = 62, normalized size = 0.72

$$\frac{1611}{128}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{571\,x^{7}+2664\,x^{5}+3959\,x^{3}+1882\,x}{64\,{\left(x^{4}+3\,x^{2}+2\right)}^{2}}+\frac{51\,x^{2}-4}{24\,x^{3}}-\frac{113}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x^
3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)
```

Mupad [B]
time = 0.92, size = 71, normalized size = 0.83

$$\frac{\frac{707\,x^{10}}{64}+\frac{1301\,x^{8}}{24}+\frac{5663\,x^{6}}{64}+\frac{5063\,x^{4}}{96}+\frac{13\,x^{2}}{2}-\frac{2}{3}}{x^{11}+6\,x^{9}+13\,x^{7}+12\,x^{5}+4\,x^{3}}-\frac{113\,\mathrm{atan}(x)}{8}+\frac{1611\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] ((13*x^2)/2 + (5063*x^4)/96 + (5663*x^6)/64 + (1301*x^8)/24 + (707*x^10)/64
 - 2/3)/(4*x^3 + 12*x^5 + 13*x^7 + 6*x^9 + x^11) - (113*atan(x))/8 + (1611*
2^(1/2)*atan((2^(1/2)*x)/2))/128
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x(3-5x^2)}{32(2+3x^2+x^4)^2}-\frac{x(1771+999x^2)}{128(2+3x^2+x^4)}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

```
[Out] -1/10/x^5+17/24/x^3-93/16/x-1/32*x*(-5*x^2+3)/(x^4+3*x^2+2)^2-1/128*x*(999*
x^2+1771)/(x^4+3*x^2+2)+29/8*arctan(x)-2207/256*arctan(1/2*x*2^(1/2))*2^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1683, 1678, 209}

$$\frac{29\text{ArcTan}(x)}{8}-\frac{2207\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}-\frac{x(999x^2+1771)}{128(x^4+3x^2+2)}-\frac{93}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/10*1/x^5 + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 + 3*x^2 + x^
4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) + (29*ArcTan[x])/8 - (
2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
```

```
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = -\frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+20x^2-34x^4+\frac{81x^6}{4}-\frac{25x^8}{4}}{x^6\left(2+3x^2+x^4\right)^2}\,dx$$

$$= -\frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{x(1771+999x^2)}{128\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32-88x^2+184x^4+\frac{681x^6}{4}}{x^6\left(2+3x^2+x^4\right)}$$

$$= -\frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{x(1771+999x^2)}{128\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^6} - \frac{68}{x^4} + \frac{186}{x^2} + \frac{116}{1+x^2}\right.$$

$$= -\frac{1}{10x^5} + \frac{17}{24x^3} - \frac{93}{16x} - \frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{x(1771+999x^2)}{128\left(2+3x^2+x^4\right)} + \frac{29}{8}\int \frac{1}{1+}$$

$$= -\frac{1}{10x^5} + \frac{17}{24x^3} - \frac{93}{16x} - \frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{x(1771+999x^2)}{128\left(2+3x^2+x^4\right)} + \frac{29}{8}\tan^{-1}($$

Mathematica [A]
time = 0.05, size = 73, normalized size = 0.78

$$\frac{-\frac{2\left(768-3136x^2+30816x^4+170702x^6+246477x^8+137120x^{10}+26145x^{12}\right)}{x^5\left(2+3x^2+x^4\right)^2}+13920\tan^{-1}(x)-33105\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 +
26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/3840
```

Maple [A]
time = 0.04, size = 68, normalized size = 0.73

method	result	size

risch	$\frac{-\frac{1743}{128}x^{12}-\frac{857}{12}x^{10}-\frac{82159}{640}x^{8}-\frac{85351}{960}x^{6}-\frac{321}{20}x^{4}+\frac{49}{30}x^{2}-\frac{2}{5}}{x^{5}\left(x^{4}+3x^{2}+2\right)^{2}}-\frac{2207\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{256}+\frac{29\arctan(x)}{8}$	66
default	$\frac{-\frac{43}{8}x^{3}-\frac{45}{8}x}{\left(x^{2}+1\right)^{2}}+\frac{29\arctan(x)}{8}-\frac{1}{10x^{5}}+\frac{17}{24x^{3}}-\frac{93}{16x}-\frac{\frac{311}{8}x^{3}+\frac{337}{4}x}{16\left(x^{2}+2\right)^{2}}-\frac{2207\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{256}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-43/8*x^3-45/8*x)/(x^2+1)^2+29/8*arctan(x)-1/10/x^5+17/24/x^3-93/16/x-1/16
*(311/8*x^3+337/4*x)/(x^2+2)^2-2207/256*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 77, normalized size = 0.83

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{26145\,x^{12}+137120\,x^{10}+246477\,x^{8}+170702\,x^{6}+30816\,x^{4}-3136\,x^{2}+768}{1920\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)}+\frac{29}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 137120*x^10
+ 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^13 + 6*x^11 + 13
*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

Fricas [A]
time = 0.36, size = 124, normalized size = 1.33

$$-\frac{52290\,x^{12}+274240\,x^{10}+492954\,x^{8}+341404\,x^{6}+61632\,x^{4}+33105\,\sqrt{2}\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-6272\,x^{2}-13920\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)\arctan\left(x\right)+1536}{3840\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 + 3
3105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)*x
) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x) +
1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
```

Sympy [A]
time = 0.12, size = 82, normalized size = 0.88

$$\frac{29\operatorname{atan}\left(x\right)}{8}-\frac{2207\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256}+\frac{-26145x^{12}-137120x^{10}-246477x^{8}-170702x^{6}-30816x^{4}+3136x^{2}-768}{1920x^{13}+11520x^{11}+24960x^{9}+23040x^{7}+7680x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

```
[Out] 29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 + (-26145*x**12 - 137120*
x**10 - 246477*x**8 - 170702*x**6 - 30816*x**4 + 3136*x**2 - 768)/(1920*x**
13 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7680*x**5)
```

Giac [A]
time = 4.80, size = 67, normalized size = 0.72

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{999\,x^{7}+4768\,x^{5}+7291\,x^{3}+3554\,x}{128\,(x^{4}+3\,x^{2}+2)^{2}}-\frac{1395\,x^{4}-170\,x^{2}+24}{240\,x^{5}}+\frac{29}{8}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x^5 + 7291*
x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 + 2
9/8*arctan(x)
```

Mupad [B]
time = 0.93, size = 77, normalized size = 0.83

$$\frac{29\,\mathrm{atan}(x)}{8}-\frac{2207\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256}-\frac{\frac{1743\,x^{12}}{128}+\frac{857\,x^{10}}{12}+\frac{82159\,x^{8}}{640}+\frac{85351\,x^{6}}{960}+\frac{321\,x^{4}}{20}-\frac{49\,x^{2}}{30}+\frac{2}{5}}{x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] (29*atan(x))/8 - (2207*2^(1/2)*atan((2^(1/2)*x)/2))/256 - ((321*x^4)/20 - (
49*x^2)/30 + (85351*x^6)/960 + (82159*x^8)/640 + (857*x^10)/12 + (1743*x^12
)/128 + 2/5)/(4*x^5 + 12*x^7 + 13*x^9 + 6*x^11 + x^13)
```

3.100 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=86

$$19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)}+\frac{201\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] 19*x^2+19/4*x^4-17/6*x^6+5/8*x^8-25/8*(7*x^2+15)/(x^4+2*x^2+3)-183/4*ln(x^4
+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$\frac{201\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{25(7x^2+15)}{8(x^4+2x^2+3)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/(8*(3 + 2*
x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (183*Log[3 + 2*
x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{-150-400x+200x^2-56x^4+40x^5}{3+2x+x^2}\,dx,\right.$$

$$= -\frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(304+152x-136x^2+40x^3-\frac{6(177+}{3+2x}\right.\right.$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)}-\frac{3}{8}\text{Subst}\left(\int \frac{177+244x}{3+2x+x^2}\,d\right.$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)}+\frac{201}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\right.$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)}-\frac{183}{4}\log\left(3+2x^2+x^4\right)-$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)}+\frac{201\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.91

$$\frac{1}{48}\left(912x^2+228x^4-136x^6+30x^8-\frac{150(15+7x^2)}{3+2x^2+x^4}+603\sqrt{2}\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)-2196\log\left(3+2x^2+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^4
) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

Maple [A]
time = 0.03, size = 74, normalized size = 0.86

method	result	size
risch	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{-\frac{175x^2}{8}-\frac{375}{8}}{x^4+2x^2+3}-\frac{183\ln(x^4+2x^2+3)}{4}+\frac{201\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	71
default	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{\frac{175x^2}{4}+\frac{375}{4}}{2(x^4+2x^2+3)}-\frac{183\ln(x^4+2x^2+3)}{4}+\frac{201\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*x^8-17/6*x^6+19/4*x^4+19*x^2-1/2*(175/4*x^2+375/4)/(x^4+2*x^2+3)-183/4*
ln(x^4+2*x^2+3)+201/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A]
time = 0.53, size = 71, normalized size = 0.83

$$\frac{5}{8}\,x^{8}-\frac{17}{6}\,x^{6}+\frac{19}{4}\,x^{4}+19\,x^{2}+\frac{201}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^{2}+1\right)\right)-\frac{25\,\left(7\,x^{2}+15\right)}{8\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{183}{4}\,\log\left(x^{4}+2\,x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2 +
3)
```

Fricas [A]
time = 0.36, size = 95, normalized size = 1.10

$$\frac{30\,x^{12}-76\,x^{10}+46\,x^{8}+960\,x^{6}+2508\,x^{4}+603\,\sqrt{2}\,\left(x^{4}+2\,x^{2}+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^{2}+1\right)\right)+1686\,x^{2}-2196\,\left(x^{4}+2\,x^{2}+3\right)\log\left(x^{4}+2\,x^{2}+3\right)-2250}{48\,\left(x^{4}+2\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 +
2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 + 3
)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.07, size = 87, normalized size = 1.01

$$\frac{5x^{8}}{8}-\frac{17x^{6}}{6}+\frac{19x^{4}}{4}+19x^{2}+\frac{-175x^{2}-375}{8x^{4}+16x^{2}+24}-\frac{183\log\left(x^{4}+2x^{2}+3\right)}{4}+\frac{201\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 + (-175*x**2 - 375)/(8*x**4 + 16
*x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)*atan(sqrt(2)*x**2/
2 + sqrt(2)/2)/16
```

Giac [A]
time = 3.37, size = 76, normalized size = 0.88

$$\frac{5}{8}\,x^{8}-\frac{17}{6}\,x^{6}+\frac{19}{4}\,x^{4}+19\,x^{2}+\frac{201}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^{2}+1\right)\right)+\frac{366\,x^{4}+557\,x^{2}+723}{8\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{183}{4}\,\log\left(x^{4}+2\,x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(x^
4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.90, size = 75, normalized size = 0.87

$$\frac{201\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{\frac{175\,x^2}{8}+\frac{375}{8}}{x^4+2\,x^2+3}-\frac{183\,\ln\left(x^4+2\,x^2+3\right)}{4}+19\,x^2+\frac{19\,x^4}{4}-\frac{17\,x^6}{6}+\frac{5\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (201*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((175*x^2)/8 + 375/8)/
(2*x^2 + x^4 + 3) - (183*log(2*x^2 + x^4 + 3))/4 + 19*x^2 + (19*x^4)/4 - (1
7*x^6)/6 + (5*x^8)/8
```

3.101 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=81

$$\frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8(3+2x^2+x^4)}-\frac{455\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{2}\log\left(3+2x^2+x^4\right)$$

```
[Out] 19/2*x^2-17/4*x^4+5/6*x^6+25/8*(5*x^2+3)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2+3)
-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$-\frac{455\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{25(5x^2+3)}{8(x^4+2x^2+3)}+\frac{19}{2}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)
) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])
/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx,x,x^2\right)\\
&= \frac{25(3+5x^2)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{-150+200x-56x^3+40x^4}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25(3+5x^2)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(152-136x+40x^2-\frac{2(303-152x)}{3+2x+x^2}\right) d\right.\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{303-152x}{3+2x+x^2}\,dx,x,x\right.\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\left(3+2x^2+x^4\right)}+\frac{19}{2}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,\right.\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\left(3+2x^2+x^4\right)}+\frac{19}{2}\log\left(3+2x^2+x^4\right)+\frac{455}{4}\text{Sub}\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25(3+5x^2)}{8\left(3+2x^2+x^4\right)}-\frac{455\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{2}\log\left(3+\right.
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 0.90

$$
\frac{1}{48}\left(456x^2-204x^4+40x^6+\frac{150(3+5x^2)}{3+2x^2+x^4}-1365\sqrt{2}\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)+456\log\left(3+2x^2+x^4\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*Sq
rt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

Maple [A]
time = 0.03, size = 69, normalized size = 0.85

method	result	size
risch	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln\left(x^4+2x^2+3\right)}{2}-\frac{455\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	66
default	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/6*x^6-17/4*x^4+19/2*x^2+1/2*(125/4*x^2+75/4)/(x^4+2*x^2+3)+19/2*ln(x^4+2*
x^2+3)-455/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A]
time = 0.51, size = 66, normalized size = 0.81

$$\frac{5}{6}\,x^6-\frac{17}{4}\,x^4+\frac{19}{2}\,x^2-\frac{455}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^2+1\right)\right)+\frac{25\,\left(5\,x^2+3\right)}{8\,\left(x^4+2\,x^2+3\right)}+\frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)
```

Fricas [A]
time = 0.37, size = 90, normalized size = 1.11

$$\frac{40\,x^{10}-124\,x^8+168\,x^6+300\,x^4-1365\,\sqrt{2}\,\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^2+1\right)\right)+2118\,x^2+456\,\left(x^4+2\,x^2+3\right)\log\left(x^4+2\,x^2+3\right)+450}{48\,\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 + 3
)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^4
+ 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.07, size = 80, normalized size = 0.99

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{125x^2+75}{8x^4+16x^2+24}+\frac{19\log\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x**2 + 24)
+ 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/16
```

Giac [A]
time = 3.72, size = 71, normalized size = 0.88

$$\frac{5}{6}\,x^6-\frac{17}{4}\,x^4+\frac{19}{2}\,x^2-\frac{455}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^2+1\right)\right)-\frac{76\,x^4+27\,x^2+153}{8\,\left(x^4+2\,x^2+3\right)}+\frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 +
3)
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.85

$$\frac{19\,\ln\left(x^4+2\,x^2+3\right)}{2}+\frac{\frac{125\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3}-\frac{455\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}+\frac{19\,x^2}{2}-\frac{17\,x^4}{4}+\frac{5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (19*log(2*x^2 + x^4 + 3))/2 + ((125*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (455
*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 + (19*x^2)/2 - (17*x^4)/4 +
(5*x^6)/6
```

3.102 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=74

$$-\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25(3-x^2)}{8(3+2x^2+x^4)}+\frac{203\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] -17/2*x^2+5/4*x^4+25/8*(-x^2+3)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+203/16*a
rctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$\frac{203\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{25(3-x^2)}{8(x^4+2x^2+3)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (203*ArcTa
n[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx, x, x^2\right)$$
$$= \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{150-56x^2+40x^3}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(-136+40x+\frac{2(279+76x)}{3+2x+x^2}\right)\,dx, x, x^2\right)$$
$$= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{8}\text{Subst}\left(\int \frac{279+76x}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{19}{4}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right) + \frac{2}{-}$$
$$= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{19}{4}\log\left(3+2x^2+x^4\right) - \frac{203}{4}\text{Subst}\left(\int \frac{-}{-}\right.$$
$$= -\frac{17x^2}{2} + \frac{5x^4}{4} + \frac{25(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{203\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{19}{4}\log\left(3+2x^2+\right.$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.89

$$\frac{1}{16}\left(-136x^2+20x^4-\frac{50(-3+x^2)}{3+2x^2+x^4}+203\sqrt{2}\,\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)+76\log\left(3+2x^2+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt[2]*ArcTan
[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

Maple [A]
time = 0.04, size = 64, normalized size = 0.86

method	result	size
risch	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{289}{20}+\frac{-\frac{25x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln(x^4+2x^2+3)}{4}+\frac{203\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	62
default	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{-\frac{25x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln(x^4+2x^2+3)}{4}+\frac{203\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x^4-17/2*x^2+1/2*(-25/4*x^2+75/4)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+20
3/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A]
time = 0.51, size = 59, normalized size = 0.80

$$\frac{5}{4}\,x^4-\frac{17}{2}\,x^2+\frac{203}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^2+1\right)\right)-\frac{25\,\left(x^2-3\right)}{8\,\left(x^4+2\,x^2+3\right)}+\frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x
^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A]
time = 0.37, size = 85, normalized size = 1.15

$$\frac{20\,x^8-96\,x^6-212\,x^4+203\,\sqrt{2}\,\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^2+1\right)\right)-458\,x^2+76\,\left(x^4+2\,x^2+3\right)\log\left(x^4+2\,x^2+3\right)+150}{16\,\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*
sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) +
150)/(x^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.07, size = 73, normalized size = 0.99

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{75-25x^2}{8x^4+16x^2+24}+\frac{19\log\left(x^4+2x^2+3\right)}{4}+\frac{203\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**4/4 - 17*x**2/2 + (75 - 25*x**2)/(8*x**4 + 16*x**2 + 24) + 19*log(x**4
 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Giac [A]
time = 3.09, size = 66, normalized size = 0.89

$$\frac{5}{4}\,x^4-\frac{17}{2}\,x^2+\frac{203}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^2+1\right)\right)-\frac{38\,x^4+101\,x^2+39}{8\,\left(x^4+2\,x^2+3\right)}+\frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(38
*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.05, size = 65, normalized size = 0.88

$$\frac{19\,\ln\left(x^4+2\,x^2+3\right)}{4}-\frac{\frac{25\,x^2}{8}-\frac{75}{8}}{x^4+2\,x^2+3}+\frac{203\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{17\,x^2}{2}+\frac{5\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (19*log(2*x^2 + x^4 + 3))/4 - ((25*x^2)/8 - 75/8)/(2*x^2 + x^4 + 3) + (203*
2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - (17*x^2)/2 + (5*x^4)/4
```

3.103 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx$

Optimal. Leaf size=65

$$\frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{17}{4}\log\left(3+2x^2+x^4\right)$$

```
[Out] 5/2*x^2-25/8*(x^2+3)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1/2*(x
^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1674, 1671, 648, 632, 210, 642}

$$-\frac{17\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5x^2}{2} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1 + x^2)/Sqr
t[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(4+x+3x^2+5x^3)}{(3+2x+x^2)^2}\,dx,x,x^2\right)\\
&= -\frac{25(3+x^2)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{-50-56x+40x^2}{3+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{25(3+x^2)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(40-\frac{34(5+4x)}{3+2x+x^2}\right)\,dx,x,x^2\right)\\
&= \frac{5x^2}{2}-\frac{25(3+x^2)}{8\left(3+2x^2+x^4\right)}-\frac{17}{8}\text{Subst}\left(\int \frac{5+4x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{5x^2}{2}-\frac{25(3+x^2)}{8\left(3+2x^2+x^4\right)}-\frac{17}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx,x,x^2\right)-\frac{17}{4}\text{Subst}\left(\right.\\
&= \frac{5x^2}{2}-\frac{25(3+x^2)}{8\left(3+2x^2+x^4\right)}-\frac{17}{4}\log\left(3+2x^2+x^4\right)+\frac{17}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,d\right.\\
&= \frac{5x^2}{2}-\frac{25(3+x^2)}{8\left(3+2x^2+x^4\right)}-\frac{17\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{17}{4}\log\left(3+2x^2+x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.94

$$\frac{1}{16}\left(40x^2-\frac{50(3+x^2)}{3+2x^2+x^4}-17\sqrt{2}\,\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)-68\log\left(3+2x^2+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/Sq
rt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

Maple [A]
time = 0.03, size = 59, normalized size = 0.91

method	result	size
risch	$\frac{5x^2}{2}+\frac{-\frac{25x^2}{8}-\frac{75}{8}}{x^4+2x^2+3}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	56
default	$\frac{5x^2}{2}-\frac{\frac{25x^2}{4}+\frac{75}{4}}{2\left(x^4+2x^2+3\right)}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*x^2-1/2*(25/4*x^2+75/4)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*2^(1/2
)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A]
time = 0.51, size = 54, normalized size = 0.83

$$\frac{5}{2}\,x^{2}-\frac{17}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^{2}+1\right)\right)-\frac{25\,\left(x^{2}+3\right)}{8\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{17}{4}\,\log\left(x^{4}+2\,x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A]
time = 0.37, size = 80, normalized size = 1.23

$$\frac{40\,x^{6}+80\,x^{4}-17\,\sqrt{2}\,\left(x^{4}+2\,x^{2}+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^{2}+1\right)\right)+70\,x^{2}-68\,\left(x^{4}+2\,x^{2}+3\right)\log\left(x^{4}+2\,x^{2}+3\right)-150}{16\,\left(x^{4}+2\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^
2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4 +
2*x^2 + 3)
```

Sympy [A]
time = 0.07, size = 68, normalized size = 1.05

$$\frac{5x^{2}}{2}+\frac{-25x^{2}-75}{8x^{4}+16x^{2}+24}-\frac{17\log\left(x^{4}+2x^{2}+3\right)}{4}-\frac{17\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**2/2 + (-25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 + 2*x**2 +
 3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Giac [A]
time = 3.31, size = 54, normalized size = 0.83

$$\frac{5}{2}\,x^{2}-\frac{17}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(x^{2}+1\right)\right)-\frac{25\,\left(x^{2}+3\right)}{8\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{17}{4}\,\log\left(x^{4}+2\,x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.92, size = 60, normalized size = 0.92

$$\frac{5\,x^2}{2}-\frac{\frac{25\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3}-\frac{17\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{17\,\ln\left(x^4+2\,x^2+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (5*x^2)/2 - ((25*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (17*2^(1/2)*atan(2^(1/2
)/2 + (2^(1/2)*x^2)/2))/16 - (17*log(2*x^2 + x^4 + 3))/4
```

3.104 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=58

$$\frac{25(1+x^2)}{8(3+2x^2+x^4)} - \frac{23\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4}\log\left(3+2x^2+x^4\right)$$

[Out] 25/8*(x^2+1)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1677, 1674, 648, 632, 210, 642}

$$-\frac{23\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{25(x^2+1)}{8(x^4+2x^2+3)} + \frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]

[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{4+x+3x^2+5x^3}{(3+2x+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{25(1+x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{16}\mathrm{Subst}\left(\int \frac{-6+40x}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{25(1+x^2)}{8\,(3+2x^2+x^4)} + \frac{5}{4}\mathrm{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right) - \frac{23}{8}\mathrm{Subst}\left(\int \frac{}{3+2\cdots}\right. \\
&= \frac{25(1+x^2)}{8\,(3+2x^2+x^4)} + \frac{5}{4}\log\left(3+2x^2+x^4\right) + \frac{23}{4}\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2(1\cdots\right. \\
&= \frac{25(1+x^2)}{8\,(3+2x^2+x^4)} - \frac{23\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 1.00

$$\frac{25(1+x^2)}{8\,(3+2x^2+x^4)} - \frac{23\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4}\log\left(3+2x^2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Maple [A]
time = 0.02, size = 54, normalized size = 0.93

method	result	size
risch	$\frac{\frac{25x^2}{8}+\frac{25}{8}}{x^4+2x^2+3}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	51
default	$\frac{\frac{25x^2}{4}+\frac{25}{4}}{2x^4+4x^2+6}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(25/4*x^2+25/4)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*2^(1/2)*arctan(
1/4*(2*x^2+2)*2^(1/2))
```

Maxima [A]
time = 0.50, size = 49, normalized size = 0.84

$$-\frac{23}{16}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{5}{4}\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Fricas [A]
time = 0.36, size = 70, normalized size = 1.21

$$-\frac{23\sqrt{2}\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-50\,x^2-20\left(x^4+2\,x^2+3\right)\log\left(x^4+2\,x^2+3\right)-50}{16\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(23*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 50*x^2
- 20*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.07, size = 60, normalized size = 1.03

$$\frac{25x^2 + 25}{8x^4 + 16x^2 + 24} + \frac{5\log(x^4 + 2x^2 + 3)}{4} - \frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)/4 - 23*sq
rt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Giac [A]
time = 4.28, size = 49, normalized size = 0.84

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{25(x^2+1)}{8(x^4+2x^2+3)} + \frac{5}{4}\log(x^4+2x^2+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.05, size = 69, normalized size = 1.19

$$\frac{5\ln(x^4+2x^2+3)}{4} + \frac{25x^2}{8(x^4+2x^2+3)} + \frac{25}{8(x^4+2x^2+3)} - \frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (5*log(2*x^2 + x^4 + 3))/4 + (25*x^2)/(8*(2*x^2 + x^4 + 3)) + 25/(8*(2*x^2
+ x^4 + 3)) - (23*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16
```

3.105 $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} \, dx$

Optimal. Leaf size=66

$$\frac{25(1-x^2)}{24(3+2x^2+x^4)}+\frac{89\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)$$

```
[Out] 25/24*(-x^2+1)/(x^4+2*x^2+3)+4/9*ln(x)-1/9*ln(x^4+2*x^2+3)+89/144*arctan(1/
2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$,
Rules used = {1677, 1660, 814, 648, 632, 210, 642}

$$\frac{89\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{25(1-x^2)}{24(x^4+2x^2+3)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt[2]])/(72*
Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x(3+2x+x^2)^2}\,dx,x,x^2\right)$$
$$= \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}+\frac{70x}{3}}{x(3+2x+x^2)}\,dx,x,x^2\right)$$
$$= \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x}-\frac{2(-73+16x)}{9(3+2x+x^2)}\right)dx,x,x^2\right)$$
$$= \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{4\log(x)}{9} - \frac{1}{72}\text{Subst}\left(\int \frac{-73+16x}{3+2x+x^2}\,dx,x,x^2\right)$$
$$= \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{4\log(x)}{9} - \frac{1}{9}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right) + \frac{89}{72}\text{Subst}\Big($$
$$= \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{4\log(x)}{9} - \frac{1}{9}\log(3+2x^2+x^4) - \frac{89}{36}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx\right.$$
$$= \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{89\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{4\log(x)}{9} - \frac{1}{9}\log(3+2x^2+x^4)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 93, normalized size = 1.41

$$\frac{1}{288}\left(-\frac{300(-1+x^2)}{3+2x^2+x^4}+128\log(x)-\sqrt{2}\left(89i+16\sqrt{2}\right)\log\left(1-i\sqrt{2}+x^2\right)+\sqrt{2}\left(89i-16\sqrt{2}\right)\log\left(1+i\sqrt{2}+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-300*(-1 + x^2))/(3 + 2*x^2 + x^4) + 128*Log[x] - Sqrt[2]*(89*I + 16*Sqrt
[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(89*I - 16*Sqrt[2])*Log[1 + I*Sqrt[
2] + x^2])/288
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.88

method	result	size
default	$-\frac{\frac{75x^2}{4}-\frac{75}{4}}{18(x^4+2x^2+3)}-\frac{\ln(x^4+2x^2+3)}{9}+\frac{89\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{144}+\frac{4\ln(x)}{9}$	58
risch	$\frac{-\frac{25x^2}{24}+\frac{25}{24}}{x^4+2x^2+3}+\frac{4\ln(x)}{9}-\frac{\ln(7921x^4+15842x^2+23763)}{9}+\frac{89\sqrt{2}\,\arctan\left(\frac{(89x^2+89)\sqrt{2}}{178}\right)}{144}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^(1/2)*arct
an(1/4*(2*x^2+2)*2^(1/2))+4/9*ln(x)
```

Maxima [**A**]
time = 0.49, size = 55, normalized size = 0.83

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{25\left(x^2-1\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{2}{9}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^2
 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Fricas [**A**]
time = 0.37, size = 84, normalized size = 1.27

$$\frac{89\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-150x^2-16\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right)+64\left(x^4+2x^2+3\right)\log\left(x\right)+150}{144\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^2
 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x)
+ 150)/(x^4 + 2*x^2 + 3)
```

Sympy [**A**]
time = 0.07, size = 65, normalized size = 0.98

$$\frac{25-25x^2}{24x^4+48x^2+72}+\frac{4\log\left(x\right)}{9}-\frac{\log\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25 - 25*x**2)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4 + 2*x**2 +
3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

Giac [**A**]
time = 5.42, size = 62, normalized size = 0.94

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{8x^4-59x^2+99}{72\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{2}{9}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/(
x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Mupad [B]
time = 0.91, size = 59, normalized size = 0.89

$$\frac{4\,\ln\left(x\right)}{9}-\frac{\ln\left(x^4+2\,x^2+3\right)}{9}-\frac{\frac{25\,x^2}{24}-\frac{25}{24}}{x^4+2\,x^2+3}+\frac{89\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (4*log(x))/9 - log(2*x^2 + x^4 + 3)/9 - ((25*x^2)/24 - 25/24)/(2*x^2 + x^4
+ 3) + (89*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/144
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{2}{9x^2}-\frac{25(5+x^2)}{72(3+2x^2+x^4)}-\frac{71\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

```
[Out] -2/9/x^2-25/72*(x^2+5)/(x^4+2*x^2+3)-13/27*ln(x)+13/108*ln(x^4+2*x^2+3)-71/
432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1660, 1642, 648, 632, 210, 642}

$$-\frac{71\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{2}{9x^2}-\frac{25(x^2+5)}{72(x^4+2x^2+3)}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(1 + x^2)/S
qrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2*x^2 + x^4])/108
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2(3+2x+x^2)^2}\,dx, x, x^2\right)$$
$$= -\frac{25(5+x^2)}{72(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}-\frac{50x^2}{9}}{x^2(3+2x+x^2)}\,dx, x, x^2\right)$$
$$= -\frac{25(5+x^2)}{72(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^2}-\frac{104}{27x}+\frac{2(-19+52x)}{27(3+2x+x^2)}\right)\,dx, x, x^2\right)$$
$$= -\frac{2}{9x^2} - \frac{25(5+x^2)}{72(3+2x^2+x^4)} - \frac{13\log(x)}{27} + \frac{1}{216}\text{Subst}\left(\int \frac{-19+52x}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{2}{9x^2} - \frac{25(5+x^2)}{72(3+2x^2+x^4)} - \frac{13\log(x)}{27} + \frac{13}{108}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{2}{9x^2} - \frac{25(5+x^2)}{72(3+2x^2+x^4)} - \frac{13\log(x)}{27} + \frac{13}{108}\log\left(3+2x^2+x^4\right) + \frac{71}{108}\text{Subst}\Big($$
$$= -\frac{2}{9x^2} - \frac{25(5+x^2)}{72(3+2x^2+x^4)} - \frac{71\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}} - \frac{13\log(x)}{27} + \frac{13}{108}\log\left(3+2x^2\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 97, normalized size = 1.37

$$\frac{1}{864}\left(-\frac{192}{x^2}-\frac{300(5+x^2)}{3+2x^2+x^4}-416\log(x)+\sqrt{2}\left(71i+52\sqrt{2}\right)\log\left(1-i\sqrt{2}+x^2\right)+\sqrt{2}\left(-71i+52\sqrt{2}\right)\log\left(1+i\sqrt{2}+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
[Out] (-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(71*I
+ 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(-71*I + 52*Sqrt[2])*Log[1
 + I*Sqrt[2] + x^2])/864
```

Maple [A]
time = 0.03, size = 63, normalized size = 0.89

method	result	size
default	$\frac{-\frac{75x^2}{4}-\frac{375}{4}}{54x^4+108x^2+162}+\frac{13\ln(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}-\frac{2}{9x^2}-\frac{13\ln(x)}{27}$	63
risch	$\frac{-\frac{41}{72}x^4-\frac{157}{72}x^2-\frac{2}{3}}{x^2(x^4+2x^2+3)}-\frac{13\ln(x)}{27}+\frac{13\ln(5041x^4+10082x^2+15123)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(71x^2+71)\sqrt{2}}{142}\right)}{432}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*x^2+3)-71/432*2^(1/2)*
arctan(1/4*(2*x^2+2)*2^(1/2))-2/9/x^2-13/27*ln(x)
```

Maxima [A]
time = 0.51, size = 66, normalized size = 0.93

$$-\frac{71}{432}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,(x^2+1)\right)-\frac{41\,x^4+157\,x^2+48}{72\,(x^6+2\,x^4+3\,x^2)}+\frac{13}{108}\log\left(x^4+2\,x^2+3\right)-\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Fricas [A]
time = 0.36, size = 105, normalized size = 1.48

$$-\frac{246\,x^4+71\,\sqrt{2}\,(x^6+2\,x^4+3\,x^2)\arctan\left(\frac{1}{2}\sqrt{2}\,(x^2+1)\right)+942\,x^2-52\,(x^6+2\,x^4+3\,x^2)\log\left(x^4+2\,x^2+3\right)+208\,(x^6+2\,x^4+3\,x^2)\log\left(x\right)+288}{432\,(x^6+2\,x^4+3\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2
+ 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^6
+ 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)
```

Sympy [A]
time = 0.08, size = 76, normalized size = 1.07

$$\frac{-41x^4-157x^2-48}{72x^6+144x^4+216x^2}-\frac{13\log\left(x\right)}{27}+\frac{13\log\left(x^4+2x^2+3\right)}{108}-\frac{71\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-41*x**4 - 157*x**2 - 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*log(x)/27 +
 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/432
```

Giac [A]
time = 5.38, size = 66, normalized size = 0.93

$$-\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{41\,x^4+157\,x^2+48}{72\left(x^6+2\,x^4+3\,x^2\right)}+\frac{13}{108}\log\left(x^4+2\,x^2+3\right)-\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Mupad [B]
time = 0.06, size = 68, normalized size = 0.96

$$\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}-\frac{13\,\ln\left(x\right)}{27}-\frac{\frac{41\,x^4}{72}+\frac{157\,x^2}{72}+\frac{2}{3}}{x^6+2\,x^4+3\,x^2}-\frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (13*log(2*x^2 + x^4 + 3))/108 - (13*log(x))/27 - ((157*x^2)/72 + (41*x^4)/7
2 + 2/3)/(3*x^2 + 2*x^4 + x^6) - (71*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)
/2))/432
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2} dx$

Optimal. Leaf size=80

$$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{125\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

```
[Out] -1/9/x^4+13/54/x^2+25/216*(5*x^2+7)/(x^4+2*x^2+3)+13/27*ln(x)-13/108*ln(x^4
+2*x^2+3)+125/432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1660, 1642, 648, 632, 210, 642}

$$\frac{125\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(5x^2+7)}{216\left(x^4+2x^2+3\right)}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -1/9*1/x^4 + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (125*
ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/27 - (13*Log[3 + 2*x
^2 + x^4])/108
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3(3+2x+x^2)^2}\,dx,x,x^2\right)$$
$$= \frac{25(7+5x^2)}{216(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{250x^3}{27}}{x^3(3+2x+x^2)}\,dx,x,x^2\right)$$
$$= \frac{25(7+5x^2)}{216(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^3}-\frac{104}{27x^2}+\frac{104}{27x}-\frac{2(-73+52x)}{27(3+2x+x^2)}\right) dx\right.$$
$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216(3+2x^2+x^4)}+\frac{13\log(x)}{27}-\frac{1}{216}\text{Subst}\left(\int \frac{-73+52x}{3+2x+x^2}\,dx\right.$$
$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216(3+2x^2+x^4)}+\frac{13\log(x)}{27}-\frac{13}{108}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx\right.$$
$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216(3+2x^2+x^4)}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)-\frac{125}{108}$$
$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216(3+2x^2+x^4)}+\frac{125\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 105, normalized size = 1.31

$$\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}+\frac{100(7+5x^2)}{3+2x^2+x^4}+416\log(x)-\sqrt{2}\left(125i+52\sqrt{2}\right)\log\left(1-i\sqrt{2}+x^2\right)+\sqrt{2}\left(125i-52\sqrt{2}\right)\log\left(1+i\sqrt{2}+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^4 + 208/x^2 + (100*(7 + 5*x^2))/(3 + 2*x^2 + x^4) + 416*Log[x] - Sqr
t[2]*(125*I + 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(125*I - 52*Sq
rt[2])*Log[1 + I*Sqrt[2] + x^2])/864
```

Maple [A]
time = 0.03, size = 68, normalized size = 0.85

method	result	size
default	$-\frac{-\frac{125x^2}{4}-\frac{175}{4}}{54(x^4+2x^2+3)}-\frac{13\ln(x^4+2x^2+3)}{108}+\frac{125\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{13\ln(x)}{27}$	68
risch	$\frac{\frac{59}{72}x^6+\frac{85}{72}x^4+\frac{1}{2}x^2-\frac{1}{3}}{x^4(x^4+2x^2+3)}+\frac{13\ln(x)}{27}-\frac{13\ln(15625x^4+31250x^2+46875)}{108}+\frac{125\sqrt{2}\arctan\left(\frac{(125x^2+125)\sqrt{2}}{250}\right)}{432}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/108*ln(x^4+2*x^2+3)+125/432*2^(1/
2)*arctan(1/4*(2*x^2+2)*2^(1/2))-1/9/x^4+13/54/x^2+13/27*ln(x)
```

Maxima [A]
time = 0.51, size = 71, normalized size = 0.89

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{59\,x^6+85\,x^4+36\,x^2-24}{72\left(x^8+2\,x^6+3\,x^4\right)}-\frac{13}{108}\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36*
x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log(x
^2)
```

Fricas [A]
time = 0.36, size = 110, normalized size = 1.38

$$\frac{354\,x^6+510\,x^4+125\,\sqrt{2}\left(x^8+2\,x^6+3\,x^4\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+216\,x^2-52\left(x^8+2\,x^6+3\,x^4\right)\log\left(x^4+2\,x^2+3\right)+208\left(x^8+2\,x^6+3\,x^4\right)\log\left(x\right)-144}{432\left(x^8+2\,x^6+3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sqr
t(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3) +
 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)
```

Sympy [A]
time = 0.09, size = 80, normalized size = 1.00

$$\frac{13\log\left(x\right)}{27}-\frac{13\log\left(x^4+2x^2+3\right)}{108}+\frac{125\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}+\frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

```
[Out] 13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*x*
*6 + 216*x**4)
```

Giac [A]
time = 4.30, size = 79, normalized size = 0.99

$$\frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^{2}+1\right)\right)+\frac{26\,x^{4}+177\,x^{2}+253}{216\left(x^{4}+2\,x^{2}+3\right)}-\frac{39\,x^{4}-26\,x^{2}+12}{108\,x^{4}}-\frac{13}{108}\log\left(x^{4}+2\,x^{2}+3\right)+\frac{13}{54}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 + 2
53)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4 +
 2*x^2 + 3) + 13/54*log(x^2)
```

Mupad [B]
time = 0.06, size = 72, normalized size = 0.90

$$\frac{13\,\ln\left(x\right)}{27}-\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}+\frac{\frac{59\,x^6}{72}+\frac{85\,x^4}{72}+\frac{x^2}{2}-\frac{1}{3}}{x^8+2\,x^6+3\,x^4}+\frac{125\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (13*log(x))/27 - (13*log(2*x^2 + x^4 + 3))/108 + (x^2/2 + (85*x^4)/72 + (59
*x^6)/72 - 1/3)/(3*x^4 + 2*x^6 + x^8) + (125*2^(1/2)*atan(2^(1/2)/2 + (2^(1
/2)*x^2)/2))/432
```

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=87

$$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}-\frac{1237\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x^4\right)$$

```
[Out] -2/27/x^6+13/108/x^4-13/54/x^2+25/648*(-7*x^2+1)/(x^4+2*x^2+3)+61/243*ln(x)
-61/972*ln(x^4+2*x^2+3)-1237/3888*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1677, 1660, 1642, 648, 632, 210, 642}

$$-\frac{1237\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(x^4+2x^2+3)}-\frac{61}{972}\log\left(x^4+2x^2+3\right)+\frac{61\log(x)}{243}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*(3 + 2*x^2
 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[2]) + (61*Log[x])/24
3 - (61*Log[3 + 2*x^2 + x^4])/972
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^4(3+2x+x^2)^2}\,dx, x, x^2\right)$$
$$= \frac{25(1-7x^2)}{648(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{800x^3}{81}-\frac{350x^4}{81}}{x^4(3+2x+x^2)}\,dx, x, x^2\right)$$
$$= \frac{25(1-7x^2)}{648(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^4}-\frac{104}{27x^3}+\frac{104}{27x^2}+\frac{488}{243x}-\frac{2(1481+$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}+\frac{61\log(x)}{243}-\frac{\text{Subst}\left(\int \frac{1481+244x}{3+2x+x^2}\right.}{1944}$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}+\frac{61\log(x)}{243}-\frac{61}{972}\text{Subst}\left(\int \frac{}{3+}\right.$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2\right.$$
$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)}-\frac{1237\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(}{243}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 110, normalized size = 1.26

$$\frac{-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}-\frac{300(-1+7x^2)}{3+2x^2+x^4}+1952\log(x)+\sqrt{2}\left(1237i-244\sqrt{2}\right)\log\left(1-i\sqrt{2}+x^2\right)-\sqrt{2}\left(1237i+244\sqrt{2}\right)\log\left(1+i\sqrt{2}+x^2\right)}{7776}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 195
2*Log[x] + Sqrt[2]*(1237*I - 244*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] - Sqrt[2
]*(1237*I + 244*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/7776
```

Maple [A]
time = 0.03, size = 73, normalized size = 0.84

method	result
default	$-\frac{\frac{525x^2}{4}-\frac{75}{4}}{486(x^4+2x^2+3)}-\frac{61\ln(x^4+2x^2+3)}{972}-\frac{1237\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{3888}-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{61\ln(x)}{243}$
risch	$\frac{-\frac{331}{648}x^8-\frac{209}{648}x^6-\frac{5}{9}x^4+\frac{23}{108}x^2-\frac{2}{9}}{x^6(x^4+2x^2+3)}+\frac{61\ln(x)}{243}-\frac{61\ln(1530169x^4+3060338x^2+4590507)}{972}-\frac{1237\sqrt{2}\arctan\left(\frac{(1237x^2+1237)\sqrt{}}{2474}\right.}{3888}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/486*(525/4*x^2-75/4)/(x^4+2*x^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1
/2)*arctan(1/4*(2*x^2+2)*2^(1/2))-2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x
)
```

Maxima [A]
time = 0.51, size = 76, normalized size = 0.87

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{331\,x^8+209\,x^6+360\,x^4-138\,x^2+144}{648\,(x^{10}+2\,x^8+3\,x^6)}-\frac{61}{972}\log\left(x^4+2\,x^2+3\right)+\frac{61}{486}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^6
 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x^2
 + 3) + 61/486*log(x^2)
```

Fricas [A]
time = 0.37, size = 115, normalized size = 1.32

$$-\frac{1986\,x^8+1254\,x^6+2160\,x^4+1237\,\sqrt{2}\,(x^{10}+2\,x^8+3\,x^6)\arctan\left(\frac{1}{2}\,\sqrt{2}\,(x^2+1)\right)-828\,x^2+244\,(x^{10}+2\,x^8+3\,x^6)\log\left(x^4+2\,x^2+3\right)-976\,(x^{10}+2\,x^8+3\,x^6)\log\left(x\right)+864}{3888\,(x^{10}+2\,x^8+3\,x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x^
6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*log
(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x^8
+ 3*x^6)
```

Sympy [A]
time = 0.10, size = 85, normalized size = 0.98

$$\frac{61\log\left(x\right)}{243}-\frac{61\log\left(x^4+2x^2+3\right)}{972}-\frac{1237\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}+\frac{-331x^8-209x^6-360x^4+138x^2-144}{648x^{10}+1296x^8+1944x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

```
[Out] 61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*x
**2/2 + sqrt(2)/2)/3888 + (-331*x**8 - 209*x**6 - 360*x**4 + 138*x**2 - 144
)/(648*x**10 + 1296*x**8 + 1944*x**6)
```

Giac [A]
time = 4.09, size = 84, normalized size = 0.97

$$-\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{122\,x^4-281\,x^2+441}{1944\left(x^4+2\,x^2+3\right)}-\frac{671\,x^6+702\,x^4-351\,x^2+216}{2916\,x^6}-\frac{61}{972}\log\left(x^4+2\,x^2+3\right)+\frac{61}{486}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x^
2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x^6
 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

Mupad [B]
time = 0.07, size = 78, normalized size = 0.90

$$\frac{61\,\ln\left(x\right)}{243}-\frac{61\,\ln\left(x^4+2\,x^2+3\right)}{972}-\frac{\frac{331\,x^8}{648}+\frac{209\,x^6}{648}+\frac{5\,x^4}{9}-\frac{23\,x^2}{108}+\frac{2}{9}}{x^{10}+2\,x^8+3\,x^6}-\frac{1237\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^7*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (61*log(x))/243 - (61*log(2*x^2 + x^4 + 3))/972 - ((5*x^4)/9 - (23*x^2)/108
 + (209*x^6)/648 + (331*x^8)/648 + 2/9)/(3*x^6 + 2*x^8 + x^10) - (1237*2^(1
/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/3888
```

3.109 $\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=248

$$38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.$$

```
[Out] 38*x+19/3*x^3-17/5*x^5+5/7*x^7+25/8*x*(5*x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/64*ln(x^2+
3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/32*arctan
((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1236582*3^(1/2))^
(1/2)-1/32*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1
236582*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{25(5x^2+3)\,x}{8\left(x^4+2x^2+3\right)}+38x$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(262771 + 618291*Sqrt[3])/2]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-
262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3]
)]*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \frac{-450-1650x^2+1200x^4-336x^8+240x^{10}}{3+2x^2+x^4}\,dx$$

$$= \frac{25x(3+5x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \left(1824+912x^2-816x^4+240x^6-\frac{6(987+1339x}{3+2x^2+x^4}\right.$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}-\frac{1}{8}\int \frac{987+1339x^2}{3+2x^2+x^4}\,dx$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}-\frac{\int \frac{987\sqrt{2\left(-1+\sqrt{3}\right)}-\left(98\right.}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right.}}}{16\sqrt{6\left(-1+\sqrt{\right.}}}$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}-\frac{1}{32}\left(1339+329\sqrt{3}\right)\int$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+61829\right.}$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\right.}$$

__Mathematica__ [C] Result contains complex when optimal does not.
time = 0.11, size = 145, normalized size = 0.58

$$38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)}-\frac{\left(352i+1339\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(-352i+1339\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqr
t[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2
]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [A]
time = 0.09, size = 296, normalized size = 1.19

method	result
risch	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x+\frac{\frac{125}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-1339\_R^2-987\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{-\frac{125}{8}x^3-\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(-505\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-176\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/7*x^7-17/5*x^5+19/3*x^3+38*x-(-125/8*x^3-75/8*x)/(x^4+2*x^2+3)+1/64*(-505
*(-2+2*3^(1/2))^(1/2)*3^(1/2)-176*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))+1/16*(-658*3^(1/2)+1/2*(-505*(-2+2*3^(1/2))^(1/2)*3^(1/
2)-176*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arct
an((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-1/64*(-505*(-2+2*3^(1/2)
)^(1/2)*3^(1/2)-176*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(
1/2))-1/16*(658*3^(1/2)-1/2*(-505*(-2+2*3^(1/2))^(1/2)*3^(1/2)-176*(-2+2*3^
(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
 - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(175) = 350.
time = 0.38, size = 521, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/338902147590720*(242072962564800*x^11 - 668121376678848*x^9 + 56806455215
2064*x^7 + 13714240239171136*x^5 - 102773860*14158657803^(1/4)*sqrt(68699)*
sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262771*sqrt(3) + 1854873)*arctan(1/3
1450895547323130263119373 82*14158657803^(3/4)*sqrt(734099)*sqrt(206097)*sqr
t(68699)*sqrt(14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(2
62771*sqrt(3) + 1854873) + 151295601603*x^2 + 151295601603*sqrt(3))*(329*sq
rt(3)*sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(3) + 1854873) - 1/2078771306
9048994*14158657803^(3/4)*sqrt(68699)*(329*sqrt(3)*sqrt(2)*x - 1339*sqrt(2)
*x)*sqrt(262771*sqrt(3) + 1854873) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 1
02773860*14158657803^(1/4)*sqrt(68699)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sq
rt(262771*sqrt(3) + 1854873)*arctan(1/9435268664196939078935812146*14158657
803^(3/4)*sqrt(734099)*sqrt(68699)*sqrt(-1854873*14158657803^(1/4)*sqrt(686
99)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 2806341264321
61419*x^2 + 280634126432161419*sqrt(3))*(329*sqrt(3)*sqrt(2) - 1339*sqrt(2)
)*sqrt(262771*sqrt(3) + 1854873) - 1/20787713069048994*14158657803^(3/4)*sq
rt(68699)*(329*sqrt(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sqrt(262771*sqrt(3) + 18
54873) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 35*14158657803^(1/4)*sqrt(686
99)*(1854873*x^4 + 3709746*x^2 - 262771*sqrt(3)*(x^4 + 2*x^2 + 3) + 5564619
)*sqrt(262771*sqrt(3) + 1854873)*log(1854873/734099*14158657803^(1/4)*sqrt(
68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 3822837606
81*x^2 + 382283760681*sqrt(3)) - 35*14158657803^(1/4)*sqrt(68699)*(1854873*
x^4 + 3709746*x^2 - 262771*sqrt(3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771
*sqrt(3) + 1854873)*log(-1854873/734099*14158657803^(1/4)*sqrt(68699)*(1339
*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 382283760681*x^2 + 382
283760681*sqrt(3)) + 37491050077223400*x^3 + 41812052459005080*x)/(x^4 + 2*
x^2 + 3)
```

Sympy [A]
time = 0.33, size = 71, normalized size = 0.29

$$\frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3 + 75x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t\log\left(-\frac{16547840t^3}{453886804809} - \frac{11974973632t}{453886804809} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x*
*2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambda(
_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 + x))
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(175) = 350.
time = 3.74, size = 585, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*sq
rt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3
) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6*sq
rt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*3^(1
/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3
) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*
(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(
-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532
*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*s
qrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/41472*sqrt(2)*(24102*3^(3/4)*sq
rt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*
sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3
) + 1/2) + sqrt(3)) - 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 1339
*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 35532*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.11, size = 171, normalized size = 0.69

$$38\,x+\frac{\frac{125\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}+\frac{19\,x^3}{3}-\frac{17\,x^5}{5}+\frac{5\,x^7}{7}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771-\sqrt{2}\,734099{}\mathrm{i}}\,734099{}\mathrm{i}}{64\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713{}\mathrm{i}}{128}\right)}+\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771-\sqrt{2}\,734099{}\mathrm{i}}}{128\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713{}\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771-\sqrt{2}\,734099{}\mathrm{i}}\,1{}\mathrm{i}}{16}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771+\sqrt{2}\,734099{}\mathrm{i}}\,734099{}\mathrm{i}}{64\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713{}\mathrm{i}}{128}\right)}-\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771+\sqrt{2}\,734099{}\mathrm{i}}}{128\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713{}\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771+\sqrt{2}\,734099{}\mathrm{i}}\,1{}\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 38*x + (atan((x*(- 2^(1/2)*734099i - 262771)^(1/2)*734099i)/(64*((2^(1/2)*7
24555713i)/128 - 1112159985/64)) + (734099*2^(1/2)*x*(- 2^(1/2)*734099i - 2
62771)^(1/2))/(128*((2^(1/2)*724555713i)/128 - 1112159985/64)))*(- 2^(1/2)*
734099i - 262771)^(1/2)*1i)/16 - (atan((x*(2^(1/2)*734099i - 262771)^(1/2)*
734099i)/(64*((2^(1/2)*724555713i)/128 + 1112159985/64)) - (734099*2^(1/2)*
x*(2^(1/2)*734099i - 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 + 111215
9985/64)))*(2^(1/2)*734099i - 262771)^(1/2)*1i)/16 + ((75*x)/8 + (125*x^3)/
8)/(2*x^2 + x^4 + 3) + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7
```

3.110 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=237

$$19x-\frac{17x^3}{3}+x^5+\frac{25x(3-x^2)}{8(3+2x^2+x^4)}+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{16}\sqrt{\frac{3}{2}$$

```
[Out] 19*x-17/3*x^3+x^5+25/8*x*(-x^2+3)/(x^4+2*x^2+3)+3/32*arctan((-2*x+(-2+2*3^(
1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2)-3/32*arctan(
(2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2
)+3/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)-3
/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{16}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+x^5-\frac{17x^3}{3}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25(3-x^2)x}{8(x^4+2x^2+3)}+19x$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (3*Sqrt[
(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011
*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(
3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{48}\int \frac{-450+1050x^2-336x^6+240x^8}{3+2x^2+x^4}\,dx$$

$$= \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{1}{48}\int \left(912-816x^2+240x^4-\frac{54(59-31x^2)}{3+2x^2+x^4}\right) dx$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} - \frac{9}{8}\int \frac{59-31x^2}{3+2x^2+x^4}\,dx$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} - \frac{1}{32}\left(3\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{59\sqrt{2\left(-\right.}}{\sqrt{3}}$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} - \frac{1}{16}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\int$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}\right.$$

$$= 19x - \frac{17x^3}{3} + x^5 + \frac{25x(3-x^2)}{8\,(3+2x^2+x^4)} + \frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\tan^{-1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 132, normalized size = 0.56

$$19x - \frac{17x^3}{3} + x^5 - \frac{25x(-3+x^2)}{8\,(3+2x^2+x^4)} + \frac{9\left(90i+31\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{9\left(-90i+31\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90*
I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]])
 + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I
)*Sqrt[2]])
```

Maple [A]
time = 0.04, size = 288, normalized size = 1.22

method	result
risch	$x^5 - \frac{17x^3}{3} + 19x + \frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3} + \frac{9\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)} \frac{\left(31\_R^2-59\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$x^5 - \frac{17x^3}{3} + 19x + \frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3} + \frac{3\left(76\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+135\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\cdots\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)+3/64*(76*(-2+2*3^(1/2))^
(1/2)*3^(1/2)+135*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/
2))+3/16*(-118*3^(1/2)+1/2*(76*(-2+2*3^(1/2))^(1/2)*3^(1/2)+135*(-2+2*3^(1/
2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))+3/64*(-76*(-2+2*3^(1/2))^(1/2)*3^(1/2)-135
*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))+3/16*(-118*3^
(1/2)-1/2*(-76*(-2+2*3^(1/2))^(1/2)*3^(1/2)-135*(-2+2*3^(1/2))^(1/2))*(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*integrate(
(31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(166) = 332$.
time = 0.37, size = 483, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/287671488*(287671488*x^9 - 1054795456*x^7 + 3068495872*x^5 + 3588*6779732
67^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43440359*sqrt(3) + 7533036
```

```
3)*arctan(1/1822344999502852422*677973267^(3/4)*sqrt(15033)*sqrt(299)*sqrt(
4494867*x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3)
+ 75330363) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqrt(2))*sqrt(-4344
0359*sqrt(3) + 75330363) - 1/405428013666*677973267^(3/4)*(59*sqrt(3)*sqrt(
2)*x + 93*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) - 1/2*sqrt(3)*sqrt(
2) + 1/2*sqrt(2)) + 3588*677973267^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*
sqrt(-43440359*sqrt(3) + 75330363)*arctan(1/1822344999502852422*677973267^(
3/4)*sqrt(15033)*sqrt(299)*sqrt(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x
 + 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3))*(59*sqrt(3)*
sqrt(2) + 93*sqrt(2))*sqrt(-43440359*sqrt(3) + 75330363) - 1/405428013666*6
77973267^(3/4)*(59*sqrt(3)*sqrt(2)*x + 93*sqrt(2)*x)*sqrt(-43440359*sqrt(3)
 + 75330363) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 5142127848*x^3 - 3*6779
73267^(1/4)*(15033*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099
)*sqrt(-43440359*sqrt(3) + 75330363)*log(2033919801*x^2 + 135297/299*677973
267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 203391
9801*sqrt(3)) + 3*677973267^(1/4)*(15033*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^
4 + 2*x^2 + 3) + 45099)*sqrt(-43440359*sqrt(3) + 75330363)*log(2033919801*x
^2 - 135297/299*677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3
) + 75330363) + 2033919801*sqrt(3)) + 19094195016*x)/(x^4 + 2*x^2 + 3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(199) = 398$.
time = 0.73, size = 1205, normalized size = 5.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x**5 - 17*x**3/3 + 19*x + (-25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) - 3*sqr
t(26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-304*sqrt(2)*sqrt(8669 + 5
011*sqrt(3))/299 - 433349*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/1498289 + 152*s
qrt(3)*sqrt(8669 + 5011*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498289)
 - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521 -
993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993637694
9404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) + 3*sqrt(26007/2
048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-152*sqrt(3)*sqrt(8669 + 5011*sqrt(
3))*sqrt(43440359*sqrt(3) + 75240962)/1498289 + 433349*sqrt(6)*sqrt(8669 +
5011*sqrt(3))/1498289 + 304*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/299) - 288291
8249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521 - 993398584
*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 49936376949404567/2
244869927521 + 17261871038090*sqrt(3)/1343965233) - 2*sqrt(-27*sqrt(2)*sqrt
(43440359*sqrt(3) + 75240962)/1024 + 234063/2048 + 405891*sqrt(3)/2048)*ata
n(2996578*sqrt(3)*x/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) +
75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*s
```

```
qrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))) -
 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)*sqrt(-2*sqrt(2)*s
qrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(4344035
9*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8
669 + 15033*sqrt(3))) - 1300047*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/(17641*sq
rt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt
(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*
sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))) + 456*sqrt(8669 + 5011*sqrt(3)
)*sqrt(43440359*sqrt(3) + 75240962)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(434
40359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(
3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 1
5033*sqrt(3)))) - 2*sqrt(-27*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/1024
 + 234063/2048 + 405891*sqrt(3)/2048)*atan(2996578*sqrt(3)*x/(17641*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))
+ 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(
3) + 75240962) + 8669 + 15033*sqrt(3))) - 456*sqrt(8669 + 5011*sqrt(3))*sqr
t(43440359*sqrt(3) + 75240962)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359
*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) +
75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*
sqrt(3))) + 1300047*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)*sqrt(-
2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*s
qrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75
240962) + 8669 + 15033*sqrt(3))) + 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3)
)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 +
 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqr
t(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(166) = 332.
time = 3.01, size = 576, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] x^5 - 17/3*x^3 - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
+ 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 558*
3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqrt(3) +
 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
```

```
n(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2
)) - 1/4608*sqrt(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 31*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18
)^(3/2) + 558*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqr
t(2)*sqrt(-6*sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2
*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/4608*sqrt(2)*(558*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 31*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 21
24*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
1/2) + sqrt(3)) + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.94, size = 164, normalized size = 0.69

$$19\,x+\frac{\frac{75\,x}{8}-\frac{25\,x^3}{8}}{x^4+2\,x^2+3}-\frac{17\,x^3}{3}+x^5-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\,\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}-\frac{24219\,\sqrt{2}\,x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}}{128\,\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\,\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}+\frac{24219\,\sqrt{2}\,x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}}{128\,\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 19*x + ((75*x)/8 - (25*x^3)/8)/(2*x^2 + x^4 + 3) - (atan((x*(26007 - 2^(1/2
)*897i)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 - 1380483/16)) - (24219*2
^(1/2)*x*(26007 - 2^(1/2)*897i)^(1/2))/(128*((2^(1/2)*4286763i)/128 - 13804
83/16)))*(26007 - 2^(1/2)*897i)^(1/2)*3i)/16 + (atan((x*(2^(1/2)*897i + 260
07)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 + 1380483/16)) + (24219*2^(1/
2)*x*(2^(1/2)*897i + 26007)^(1/2))/(128*((2^(1/2)*4286763i)/128 + 1380483/1
6)))*(2^(1/2)*897i + 26007)^(1/2)*3i)/16 - (17*x^3)/3 + x^5
```

3.111 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=232

$$-17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{16}\sqrt{\frac{1}{2}\left(\right.}$$

```
[Out] -17*x+5/3*x^3-25/8*x*(x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1
/2))^(1/2))*(-28790+52998*3^(1/2))^(1/2)+1/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2
))^(1/2))*(-28790+52998*3^(1/2))^(1/2)-1/32*arctan((-2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)+1/32*arctan((2*x+(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{5x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{25(x^2+3)x}{8(x^4+2x^2+3)}-17x$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqrt[(14395 +
 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[S
qrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3
])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \frac{450-150x^2-336x^4+240x^6}{3+2x^2+x^4}\,dx$$

$$= -\frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \left(-816+240x^2+\frac{6(483+127x^2)}{3+2x^2+x^4}\right)dx$$

$$= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}+\frac{1}{8}\int \frac{483+127x^2}{3+2x^2+x^4}\,dx$$

$$= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}+\frac{\int \frac{483\sqrt{2\left(-1+\sqrt{3}\right)}-\left(483-127\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,d}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}+\frac{1}{32}\left(127+161\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1\right.}}$$

$$= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{}\right.$$

$$= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 129, normalized size = 0.56

$$-17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)}+\frac{\left(-356i+127\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}+\frac{\left(356i+127\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 127
*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) + ((3
56*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[
2]])
```

Maple [A]
time = 0.04, size = 285, normalized size = 1.23

method	result
risch	$\frac{5x^3}{3} - 17x + \frac{-\frac{25}{8}x^3 - \frac{75}{8}x}{x^4+2x^2+3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)} \frac{\left(127\_R^2+483\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^3}{3} - 17x + \frac{-\frac{25}{8}x^3 - \frac{75}{8}x}{x^4+2x^2+3} + \frac{\left(-17\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-178\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{64}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)

[Out] 5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)+1/64*(-17*(-2+2*3^(1/2))^(1/2)*3^(1/2)-178*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))+1/16*(322*3^(1/2)+1/2*(-17*(-2+2*3^(1/2))^(1/2)*3^(1/2)-178*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1/64*(17*(-2+2*3^(1/2))^(1/2)*3^(1/2)+178*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))+1/16*(322*3^(1/2)-1/2*(17*(-2+2*3^(1/2))^(1/2)*3^(1/2)+178*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] 5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integrate((127*x^2 + 483)/(x^4 + 2*x^2 + 3), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(163) = 326$.
time = 0.37, size = 510, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

```
[Out] 1/1295793216*(2159655360*x^7 - 17709173952*x^5 - 123268*143883^(1/4)*sqrt(2
19)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(14395*sqrt(3) + 79497)*arctan(1/
5985002162114670 6*143883^(3/4)*sqrt(30817)*sqrt(219)*sqrt(73)*sqrt(143883^(
1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 202467
69*x^2 + 20246769*sqrt(3))*(161*sqrt(3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*s
qrt(3) + 79497) - 1/8868084822*143883^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*
x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3) + 79497) + 1/2*sqrt(3)*sqrt(2) - 1/2*
sqrt(2)) - 123268*143883^(1/4)*sqrt(219)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*
sqrt(14395*sqrt(3) + 79497)*arctan(1/7241852616158751426*143883^(3/4)*sqrt(
30817)*sqrt(219)*sqrt(-1068793*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*
x)*sqrt(14395*sqrt(3) + 79497) + 21639604979817*x^2 + 21639604979817*sqrt(3
))*(161*sqrt(3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*sqrt(3) + 79497) - 1/8868
084822*143883^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*x - 127*sqrt(2)*x)*sqrt(
14395*sqrt(3) + 79497) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 143883^(1/4)*
sqrt(219)*(79497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 23849
1)*sqrt(14395*sqrt(3) + 79497)*log(1068793/30817*143883^(1/4)*sqrt(219)*(12
7*sqrt(3)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 702197001*x^2 + 70219700
1*sqrt(3)) + 143883^(1/4)*sqrt(219)*(79497*x^4 + 158994*x^2 - 14395*sqrt(3)
*(x^4 + 2*x^2 + 3) + 238491)*sqrt(14395*sqrt(3) + 79497)*log(-1068793/30817
*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqrt(14395*sqrt(3) + 79497)
 + 702197001*x^2 + 702197001*sqrt(3)) - 41627357064*x^3 - 78233515416*x)/(x
^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.34, size = 60, normalized size = 0.26

$$\frac{5x^3}{3} - 17x + \frac{-25x^3 - 75x}{8x^4 + 16x^2 + 24} + \text{RootSum}\left(1048576t^4 + 29480960t^2 + 2106591003, \left(t \mapsto t\log\left(\frac{557056t^3}{816619683} + \frac{166600064t}{816619683} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**3/3 - 17*x + (-25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(10485
76*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*log(557056*_t**3/8166
19683 + 166600064*_t/816619683 + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(163) = 326.
time = 3.69, size = 573, normalized size = 2.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(3
```

```
) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 22
86*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18)
)*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3
) + 1/2)) - 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqr
t(3) + 18) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*s
qrt(3) + 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1738
8*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18
))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/41472*sqrt
(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 127*3^(3/4)
*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 17388*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2
 + 3)
```

Mupad [B]
time = 0.09, size = 162, normalized size = 0.70

$$\frac{5\,x^3}{3}-\frac{\frac{25\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}-17\,x+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\,\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}-\frac{30817\,\sqrt{2}\,x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}}{128\,\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\,\left(\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}+\frac{30817\,\sqrt{2}\,x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}}{128\,\left(\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (atan((x*(- 2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/
128 - 1571667/64)) - (30817*2^(1/2)*x*(- 2^(1/2)*30817i - 14395)^(1/2))/(12
8*((2^(1/2)*14884611i)/128 - 1571667/64)))*(- 2^(1/2)*30817i - 14395)^(1/2)
*1i)/16 - ((75*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) - 17*x - (atan((x*(2^(1
/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/128 + 1571667/64
)) + (30817*2^(1/2)*x*(2^(1/2)*30817i - 14395)^(1/2))/(128*((2^(1/2)*148846
11i)/128 + 1571667/64)))*(2^(1/2)*30817i - 14395)^(1/2)*1i)/16 + (5*x^3)/3
```

3.112 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=225

$$5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+\right.}$$

```
[Out] 5*x+25/8*x*(x^2+1)/(x^4+2*x^2+3)-1/192*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2
))*(-115746+77394*3^(1/2))^(1/2)+1/192*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2
))*(-115746+77394*3^(1/2))^(1/2)+1/96*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)-1/96*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1682, 1690, 1183, 648, 632, 210, 642}

$$\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25(x^2+1)x}{8(x^4+2x^2+3)}+5x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 12899*Sqrt[3]
)/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sq
rt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt
[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{25x(1+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \frac{-150-186x^2+240x^4}{3+2x^2+x^4}\,dx$$
$$= \frac{25x(1+x^2)}{8(3+2x^2+x^4)} + \frac{1}{48}\int \left(240 - \frac{6(145+111x^2)}{3+2x^2+x^4}\right)dx$$
$$= 5x + \frac{25x(1+x^2)}{8(3+2x^2+x^4)} - \frac{1}{8}\int \frac{145+111x^2}{3+2x^2+x^4}\,dx$$
$$= 5x + \frac{25x(1+x^2)}{8(3+2x^2+x^4)} - \frac{\int \frac{145\sqrt{2\left(-1+\sqrt{3}\right)}-\left(145-111\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}} - \frac{\int \frac{145}{}}{}$$
$$= 5x + \frac{25x(1+x^2)}{8(3+2x^2+x^4)} - \frac{1}{96}\left(333+145\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x}$$
$$= 5x + \frac{25x(1+x^2)}{8(3+2x^2+x^4)} - \frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(\right.}\right.$$
$$= 5x + \frac{25x(1+x^2)}{8(3+2x^2+x^4)} + \frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\right.}}{\sqrt{2\left(1\right.}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 121, normalized size = 0.54

$$5x + \frac{25(x+x^3)}{8(3+2x^2+x^4)} - \frac{\left(-34i+111\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{\left(34i+111\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan[
x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[2]
)*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [A]
time = 0.04, size = 281, normalized size = 1.25

method	result
risch	$5x+\frac{\frac{25}{8}x^3+\frac{25}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-111\_R^2-145\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$5x-\frac{-\frac{25}{8}x^3-\frac{25}{8}x}{x^4+2x^2+3}-\frac{\left(94\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-51\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{192}-\frac{\left(290\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x-(-25/8*x^3-25/8*x)/(x^4+2*x^2+3)-1/192*(94*(-2+2*3^(1/2))^(1/2)*3^(1/2)
-51*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))-1/48*(290*
3^(1/2)+1/2*(94*(-2+2*3^(1/2))^(1/2)*3^(1/2)-51*(-2+2*3^(1/2))^(1/2))*(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))-1/192*(-94*(-2+2*3^(1/2))^(1/2)*3^(1/2)+51*(-2+2*3^(1/2))^
(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))-1/48*(290*3^(1/2)-1/2*(-94*(-
2+2*3^(1/2))^(1/2)*3^(1/2)+51*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^4
 + 2*x^2 + 3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(158) = 316$.
time = 0.37, size = 461, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/19736089152*(98680445760*x^5 + 31876*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2
 + 3)*sqrt(248834609*sqrt(3) + 499152603)*arctan(1/2453286601800494203302*4
```

```
99152603^(3/4)*sqrt(7969)*sqrt(11933241279921*x^2 + 38697*499152603^(1/4)*(
145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 11933241279921
*sqrt(3))*(111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqrt(248834609*sqrt(3) + 4991
52603) - 1/7955494186614*499152603^(3/4)*(111*sqrt(3)*sqrt(2)*x - 145*sqrt(
2)*x)*sqrt(248834609*sqrt(3) + 499152603) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(
2)) + 31876*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(248834609*sqrt(3
) + 499152603)*arctan(1/245328660180049420330 2*499152603^(3/4)*sqrt(7969)*s
qrt(11933241279921*x^2 - 38697*499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt
(248834609*sqrt(3) + 499152603) + 11933241279921*sqrt(3))*(111*sqrt(3)*sqrt
(2) - 145*sqrt(2))*sqrt(248834609*sqrt(3) + 499152603) - 1/7955494186614*49
9152603^(3/4)*(111*sqrt(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(248834609*sqrt(3
) + 499152603) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 259036170120*x^3 + 49
9152603^(1/4)*(19291*x^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57
873)*sqrt(248834609*sqrt(3) + 499152603)*log(1497457809*x^2 + 38697/7969*49
9152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) +
 1497457809*sqrt(3)) - 499152603^(1/4)*(19291*x^4 + 38582*x^2 - 12899*sqrt(
3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(248834609*sqrt(3) + 499152603)*log(14974
57809*x^2 - 38697/7969*499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(2488346
09*sqrt(3) + 499152603) + 1497457809*sqrt(3)) + 357716615880*x)/(x^4 + 2*x^
2 + 3)
```

Sympy [**A**]
time = 0.33, size = 51, normalized size = 0.23

$$5x + \frac{25x^3 + 25x}{8x^4 + 16x^2 + 24} + \operatorname{RootSum}\left(3145728t^4 + 39507968t^2 + 166384201, \left(t \mapsto t\log\left(-\frac{9240576t^3}{102792131} - \frac{95003488t}{102792131} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 39
507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 9500
3488*_t/102792131 + x)))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(158) = 316.
time = 5.36, size = 566, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6
*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*
```

```
sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/
4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/6912
*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(2)*s
qrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
- 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/13824*sqrt
(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^
(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqr
t(3) + 18) - 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/13824*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2)
 + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sq
rt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 1740*3^(1/4)*sqrt
(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
 + 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3)
```

Mupad [B]
time = 0.96, size = 156, normalized size = 0.69

$$5\,x+\frac{\frac{25\,x^3}{8}+\frac{25\,x}{8}}{x^4+2\,x^2+3}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873-\sqrt{2}\,23907{}\mathrm{i}}\,7969{}\mathrm{i}}{576\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505{}\mathrm{i}}{384}\right)}+\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873-\sqrt{2}\,23907{}\mathrm{i}}}{1152\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505{}\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873-\sqrt{2}\,23907{}\mathrm{i}}\,1{}\mathrm{i}}{48}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873+\sqrt{2}\,23907{}\mathrm{i}}\,7969{}\mathrm{i}}{576\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505{}\mathrm{i}}{384}\right)}-\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873+\sqrt{2}\,23907{}\mathrm{i}}}{1152\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505{}\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873+\sqrt{2}\,23907{}\mathrm{i}}\,1{}\mathrm{i}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 5*x + ((25*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) + (atan((x*(- 2^(1/2)*23907
i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 - 374543/96)) + (7969*
2^(1/2)*x*(- 2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384 -
 374543/96)))*(- 2^(1/2)*23907i - 57873)^(1/2)*1i)/48 - (atan((x*(2^(1/2)*2
3907i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 + 374543/96)) - (7
969*2^(1/2)*x*(2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384
 + 374543/96)))*(2^(1/2)*23907i - 57873)^(1/2)*1i)/48
```

3.113 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=224

$$\frac{25x(1-x^2)}{24\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+\right.}$$

```
[Out] 25/24*x*(-x^2+1)/(x^4+2*x^2+3)-1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/288*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/576*ln(x^2+
3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)-1/576*ln(x^2+3^
(1/2)+x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1692, 1183, 648, 632, 210, 642}

$$-\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{48}\sqrt{\frac{1}{6}\left(12897\sqrt{3}-11567\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x(1-x^2)}{24\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*Sqrt[3])/6]
*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(
-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/48 + (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} + \frac{1}{48}\int \frac{14+190x^2}{3+2x^2+x^4}\,dx$$

$$= \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} + \frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}-\left(14-190\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{\int \frac{14\sqrt{2\left(-1+\right.}}{\sqrt{3}+\sqrt{2}}}{96\sqrt{6}}$$

$$= \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} + \frac{\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}} + \frac{1}{288}\left(28\right.$$

$$= \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} + \frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\,\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x\right.$$

$$= \frac{25x(1-x^2)}{24\,(3+2x^2+x^4)} - \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}}{\sqrt{2\left(1+\sqrt{3}\right.}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 115, normalized size = 0.51

$$\frac{1}{48}\left(-\frac{50x(-1+x^2)}{3+2x^2+x^4}+\frac{\left(95+44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(95-44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
[Out] ((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sqr
t[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/Sq
rt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

Maple [A]
time = 0.04, size = 277, normalized size = 1.24

method	result
risch	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(95\_R^2+7\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{96}$
default	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+132\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{576}+\frac{\left(14\sqrt{3}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+1/576*(139*(-2+2*3^(1/2))^(1/2)*3^(1/2)+
132*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))+1/144*(14*
3^(1/2)+1/2*(139*(-2+2*3^(1/2))^(1/2)*3^(1/2)+132*(-2+2*3^(1/2))^(1/2))*(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))+1/576*(-139*(-2+2*3^(1/2))^(1/2)*3^(1/2)-132*(-2+2*3^(1/
2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))+1/144*(14*3^(1/2)-1/2*(-1
39*(-2+2*3^(1/2))^(1/2)*3^(1/2)-132*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1
/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*x
^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(155) = 310$.
time = 0.37, size = 461, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/33461214912*(54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-1491795
99*sqrt(3) + 498997827)*arctan(1/29015889224422097862*6160467^(3/4)*sqrt(13
```

```
513)*sqrt(1433)*sqrt(174277161*x^2 + 6160467^(1/4)*(7*sqrt(3)*x - 285*x)*sq
rt(-149179599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sqrt(3)*sqrt(2)
 - 7*sqrt(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/499478343426*6160467
^(3/4)*(95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sqrt(3) + 49899
7827) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 54052*6160467^(1/4)*sqrt(2)*(x
^4 + 2*x^2 + 3)*sqrt(-149179599*sqrt(3) + 498997827)*arctan(1/2901588922442
2097862*6160467^(3/4)*sqrt(13513)*sqrt(1433)*sqrt(174277161*x^2 - 6160467^(
1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161
*sqrt(3))*(95*sqrt(3)*sqrt(2) - 7*sqrt(2))*sqrt(-149179599*sqrt(3) + 498997
827) - 1/499478343426*6160467^(3/4)*(95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sq
rt(-149179599*sqrt(3) + 498997827) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 3
4855432200*x^3 - 6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3)*(x^4
+ 2*x^2 + 3) + 34701)*sqrt(-149179599*sqrt(3) + 498997827)*log(1496993481*x
^2 + 116073/13513*6160467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(
3) + 498997827) + 1496993481*sqrt(3)) + 6160467^(1/4)*(11567*x^4 + 23134*x^
2 + 12897*sqrt(3)*(x^4 + 2*x^2 + 3) + 34701)*sqrt(-149179599*sqrt(3) + 4989
97827)*log(1496993481*x^2 - 116073/13513*6160467^(1/4)*(7*sqrt(3)*x - 285*x
)*sqrt(-149179599*sqrt(3) + 498997827) + 1496993481*sqrt(3)) - 34855432200*
x)/(x^4 + 2*x^2 + 3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(178) = 356$.
time = 0.72, size = 1185, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-25*x**3 + 25*x)/(24*x**4 + 48*x**2 + 72) + sqrt(11567/55296 + 1433*sqrt(3
)/6144)*log(x**2 + x*(-556*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/13513 - 1040
345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 278*sqrt(3)*sqrt(11567
+ 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161) - 476102762
00401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)/30372528846219921 - 43908
31246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658)/7065021829779 + 128104648
1635939181/30372528846219921 + 200684595453464*sqrt(3)/7065021829779) - sqr
t(11567/55296 + 1433*sqrt(3)/6144)*log(x**2 + x*(-278*sqrt(3)*sqrt(11567 +
12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161 + 1040345*sqrt
(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 556*sqrt(2)*sqrt(11567 + 12897*
sqrt(3))/13513) - 47610276200401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658
)/30372528846219921 - 4390831246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658
)/7065021829779 + 1281046481635939181/30372528846219921 + 200684595453464*s
qrt(3)/7065021829779) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqrt(3)*x/(94591*s
qrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*
```

sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149
179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 7170732*sqrt(6)*sqr
t(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt
(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 31
6396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 3869
1*sqrt(3))) - 3121035*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sq
rt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))
+ 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sq
rt(3) + 316396658) + 11567 + 38691*sqrt(3))) + 834*sqrt(11567 + 12897*sqrt(
3))*sqrt(149179599*sqrt(3) + 316396658)/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt
(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(1491795
99*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
 + 11567 + 38691*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3) + 3163
96658)/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqrt(3)*x/(9
4591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 +
38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sq
rt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 834*sqrt(1156
7 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/(94591*sqrt(2)*sqrt(
-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 2
78*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(
3) + 316396658) + 11567 + 38691*sqrt(3))) + 3121035*sqrt(2)*sqrt(11567 + 12
897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396
658) + 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqr
t(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)))
+ 7170732*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2
)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(1
49179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 3163
96658) + 11567 + 38691*sqrt(3))))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(155) = 310.
time = 4.03, size = 565, normalized size = 2.52

$$-\frac{1}{62208}\sqrt{2}\left(95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1710\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1710\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+95\cdot 3^{\frac{3}{4}}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}-252\cdot 3^{\frac{1}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}+252\cdot 3^{\frac{1}{4}}\sqrt{-6\sqrt{3}+18}\right)\arctan\left(\frac{3^{\frac{3}{4}}\left(x+3^{\frac{1}{4}}\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}\right)}{3\sqrt{\frac{1}{6}\sqrt{3}+\frac{1}{2}}}\right)-\frac{1}{62208}\sqrt{2}\left(95\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1710\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1710\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+95\cdot 3^{\frac{3}{4}}\left(-6\sqrt{3}+18\right)^{\frac{3}{2}}-252\cdot 3^{\frac{1}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}+252\cdot 3^{\frac{1}{4}}\sqrt{-6\sqrt{3}+18}\right)\arctan\left(\frac{3^{\frac{3}{4}}\left(x-3^{\frac{1}{4}}\sqrt{-\frac{1}{6}\sqrt{3}+\frac{1}{2}}\right)}{3\sqrt{\frac{1}{6}\sqrt{3}+\frac{1}{2}}}\right)-\frac{1}{124416}\sqrt{2}\left(1710\cdot 3^{\frac{1}{4}}\sqrt{2}\ldots\right.$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] -1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/62
208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*

```
sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/124416*
sqrt(2)*(1710*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3
/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1
710*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/124416*sqrt(2)*(1710*3^(3/4)*sqrt(2)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)
^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(6*sqrt(3) +
18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 252*3^(1/4)
*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqr
t(3)) - 25/24*(x^3 - x)/(x^4 + 2*x^2 + 3)
```

Mupad [B]

time = 0.13, size = 153, normalized size = 0.68

$$\frac{\frac{25\,x}{24}-\frac{25\,x^3}{24}}{x^4+2\,x^2+3}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701-\sqrt{2}\,40539{}\mathrm{i}}\,13513{}\mathrm{i}}{15552\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591{}\mathrm{i}}{10368}\right)}+\frac{13513\,\sqrt{2}\,x\,\sqrt{34701-\sqrt{2}\,40539{}\mathrm{i}}}{31104\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591{}\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701-\sqrt{2}\,40539{}\mathrm{i}}\,1{}\mathrm{i}}{144}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701+\sqrt{2}\,40539{}\mathrm{i}}\,13513{}\mathrm{i}}{15552\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591{}\mathrm{i}}{10368}\right)}-\frac{13513\,\sqrt{2}\,x\,\sqrt{34701+\sqrt{2}\,40539{}\mathrm{i}}}{31104\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591{}\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701+\sqrt{2}\,40539{}\mathrm{i}}\,1{}\mathrm{i}}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] ((25*x)/24 - (25*x^3)/24)/(2*x^2 + x^4 + 3) - (atan((x*(34701 - 2^(1/2)*405
39i)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 - 1878307/5184)) + (13513
*2^(1/2)*x*(34701 - 2^(1/2)*40539i)^(1/2))/(31104*((2^(1/2)*94591i)/10368 -
 1878307/5184)))*(34701 - 2^(1/2)*40539i)^(1/2)*1i)/144 + (atan((x*(2^(1/2)
*40539i + 34701)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 + 1878307/518
4)) - (13513*2^(1/2)*x*(2^(1/2)*40539i + 34701)^(1/2))/(31104*((2^(1/2)*945
91i)/10368 + 1878307/5184)))*(2^(1/2)*40539i + 34701)^(1/2)*1i)/144
```

3.114 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=229

$$-\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+69\right.}$$

```
[Out] -4/9/x-25/72*x*(x^2+5)/(x^4+2*x^2+3)+1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2
))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/288*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/576*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)+1/576*ln(x^2+3^
(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\,\mathrm{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{48}\sqrt{\frac{1}{6}\left(699\sqrt{3}-965\right)}\,\mathrm{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{25x(x^2+5)}{72\left(x^4+2x^2+3\right)}-\frac{4}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965 + 699*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 -
(Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/48 - (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/96 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = -\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{64+\frac{170x^2}{3}-\frac{50x^4}{3}}{x^2\left(3+2x^2+x^4\right)}\,dx$$

$$= -\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(\frac{64}{3x^2}-\frac{2(-7+19x^2)}{3+2x^2+x^4}\right)\,dx$$

$$= -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{1}{24}\int \frac{-7+19x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-7-19\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{48\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int \frac{-7\sqrt{2}\cdots}{\sqrt{3}\cdots}}{\cdots}$$

$$= -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)}\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right.}\cdots}$$

$$= -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{\cdots}\right.}\right.$$

$$= -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right.}\cdots}{\sqrt{2\left(1+\sqrt{\cdots}\right.}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 126, normalized size = 0.55

$$-\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{\left(26i+19\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}}-\frac{\left(-26i+19\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*A
rcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19*S
qrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [A]
time = 0.04, size = 283, normalized size = 1.24

method	result
risch	$\frac{-\frac{19}{24}x^4-\frac{21}{8}x^2-\frac{4}{3}}{x(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4-1930\_Z^2+488601\right)}\_R\ln\left(-96\_R^3+34499\_R+361383x\right)\right)}{96}$
default	$-\frac{\frac{25}{8}x^3+\frac{125}{8}x}{9(x^4+2x^2+3)}-\frac{\left(32\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+39\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{576}-\Big(-14\sqrt{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(25/8*x^3+125/8*x)/(x^4+2*x^2+3)-1/576*(32*(-2+2*3^(1/2))^(1/2)*3^(1/2
)+39*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))-1/144*(-1
4*3^(1/2)+1/2*(32*(-2+2*3^(1/2))^(1/2)*3^(1/2)+39*(-2+2*3^(1/2))^(1/2))*(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))-1/576*(-32*(-2+2*3^(1/2))^(1/2)*3^(1/2)-39*(-2+2*3^(1/2)
)^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))-1/144*(-14*3^(1/2)-1/2*(-32
*(-2+2*3^(1/2))^(1/2)*3^(1/2)-39*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)
)/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)
)-4/9/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2 -
 7)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(160) = 320.
time = 0.41, size = 478, normalized size = 2.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/208156608*(164790648*x^4 - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x
)*sqrt(-674535*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sq
rt(517)*sqrt(233)*sqrt(1084149*x^2 + 1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqr
t(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqr
t(2))*sqrt(-674535*sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt
(3)*sqrt(2)*x + 7*sqrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) - 1/2*sqrt(3)*
sqrt(2) + 1/2*sqrt(2)) - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x)*sqr
t(-674535*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sqrt(51
7)*sqrt(233)*sqrt(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-67
4535*sqrt(3) + 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))
*sqrt(-674535*sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*s
qrt(2)*x + 7*sqrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) + 1/2*sqrt(3)*sqrt(
2) - 1/2*sqrt(2)) - 1465803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 +
2*x^3 + 3*x) + 2895*x)*sqrt(-674535*sqrt(3) + 1465803)*log(4397409*x^2 + 20
97/517*1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-674535*sqrt(3) + 1465803) +
 4397409*sqrt(3)) + 1465803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 +
2*x^3 + 3*x) + 2895*x)*sqrt(-674535*sqrt(3) + 1465803)*log(4397409*x^2 - 20
97/517*1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-674535*sqrt(3) + 1465803) +
 4397409*sqrt(3)) + 546411096*x^2 + 277542144)/(x^5 + 2*x^3 + 3*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(184) = 368$.
time = 0.75, size = 1192, normalized size = 5.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-19*x**4 - 63*x**2 - 32)/(24*x**5 + 48*x**3 + 72*x) - sqrt(965/55296 + 233
*sqrt(3)/18432)*log(x**2 + x*(-128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517 - 21
793*sqrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 64*sqrt(3)*sqrt(965 + 699*sqrt
(3))*sqrt(674535*sqrt(3) + 1198514)/361383) - 8882635459*sqrt(2)*sqrt(67453
5*sqrt(3) + 1198514)/130597672689 - 20458048*sqrt(6)*sqrt(674535*sqrt(3) +
1198514)/560505033 + 18567565928783/130597672689 + 46950427730*sqrt(3)/5605
05033) + sqrt(965/55296 + 233*sqrt(3)/18432)*log(x**2 + x*(-64*sqrt(3)*sqrt
(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383 + 21793*sqrt(6)*s
qrt(965 + 699*sqrt(3))/361383 + 128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517) -
8882635459*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/130597672689 - 20458048*s
qrt(6)*sqrt(674535*sqrt(3) + 1198514)/560505033 + 18567565928783/1305976726
89 + 46950427730*sqrt(3)/560505033) + 2*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) +
 1198514)/27648 + 965/55296 + 233*sqrt(3)/6144)*atan(722766*sqrt(3)*x/(-64*
sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 119851
4) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3)
 + 1198514) + 965 + 2097*sqrt(3))) + 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/
```

```
(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1
198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sq
rt(3) + 1198514) + 965 + 2097*sqrt(3))) + 65379*sqrt(2)*sqrt(965 + 699*sqrt
(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3
) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(6745
35*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) - 192*sqrt(965 + 699*sqrt(3))*
sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)))) + 2
*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/27648 + 965/55296 + 233*sqrt(
3)/6144)*atan(722766*sqrt(3)*x/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 19
2*sqrt(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*s
qrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 20
97*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) +
 965 + 2097*sqrt(3))) - 65379*sqrt(2)*sqrt(965 + 699*sqrt(3))/(-64*sqrt(674
535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965
 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 11985
14) + 965 + 2097*sqrt(3))) - 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/(-64*sqr
t(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)
+ 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) +
1198514) + 965 + 2097*sqrt(3))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(160) = 320$.
time = 3.78, size = 572, normalized size = 2.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-
6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*
sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4
)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/62208
*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(2)*s
qrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*sqrt(6*
sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x -
3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/124416*sqrt(
2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sq
rt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(
```

```
3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(
3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/
6*sqrt(3) + 1/2) + sqrt(3)) - 1/124416*sqrt(2)*(342*3^(3/4)*sqrt(2)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) +
 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt
(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*s
qrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1
/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x)
```

Mupad [B]
time = 0.14, size = 159, normalized size = 0.69

$$-\frac{\frac{19\,x^4}{24}+\frac{21\,x^2}{8}+\frac{4}{3}}{x^5+2\,x^3+3\,x}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895-\sqrt{2}\,1551{}\mathrm{i}}\,517{}\mathrm{i}}{15552\,\left(\frac{517}{162}+\frac{\sqrt{2}\,3619{}\mathrm{i}}{10368}\right)}+\frac{517\,\sqrt{2}\,x\,\sqrt{2895-\sqrt{2}\,1551{}\mathrm{i}}}{31104\,\left(\frac{517}{162}+\frac{\sqrt{2}\,3619{}\mathrm{i}}{10368}\right)}\right)\,\sqrt{2895-\sqrt{2}\,1551{}\mathrm{i}}\,1{}\mathrm{i}}{144}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895+\sqrt{2}\,1551{}\mathrm{i}}\,517{}\mathrm{i}}{15552\,\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619{}\mathrm{i}}{10368}\right)}-\frac{517\,\sqrt{2}\,x\,\sqrt{2895+\sqrt{2}\,1551{}\mathrm{i}}}{31104\,\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619{}\mathrm{i}}{10368}\right)}\right)\,\sqrt{2895+\sqrt{2}\,1551{}\mathrm{i}}\,1{}\mathrm{i}}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (atan((x*(2^(1/2)*1551i + 2895)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 -
 517/162)) - (517*2^(1/2)*x*(2^(1/2)*1551i + 2895)^(1/2))/(31104*((2^(1/2)*
3619i)/10368 - 517/162)))*(2^(1/2)*1551i + 2895)^(1/2)*1i)/144 - (atan((x*(
2895 - 2^(1/2)*1551i)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 + 517/162))
 + (517*2^(1/2)*x*(2895 - 2^(1/2)*1551i)^(1/2))/(31104*((2^(1/2)*3619i)/103
68 + 517/162)))*(2895 - 2^(1/2)*1551i)^(1/2)*1i)/144 - ((21*x^2)/8 + (19*x^
4)/24 + 4/3)/(3*x + 2*x^3 + x^5)
```

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2} \, dx$

Optimal. Leaf size=238

$$-\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216\,(3+2x^2+x^4)}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{432}$$

```
[Out] -4/27/x^3+13/27/x+25/216*x*(5*x^2+7)/(x^4+2*x^2+3)+1/5184*ln(x^2+3^(1/2)-x*
(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/5184*ln(x^2+3^(1/2)+x
*(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/2592*arctan((-2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1/2))^(1/2)+1/259
2*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1
/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x(5x^2+7)}{216\,(x^4+2x^2+3)}+\frac{13}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) - (Sqr
t[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + S
qrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])
/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 566
73*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{50x^4}{9}+\frac{250x^6}{9}}{x^4\left(3+2x^2+x^4\right)}\,dx$$

$$= \frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \left(\frac{64}{3x^4}-\frac{208}{9x^2}+\frac{2(137+229x^2)}{9\left(3+2x^2+x^4\right)}\right)dx$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{1}{216}\int \frac{137+229x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{\int \frac{137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(137-229\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,d}{432\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{1}{432}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\int \frac{}{\sqrt{3}}$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{}\right.$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216\left(3+2x^2+x^4\right)}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 131, normalized size = 0.55

$$\frac{1}{864}\left(\frac{4(-96+248x^2+351x^4+229x^6)}{x^3\left(3+2x^2+x^4\right)}+\frac{2\left(229+46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{2\left(229-46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229
+ (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2*(
229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/8
64
```

Maple [A]
time = 0.04, size = 288, normalized size = 1.21

method	result
risch	$\frac{\frac{229}{216}x^6+\frac{13}{8}x^4+\frac{31}{27}x^2-\frac{4}{9}}{x^3(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+12146\_Z^2+3211828929\right)}\_R\ln\left(825\_R^3+11161024\_R+3926135421x\right)\right)}{864}$
default	$\frac{\frac{125}{8}x^3+\frac{175}{8}x}{27x^4+54x^2+81}+\frac{\left(275\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+138\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{5184}+\frac{\Big(274\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(125/8*x^3+175/8*x)/(x^4+2*x^2+3)+1/5184*(275*(-2+2*3^(1/2))^(1/2)*3^(
1/2)+138*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))+1/129
6*(274*3^(1/2)+1/2*(275*(-2+2*3^(1/2))^(1/2)*3^(1/2)+138*(-2+2*3^(1/2))^(1/
2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1
/2))/(2+2*3^(1/2))^(1/2))+1/5184*(-275*(-2+2*3^(1/2))^(1/2)*3^(1/2)-138*(-2
+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))+1/1296*(274*3^(1/
2)-1/2*(-275*(-2+2*3^(1/2))^(1/2)*3^(1/2)-138*(-2+2*3^(1/2))^(1/2))*(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))-4/27/x^3+13/27/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*inte
grate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(167) = 334$.
time = 0.39, size = 530, normalized size = 2.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2261454002496*(2397560030424*x^6 + 3674862754056*x^4 - 277108*118956627^(
1/4)*sqrt(6297)*sqrt(2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*a
rctan(1/9849351014648629637 4*118956627^(3/4)*sqrt(6297)*sqrt(2099)*sqrt(13)
*sqrt(118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3)
+ 170019) + 1308711807*x^2 + 1308711807*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137
*sqrt(2))*sqrt(6073*sqrt(3) + 170019) - 1/16481916497358*118956627^(3/4)*sq
rt(6297)*(229*sqrt(3)*sqrt(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019
) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 277108*118956627^(1/4)*sqrt(6297)*
sqrt(2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*arctan(1/26593247
73955130002098*118956627^(3/4)*sqrt(6297)*sqrt(13)*sqrt(-1530171*118956627^
(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 2002
552854428997*x^2 + 2002552854428997*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqr
t(2))*sqrt(6073*sqrt(3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(6
297)*(229*sqrt(3)*sqrt(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) -
1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 118956627^(1/4)*sqrt(6297)*(6073*x^7 +
 12146*x^5 + 18219*x^3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(6073*sqr
t(3) + 170019)*log(1530171/69277*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x
- 687*x)*sqrt(6073*sqrt(3) + 170019) + 28906460361*x^2 + 28906460361*sqrt(3
)) + 118956627^(1/4)*sqrt(6297)*(6073*x^7 + 12146*x^5 + 18219*x^3 - 56673*s
qrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(6073*sqrt(3) + 170019)*log(-1530171/6927
7*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 17
0019) + 28906460361*x^2 + 28906460361*sqrt(3)) + 2596484225088*x^2 - 100509
0667776)/(x^7 + 2*x^5 + 3*x^3)
```

Sympy [A]
time = 0.35, size = 60, normalized size = 0.25

$$\operatorname{RootSum}\left(2293235712t^4 + 12437504t^2 + 4405801, \left(t \mapsto t\log\left(\frac{19707494400t^3}{145412423} + \frac{357152768t}{145412423} + x\right)\right)\right) + \frac{229x^6 + 351x^4 + 248x^2 - 96}{216x^7 + 432x^5 + 648x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(1970
7494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351*x
**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(167) = 334.
time = 4.47, size = 579, normalized size = 2.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/
3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/
3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/1119744*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 229*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) +
18)^(3/2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*
sqrt(2)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2
+ 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/1119744*sqrt(2)*(4122
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 229*3^(3/4)*sqrt(2)*
(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)
*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3)
+ 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*
sqrt(3) + 1/2) + sqrt(3)) + 25/216*(5*x^3 + 7*x)/(x^4 + 2*x^2 + 3) + 1/27*(
13*x^2 - 4)/x^3
```

Mupad [B]
time = 0.14, size = 165, normalized size = 0.69

$$\frac{\frac{229\,x^6}{216}+\frac{13\,x^4}{8}+\frac{31\,x^2}{27}-\frac{4}{9}}{x^7+2\,x^5+3\,x^3}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219-\sqrt{2}\,207831{}\mathrm{i}}\,69277{}\mathrm{i}}{11337408\,\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949{}\mathrm{i}}{7558272}\right)}+\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219-\sqrt{2}\,207831{}\mathrm{i}}}{22674816\,\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949{}\mathrm{i}}{7558272}\right)}\right)\,\sqrt{-18219-\sqrt{2}\,207831{}\mathrm{i}}\,1{}\mathrm{i}}{1296}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219+\sqrt{2}\,207831{}\mathrm{i}}\,69277{}\mathrm{i}}{11337408\,\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949{}\mathrm{i}}{7558272}\right)}-\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219+\sqrt{2}\,207831{}\mathrm{i}}}{22674816\,\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949{}\mathrm{i}}{7558272}\right)}\right)\,\sqrt{-18219+\sqrt{2}\,207831{}\mathrm{i}}\,1{}\mathrm{i}}{1296}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] ((31*x^2)/27 + (13*x^4)/8 + (229*x^6)/216 - 4/9)/(3*x^3 + 2*x^5 + x^7) - (a
tan((x*(- 2^(1/2)*207831i - 18219)^(1/2)*69277i)/(11337408*((2^(1/2)*949094
9i)/7558272 - 19051175/3779136)) + (69277*2^(1/2)*x*(- 2^(1/2)*207831i - 18
219)^(1/2))/(22674816*((2^(1/2)*9490949i)/7558272 - 19051175/3779136)))*(-
2^(1/2)*207831i - 18219)^(1/2)*1i)/1296 + (atan((x*(2^(1/2)*207831i - 18219
)^(1/2)*69277i)/(11337408*((2^(1/2)*9490949i)/7558272 + 19051175/3779136))
- (69277*2^(1/2)*x*(2^(1/2)*207831i - 18219)^(1/2))/(22674816*((2^(1/2)*949
0949i)/7558272 + 19051175/3779136)))*(2^(1/2)*207831i - 18219)^(1/2)*1i)/12
96
```

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2} \, dx$

<u>**Optimal**</u>. Leaf size=245

$$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648(3+2x^2+x^4)}+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}$$

```
[Out] -4/45/x^5+13/81/x^3-13/27/x+25/648*x*(-7*x^2+1)/(x^4+2*x^2+3)+1/7776*arctan
((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-6836286+4130514*3^(1/2)
)^(1/2)-1/7776*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-683
6286+4130514*3^(1/2))^(1/2)-1/15552*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*
(6836286+4130514*3^(1/2))^(1/2)+1/15552*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/
2))*(6836286+4130514*3^(1/2))^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.20, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$,
Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}-\frac{\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296}-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{25x(1-7x^2)}{648(x^4+2x^2+3)}-\frac{13}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(3 + 2*x^2
+ x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(-1139381 + 688419*Sqrt[3])/
6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x
^2])/2592 + (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 +
Sqrt[3])]*x + x^2])/2592
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 632</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{400x^4}{9}+\frac{1550x^6}{27}-\frac{350x^8}{27}}{x^6\left(3+2x^2+x^4\right)}\,dx$$

$$= \frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \left(\frac{64}{3x^6}-\frac{208}{9x^4}+\frac{208}{9x^2}-\frac{2(-463+487x^2)}{27\left(3+2x^2+x^4\right)}\right)dx$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)}-\frac{1}{648}\int \frac{-463+487x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)}-\frac{\int \frac{-463\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-463-\right.}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x}}{1296\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)}-\frac{\left(1461-463\sqrt{3}\right)\int \frac{}{\sqrt{3}-\sqrt{2\left(\right.}}}{7776}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}}{}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}}{12}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 140, normalized size = 0.57

$$\frac{-\frac{4\left(864-984x^2+3928x^4+2475x^6+2435x^8\right)}{x^5\left(3+2x^2+x^4\right)}-\frac{10i\left(-487i+475\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{10i\left(487i+475\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
[Out] ((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x^
4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1
```

```
- I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])
/Sqrt[1 + I*Sqrt[2]])/12960
```

Maple [A]
time = 0.04, size = 293, normalized size = 1.20

method	result
risch	$\frac{-\frac{487}{648}x^8-\frac{55}{72}x^6-\frac{491}{405}x^4+\frac{41}{135}x^2-\frac{4}{15}}{x^5(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4-2278762\_Z^2+473920719561\right)}\_R\ln\left(-2886\_R^3+1211171969\_R+171\right.\right.}{2592}$
default	$-\frac{\frac{175}{24}x^3-\frac{25}{24}x}{27(x^4+2x^2+3)}-\frac{\left(962\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+1425\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{15552}-\frac{\left(-926\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*(175/24*x^3-25/24*x)/(x^4+2*x^2+3)-1/15552*(962*(-2+2*3^(1/2))^(1/2)*
3^(1/2)+1425*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))-1
/3888*(-926*3^(1/2)+1/2*(962*(-2+2*3^(1/2))^(1/2)*3^(1/2)+1425*(-2+2*3^(1/2
))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/
2))^(1/2))/(2+2*3^(1/2))^(1/2))-1/15552*(-962*(-2+2*3^(1/2))^(1/2)*3^(1/2)-
1425*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))-1/3888*(-
926*3^(1/2)-1/2*(-962*(-2+2*3^(1/2))^(1/2)*3^(1/2)-1425*(-2+2*3^(1/2))^(1/2
))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))-4/45/x^5+13/81/x^3-13/27/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*x
^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(172) = 344$.
time = 0.38, size = 503, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/1478473537631040*(1111136748188760*x^8 + 1129389507912600*x^6 + 17924210
04881088*x^4 - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5)*sqrt(-
784371528639*sqrt(3) + 1421762158683)*arctan(1/6144866223568721756453718*21
6699003^(3/4)*sqrt(248569)*sqrt(2833)*sqrt(57039874137*x^2 + 216699003^(1/4
)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57
039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639*s
qrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*sq
rt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/2*
sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x
^7 + 3*x^5)*sqrt(-784371528639*sqrt(3) + 1421762158683)*arctan(1/6144866223
568721756453718*216699003^(3/4)*sqrt(248569)*sqrt(2833)*sqrt(57039874137*x^
2 - 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1
421762158683) + 57039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sq
rt(-784371528639*sqrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/
4)*(487*sqrt(3)*sqrt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 142
1762158683) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 5*216699003^(1/4)*(11393
81*x^9 + 2278762*x^7 + 3418143*x^5 + 688419*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*
sqrt(-784371528639*sqrt(3) + 1421762158683)*log(4265286476049*x^2 + 1858731
3/248569*216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3
) + 1421762158683) + 4265286476049*sqrt(3)) + 5*216699003^(1/4)*(1139381*x^
9 + 2278762*x^7 + 3418143*x^5 + 688419*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(
-784371528639*sqrt(3) + 1421762158683)*log(4265286476049*x^2 - 18587313/248
569*216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1
421762158683) + 4265286476049*sqrt(3)) - 449017889206464*x^2 + 394259610034
944)/(x^9 + 2*x^7 + 3*x^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(199) = 398$.
time = 0.76, size = 1202, normalized size = 4.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] -sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-3848*sqrt(2)*s
qrt(1139381 + 688419*sqrt(3))/248569 - 769085497*sqrt(6)*sqrt(1139381 + 688
419*sqrt(3))/171119622411 + 1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sqr
t(784371528639*sqrt(3) + 1359975610922)/171119622411) - 8677510907569510603
*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/29281925174083213452921
 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605
100269239577 + 20196165220927340076543947/29281925174083213452921 + 5094503
6826336313070*sqrt(3)/127605100269239577) + sqrt(1139381/40310784 + 2833*sq
rt(3)/165888)*log(x**2 + x*(-1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sq
```

```
rt(784371528639*sqrt(3) + 1359975610922)/171119622411 + 769085497*sqrt(6)*s
qrt(1139381 + 688419*sqrt(3))/171119622411 + 3848*sqrt(2)*sqrt(1139381 + 68
8419*sqrt(3))/248569) - 867751090756951O603*sqrt(2)*sqrt(784371528639*sqrt(
3) + 1359975610922)/29281925174083213452921 - 21752950947364*sqrt(6)*sqrt(7
84371528639*sqrt(3) + 1359975610922)/127605100269239577 + 20196165220927340
076543947/29281925174083213452921 + 5094503682633631307O*sqrt(3)/1276051002
69239577) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/2015
5392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(342239244822*sqrt(3)*x/(
-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(78437
1528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*s
qrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381
 + 2065257*sqrt(3))) + 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-
1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371
528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sq
rt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381
+ 2065257*sqrt(3))) + 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1
924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(7843715
28639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqr
t(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 +
 2065257*sqrt(3))) - 5772*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*
sqrt(3) + 1359975610922)/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*
sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065
257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3)
+ 1359975610922) + 1139381 + 2065257*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(7843
71528639*sqrt(3) + 1359975610922)/20155392 + 1139381/40310784 + 2833*sqrt(3
)/55296)*atan(342239244822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 135
9975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 11
39381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528
639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 5772*sqrt(1139
381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/(-1924*sqr
t(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*s
qrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sq
rt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 206525
7*sqrt(3))) - 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt
(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sq
rt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqr
t(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257
*sqrt(3))) - 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt(
784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqr
t(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt
(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*
sqrt(3)))) + (-2435*x**8 - 2475*x**6 - 3928*x**4 + 984*x**2 - 864)/(3240*x*
*9 + 6480*x**7 + 9720*x**5)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(172) =

344.
time = 4.45, size = 584, normalized size = 2.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(
3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
 + 18) - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*l
og(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/3359232*sqrt(2
)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 487*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 8766*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^(1/4)*sqrt(2)*sqrt(-6*
sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 25/648*(7*x^3 - x)/(x^4 + 2*x^2 + 3) -
 1/405*(195*x^4 - 65*x^2 + 36)/x^5
```

Mupad [B]
time = 0.14, size = 171, normalized size = 0.70

$$-\frac{\frac{487\,x^8}{648}+\frac{55\,x^6}{72}+\frac{491\,x^4}{405}-\frac{41\,x^2}{135}+\frac{4}{15}}{x^9+2\,x^7+3\,x^5}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143-\sqrt{2}\,745707{}\mathrm{i}}\,248569{}\mathrm{i}}{306110016\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447{}\mathrm{i}}{204073344}\right)}+\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143-\sqrt{2}\,745707{}\mathrm{i}}}{612220032\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447{}\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143-\sqrt{2}\,745707{}\mathrm{i}}\,1{}\mathrm{i}}{3888}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143+\sqrt{2}\,745707{}\mathrm{i}}\,248569{}\mathrm{i}}{306110016\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447{}\mathrm{i}}{204073344}\right)}-\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143+\sqrt{2}\,745707{}\mathrm{i}}}{612220032\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447{}\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143+\sqrt{2}\,745707{}\mathrm{i}}\,1{}\mathrm{i}}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (atan((x*(2^(1/2)*745707i + 3418143)^(1/2)*248569i)/(306110016*((2^(1/2)*11
5087447i)/204073344 - 119561689/51018336)) - (248569*2^(1/2)*x*(2^(1/2)*745
707i + 3418143)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 - 1195616
89/51018336)))*(2^(1/2)*745707i + 3418143)^(1/2)*1i)/3888 - (atan((x*(34181
43 - 2^(1/2)*745707i)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/20407
3344 + 119561689/51018336)) + (248569*2^(1/2)*x*(3418143 - 2^(1/2)*745707i)
^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 + 119561689/51018336)))*
(3418143 - 2^(1/2)*745707i)^(1/2)*1i)/3888 - ((491*x^4)/405 - (41*x^2)/135
+ (55*x^6)/72 + (487*x^8)/648 + 4/15)/(3*x^5 + 2*x^7 + x^9)
```

3.117 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=243

$$58x-9x^3+x^5-\frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2}+\frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)}+\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\right.}}{\sqrt{2\left(1\right.}}\right.$$

```
[Out] 58*x-9*x^3+x^5-25/16*x*(7*x^2+15)/(x^4+2*x^2+3)^2+1/64*x*(252*x^2+3305)/(x^
4+2*x^2+3)+3/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-
8595619+7678611*3^(1/2))^(1/2)-3/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*(-8595619+7678611*3^(1/2))^(1/2)+3/512*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)-3/512*ln(x^2+3^(1/2)+x*
(-2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1682, 1692, 1690, 1183, 648, 632, 210, 642}

$$\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{256}\sqrt{7678611\sqrt{3}-8595619}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+x^5-9x^3+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\,\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{(252x^2+3305)\,x}{64\,(x^4+2x^2+3)}-\frac{25(7x^2+15)\,x}{16\,(x^4+2x^2+3)^2}+58x$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (3*Sqrt
[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(
1 + Sqrt[3])]])/256 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[S
qrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{2250-2850x^2-4800x^4+2400x^6-672x^{10}}{(3+2x^2+x^4)^2}$$

$$= -\frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} + \frac{\int \frac{-201960+193248x^2+87552x^4-78336x}{3+2x^2+x^4}}{4608}$$

$$= -\frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} + \frac{\int \Big(267264-124416x^2+23040}{4608}$$

$$= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} - \frac{3}{64}\int \frac{4647-1}{3+2x^2}$$

$$= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} - \frac{1}{256}\sqrt{3\Big(1+}$$

$$= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} - \frac{1}{256}\Big(3\sqrt{7220}$$

$$= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} + \frac{3}{512}\sqrt{8595619}$$

$$= 58x-9x^3+x^5 - \frac{25x(15+7x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(3305+252x^2)}{64\,(3+2x^2+x^4)} + \frac{3}{256}\sqrt{-85956}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.14, size = 156, normalized size = 0.64

$$58x-9x^3+x^5-\frac{25x(15+7x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{x(3305+252x^2)}{64\left(3+2x^2+x^4\right)}+\frac{3\left(4795i+148\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}}+\frac{3\left(-4795i+148\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*Sqrt[2])*ArcTan[x/S
qrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (3*(-4795*I + 148*Sqrt
[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [A]
time = 0.04, size = 298, normalized size = 1.23

method	result
risch	$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{\left(x^4+2x^2+3\right)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(148\_R^2-4647\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{256}$
default	$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{\left(x^4+2x^2+3\right)^2}+\frac{3\left(1697\sqrt{-2+2\sqrt{3}}\sqrt{3}+4795\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\right.}{1024}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4+2*x^2+3)^
2+3/1024*(1697*(-2+2*3^(1/2))^(1/2)*3^(1/2)+4795*(-2+2*3^(1/2))^(1/2))*ln(x
^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))+3/256*(-6196*3^(1/2)+1/2*(1697*(-2+2*3^(
1/2))^(1/2)*3^(1/2)+4795*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+3/1024
*(-1697*(-2+2*3^(1/2))^(1/2)*3^(1/2)-4795*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1
/2)+x*(-2+2*3^(1/2))^(1/2))+3/256*(-6196*3^(1/2)-1/2*(-1697*(-2+2*3^(1/2))^
(1/2)*3^(1/2)-4795*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 + 4
*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 - 4647)/(x^4 + 2*x^2
+ 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(190) = 380$.
time = 0.39, size = 568, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/18808834881088512*(18808834881088512*x^13 - 94044174405442560*x^11 + 6018
82716194832384*x^9 + 2970620359031916864*x^7 + 10166469141273357744*x^5 + 5
7410392*2183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(-66002414605209*sqrt(3) + 176883200667963)*arctan(1/86354562146602196340 4
537403089353*2183743218123^(3/4)*sqrt(7176299)*sqrt(853179)*sqrt(5510400844
0689*x^2 + 2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqr
t(-66002414605209*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(154
9*sqrt(3) + 148)*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/470135
82817418600331*2183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-660024146
05209*sqrt(3) + 176883200667963) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 574
10392*2183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(
-66002414605209*sqrt(3) + 176883200667963)*arctan(1/8635456214660219634045 3
7403089353*2183743218123^(3/4)*sqrt(7176299)*sqrt(853179)*sqrt(551040084406
89*x^2 - 2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(
-66002414605209*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*
sqrt(3) + 148)*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/47013582
817418600331*2183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605
209*sqrt(3) + 176883200667963) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 13526
491159952810208*x^3 - 2183743218123^(1/4)*(8595619*sqrt(3)*sqrt(2)*(x^8 + 4
*x^6 + 10*x^4 + 12*x^2 + 9) + 23035833*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x
^2 + 9))*sqrt(-66002414605209*sqrt(3) + 176883200667963)*log(53064960200388
9*x^2 + 69107499/7176299*2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*
sqrt(2)*x)*sqrt(-66002414605209*sqrt(3) + 176883200667963) + 53064960200388
9*sqrt(3)) + 2183743218123^(1/4)*(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10
*x^4 + 12*x^2 + 9) + 23035833*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*
sqrt(-66002414605209*sqrt(3) + 176883200667963)*log(530649602003889*x^2 - 6
9107499/7176299*2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x
)*sqrt(-66002414605209*sqrt(3) + 176883200667963) + 530649602003889*sqrt(3)
) + 12291279706746325584*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(221) = 442$.

time = 0.72, size = 1204, normalized size = 4.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x)/(64*x**8
 + 256*x**6 + 640*x**4 + 768*x**2 + 576) - 3*sqrt(8595619/262144 + 7678611*
sqrt(3)/262144)*log(x**2 + x*(-6788*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))
/7176299 - 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 1697
*sqrt(2)*sqrt(8595619 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 1253
83933330562)/18368002813563) - 12180952402524688792 79*sqrt(2)*sqrt(66002414
605209*sqrt(3) + 125383933330562)/10121505820771748524102649 07 - 1343534101
96228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/39544284066890
8030011 + 18391902996311867463806959889/10121505820771748524102649 07 + 5204
579286823805792980*sqrt(3)/395442840668908030011) + 3*sqrt(8595619/262144 +
 7678611*sqrt(3)/262144)*log(x**2 + x*(-1697*sqrt(2)*sqrt(8595619 + 7678611
*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/18368002813563 + 2
313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 6788*sqrt(3)*sqr
t(8595619 + 7678611*sqrt(3))/7176299) - 1218095240252468879279*sqrt(2)*sqrt
(66002414605209*sqrt(3) + 125383933330562)/10121505820771748524102649 07 - 1
34353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/39544
2840668908030011 + 18391902996311867463806959889/10121505820771748524102649
07 + 5204579286823805792980*sqrt(3)/395442840668908030011) - 2*sqrt(-9*sqrt
(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/131072 + 77360571/262144
 + 207322497*sqrt(3)/262144)*atan(110208016881378*x/(22232174302*sqrt(-2*sq
rt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 23035833*s
qrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)*sqrt(
-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 23035
833*sqrt(3))) - 52122411468*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))/(222321
74302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595
619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) +
 8595619 + 23035833*sqrt(3))) - 6941356584*sqrt(8595619 + 7678611*sqrt(3))/
(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)
 + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 12538393333
0562) + 8595619 + 23035833*sqrt(3))) + 5091*sqrt(2)*sqrt(8595619 + 7678611*
sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/(22232174302*sqrt(-
2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 230358
33*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)*s
qrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 2
3035833*sqrt(3)))) - 2*sqrt(-9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)/131072 + 77360571/262144 + 207322497*sqrt(3)/262144)*atan(110208
```

```
016881378*x/(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 1253
83933330562) + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(660024146052
09*sqrt(3) + 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562) + 8595619 + 23035833*sqrt(3))) - 5091*sqrt(2)*sqrt(859561
9 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/(222321
74302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595
619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) +
 8595619 + 23035833*sqrt(3))) + 6941356584*sqrt(8595619 + 7678611*sqrt(3))/
(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)
 + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 12538393333
0562) + 8595619 + 23035833*sqrt(3))) + 52122411468*sqrt(3)*sqrt(8595619 + 7
678611*sqrt(3))/(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) +
125383933330562) + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414
605209*sqrt(3) + 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(
3) + 125383933330562) + 8595619 + 23035833*sqrt(3))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(190) = 380$.
time = 5.73, size = 588, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666
*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3)
+ 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*3^(
1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))*ar
ctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) - 1/27648*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x
^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/27648*sqrt(2)*(666
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(
-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
```

```
18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
qrt(3) + 1/2) + sqrt(3)) + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 841
5*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B]
time = 0.11, size = 184, normalized size = 0.76

$$58\,x+\frac{\frac{63\,x^7}{16}+\frac{3809\,x^5}{64}+\frac{3333\,x^3}{32}+\frac{8415\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}-9\,x^3+x^5-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}-\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}\,3\mathrm{i}}{256}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}+\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}\,3\mathrm{i}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] 58*x - (atan((x*(17191238 - 2^(1/2)*14352598i)^(1/2)*193760073i)/(131072*((
2^(1/2)*900403059231i)/131072 - 986432531643/131072)) - (193760073*2^(1/2)*
x*(17191238 - 2^(1/2)*14352598i)^(1/2))/(262144*((2^(1/2)*900403059231i)/13
1072 - 986432531643/131072)))*(17191238 - 2^(1/2)*14352598i)^(1/2)*3i)/256
+ (atan((x*(2^(1/2)*14352598i + 17191238)^(1/2)*193760073i)/(131072*((2^(1/
2)*900403059231i)/131072 + 986432531643/131072)) + (193760073*2^(1/2)*x*(2^
(1/2)*14352598i + 17191238)^(1/2))/(262144*((2^(1/2)*900403059231i)/131072
+ 986432531643/131072)))*(2^(1/2)*14352598i + 17191238)^(1/2)*3i)/256 + ((8
415*x)/64 + (3333*x^3)/32 + (3809*x^5)/64 + (63*x^7)/16)/(12*x^2 + 10*x^4 +
 4*x^6 + x^8 + 9) - 9*x^3 + x^5
```

3.118 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=242

$$-27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64\,(3+2x^2+x^4)}-\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.$$

```
[Out] -27*x+5/3*x^3+25/16*x*(5*x^2+3)/(x^4+2*x^2+3)^2-1/64*x*(835*x^2+1468)/(x^4+
2*x^2+3)-21/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1/2
))^(1/2)+21/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1/2
))^(1/2)-21/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(34
271+22721*3^(1/2))^(1/2)+21/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*(34271+22721*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1682, 1692, 1690, 1183, 648, 632, 210, 642}

$$-\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{5x^3}{3}-\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\,\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)-\frac{(835x^2+1468)\,x}{64\,(x^4+2x^2+3)}+\frac{25(5x^2+3)\,x}{16\,(x^4+2x^2+3)^2}-27x$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (21*Sqrt[3427
1 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/256 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{-450-1050x^2+2400x^4-672x^8+480x^{10}}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{98496+27432x^2-78336x^4+23040x^6}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{\int \left(-124416+23040x^2+\frac{1512(312}{3+2x}\right.}{4608}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{21}{64}\int \frac{312+137x^2}{3+2x^2+x}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} + \frac{1}{256}\left(7\sqrt{3\left(1+\sqrt{\right.}}\right.$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{1}{512}\left(21\sqrt{-34271}\right.$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{21}{512}\sqrt{-34271+2}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2} - \frac{x(1468+835x^2)}{64(3+2x^2+x^4)} - \frac{21}{256}\sqrt{34271+227}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.13, size = 155, normalized size = 0.64

$$-27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64(3+2x^2+x^4)}+\frac{21\left(-175i+137\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}}+\frac{21\left(175i+137\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*Sqrt[2])*ArcTan[x/Sq
rt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (21*(175*I + 137*Sqrt[2
])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Maple [A]
time = 0.04, size = 295, normalized size = 1.22

method	result
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{835}{64}x^7-\frac{1569}{32}x^5-\frac{4941}{64}x^3-\frac{513}{8}x}{(x^4+2x^2+3)^2}+\frac{21\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(137\_R^2+312)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{5x^3}{3}-27x+\frac{-\frac{835}{64}x^7-\frac{1569}{32}x^5-\frac{4941}{64}x^3-\frac{513}{8}x}{(x^4+2x^2+3)^2}+\frac{21\left(33\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-175\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\right.}{1024}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2*x^2+3)^2+
21/1024*(33*(-2+2*3^(1/2))^(1/2)*3^(1/2)-175*(-2+2*3^(1/2))^(1/2))*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))+21/256*(416*3^(1/2)+1/2*(33*(-2+2*3^(1/2))^(
1/2)*3^(1/2)-175*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+21/1024*(-33*(
-2+2*3^(1/2))^(1/2)*3^(1/2)+175*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+
2*3^(1/2))^(1/2))+21/256*(416*3^(1/2)-1/2*(-33*(-2+2*3^(1/2))^(1/2)*3^(1/2)
+175*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan
((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 312)/(x^4 + 2*x^2 + 3)
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(187) = 374$.
time = 0.38, size = 564, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/954779317248*(1591298862080*x^11 - 19413846117376*x^9 - 99660064046704*x^
7 - 285508852710816*x^5 - 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 1
0*x^4 + 12*x^2 + 9)*sqrt(778671391*sqrt(3) + 1548731523)*arctan(1/197530213
71716480527209*1548731523^(3/4)*sqrt(68163)*sqrt(13679)*sqrt(932401677*x^2
+ 1548731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*s
qrt(3) + 1548731523) + 932401677*sqrt(3))*sqrt(778671391*sqrt(3) + 15487315
23)*(104*sqrt(3) - 137) - 1/21185098503117*1548731523^(3/4)*(104*sqrt(3)*x
- 137*x)*sqrt(778671391*sqrt(3) + 1548731523) + 1/2*sqrt(3)*sqrt(2) - 1/2*s
qrt(2)) - 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9)*sqrt(778671391*sqrt(3) + 1548731523)*arctan(1/19753021371716480527209*1
548731523^(3/4)*sqrt(68163)*sqrt(13679)*sqrt(932401677*x^2 - 1548731523^(1/
4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731
523) + 932401677*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104*sqrt(3)
 - 137) - 1/21185098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)*sqrt(77
8671391*sqrt(3) + 1548731523) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 368738
756006544*x^3 + 21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 1
0*x^4 + 12*x^2 + 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sq
rt(778671391*sqrt(3) + 1548731523)*log(227663533881*x^2 + 3339987/13679*154
8731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3
) + 1548731523) + 227663533881*sqrt(3)) - 21*1548731523^(1/4)*(34271*sqrt(3
)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 68163*sqrt(2)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9))*sqrt(778671391*sqrt(3) + 1548731523)*log(2276635338
81*x^2 - 3339987/13679*1548731523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2
)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 227663533881*sqrt(3)) - 2932365
97809792*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A]
time = 0.36, size = 82, normalized size = 0.34

$$\frac{5x^3}{3} - 27x + \frac{-835x^7 - 3138x^5 - 4941x^3 - 4104x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + 21\,\mathrm{RootSum}\left(17179869184t^4 + 8983937024t^2 + 1548731523, \left(t \mapsto t\log\left(-\frac{1107296256t^3}{310800559} + \frac{438857984t}{310800559} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x**3/3 - 27*x + (-835*x**7 - 3138*x**5 - 4941*x**3 - 4104*x)/(64*x**8 + 2
56*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(17179869184*_t**4 + 89839
37024*_t**2 + 1548731523, Lambda(_t, _t*log(-1107296256*_t**3/310800559 + 4
38857984*_t/310800559 + x)))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(187) = 374$.
time = 4.57, size = 585, normalized size = 2.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 246
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18)
)*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3
) + 1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*
sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 112
32*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 1
8))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 7/110592*sq
rt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 137*3^(3/
4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2
466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt
(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)
*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 27*x - 1/64*(835*x^7 + 3138*x^5 +
4941*x^3 + 4104*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [**B**]
time = 0.94, size = 182, normalized size = 0.75

$$\frac{5\,x^3}{3}-\frac{\frac{835\,x^7}{64}+\frac{1569\,x^5}{32}+\frac{4941\,x^3}{64}+\frac{513\,x}{8}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}-27\,x+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542-\sqrt{2}\,27358{}\mathrm{i}}\,126681219{}\mathrm{i}}{131072\,\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,49405675411{}\mathrm{i}}{16384}\right)}-\frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542-\sqrt{2}\,27358{}\mathrm{i}}}{262144\,\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,49405675411{}\mathrm{i}}{16384}\right)}\right)\,\sqrt{-68542-\sqrt{2}\,27358{}\mathrm{i}}\,21{}\mathrm{i}}{256}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542+\sqrt{2}\,27358{}\mathrm{i}}\,126681219{}\mathrm{i}}{131072\,\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,49405675411{}\mathrm{i}}{16384}\right)}+\frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542+\sqrt{2}\,27358{}\mathrm{i}}}{262144\,\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,49405675411{}\mathrm{i}}{16384}\right)}\right)\,\sqrt{-68542+\sqrt{2}\,27358{}\mathrm{i}}\,21{}\mathrm{i}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (atan((x*(- 2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*494
0567541i)/16384 + 12541440681/131072)) - (126681219*2^(1/2)*x*(- 2^(1/2)*27
358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 + 12541440681/131
072)))*(- 2^(1/2)*27358i - 68542)^(1/2)*21i)/256 - ((513*x)/8 + (4941*x^3)/
64 + (1569*x^5)/32 + (835*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - 27
*x - (atan((x*(2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*
4940567541i)/16384 - 12541440681/131072)) + (126681219*2^(1/2)*x*(2^(1/2)*2
7358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 - 12541440681/13
1072)))*(2^(1/2)*27358i - 68542)^(1/2)*21i)/256 + (5*x^3)/3
```

3.119 $\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=235

$$5x+\frac{25x(3-x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x(11+58x^2)}{64\left(3+2x^2+x^4\right)}+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.$$

```
[Out] 5*x+25/16*x*(-x^2+3)/(x^4+2*x^2+3)^2+7/64*x*(58*x^2+11)/(x^4+2*x^2+3)-1/512
*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1/5
12*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1
/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(827621+117653
1*3^(1/2))^(1/2)-1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2
))*(827621+1176531*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1682, 1692, 1690, 1183, 648, 632, 210, 642}

$$\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\,\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{7(58x^2+11)x}{64\left(x^4+2x^2+3\right)}+\frac{25(3-x^2)x}{16\left(x^4+2x^2+3\right)^2}+5x$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x^2))/(64*(
3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[827621 + 1176531*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt
[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{1}{96}\int \frac{-450+1650x^2-672x^6+480x^8}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}+\frac{\int \frac{-12744-49104x^2+23040x^4}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}+\frac{\int \left(23040-\frac{72(1137+1322x^2)}{3+2x^2+x^4}\right)\,dx}{4608}$$

$$= 5x+\frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}-\frac{1}{64}\int \frac{1137+1322x^2}{3+2x^2+x^4}\,dx$$

$$= 5x+\frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}-\frac{\int \frac{1137\sqrt{2\left(-1+\sqrt{3}\right)}-(1\ldots}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{\ldots}\right.}\ldots}}{128\sqrt{6\left(-1+\ldots\right.}\ldots}$$

$$= 5x+\frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}-\frac{1}{256}\left(1322+379\sqrt{3}\right)\int \ldots$$

$$= 5x+\frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}-\frac{1}{512}\sqrt{-827621+1176531\ldots}$$

$$= 5x+\frac{25x(3-x^2)}{16(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64(3+2x^2+x^4)}+\frac{1}{256}\sqrt{827621+1176531\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.20, size = 138, normalized size = 0.59

$$\frac{1}{256}\left(\frac{4x(3411+5112x^2+4089x^4+1686x^6+320x^8)}{(3+2x^2+x^4)^2}-\frac{i\left(-2644i+185\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{i\left(2644i+185\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)^
2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sq
rt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 +
I*Sqrt[2]])/256
```

Maple [A]
time = 0.04, size = 291, normalized size = 1.24

method	result
risch	$5x+\frac{\frac{203}{32}x^7+\frac{889}{64}x^5+\frac{159}{8}x^3+\frac{531}{64}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-1322\_R^2-1137\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{256}$
default	$5x-\frac{-\frac{203}{32}x^7-\frac{889}{64}x^5-\frac{159}{8}x^3-\frac{531}{64}x}{(x^4+2x^2+3)^2}-\frac{\left(943\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+185\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\cdots}\right.}{1024}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x-(-203/32*x^7-889/64*x^5-159/8*x^3-531/64*x)/(x^4+2*x^2+3)^2-1/1024*(943
*(-2+2*3^(1/2))^(1/2)*3^(1/2)+185*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))-1/256*(1516*3^(1/2)+1/2*(943*(-2+2*3^(1/2))^(1/2)*3^(1/
2)+185*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arct
an((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-1/1024*(-943*(-2+2*3^(1/
2))^(1/2)*3^(1/2)-185*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))
^(1/2))-1/256*(1516*3^(1/2)-1/2*(-943*(-2+2*3^(1/2))^(1/2)*3^(1/2)-185*(-2+
2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 + 10*x^4 + 1
2*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(180) = 360$.
time = 0.38, size = 553, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4759173538071552*(23795867690357760*x^9 + 125374477893572448*x^7 + 304066
571830852752*x^5 - 10534088*4152675581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x
^4 + 12*x^2 + 9)*sqrt(973721762751*sqrt(3) + 4152675581883)*arctan(1/847120
6900375217227324302495633*4152675581883^(3/4)*sqrt(1316761)*sqrt(1822693750
625280321*x^2 + 392177*4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*s
qrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152675581883) + 1822693750625280321*
sqrt(3))*sqrt(973721762751*sqrt(3) + 4152675581883)*(379*sqrt(3) - 1322) -
1/5468081251875840963*4152675581883^(3/4)*(379*sqrt(3)*x - 1322*x)*sqrt(973
721762751*sqrt(3) + 4152675581883) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 1
0534088*4152675581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(973721762751*sqrt(3) + 4152675581883)*arctan(1/84712069003752172273243024
95633*4152675581883^(3/4)*sqrt(1316761)*sqrt(1822693750625280321*x^2 - 3921
77*4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(97372
1762751*sqrt(3) + 4152675581883) + 1822693750625280321*sqrt(3))*sqrt(973721
762751*sqrt(3) + 4152675581883)*(379*sqrt(3) - 1322) - 1/546808125187584096
3*4152675581883^(3/4)*(379*sqrt(3)*x - 1322*x)*sqrt(973721762751*sqrt(3) +
4152675581883) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 380138986353465216*x^
3 - 4152675581883^(1/4)*(827621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*
x^2 + 9) - 3529593*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(973721
762751*sqrt(3) + 4152675581883)*log(1384225193961*x^2 + 392177/1316761*4152
675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751
*sqrt(3) + 4152675581883) + 1384225193961*sqrt(3)) + 4152675581883^(1/4)*(8
27621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 3529593*sqrt(2)
*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(973721762751*sqrt(3) + 415267558
1883)*log(1384225193961*x^2 - 392177/1316761*4152675581883^(1/4)*(1322*sqrt
(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152675581883)
+ 1384225193961*sqrt(3)) + 253649077161907248*x)/(x^8 + 4*x^6 + 10*x^4 + 12
*x^2 + 9)
```

Sympy [A]

time = 0.35, size = 71, normalized size = 0.30

$$5x+\frac{406x^7+889x^5+1272x^3+531x}{64x^8+256x^6+640x^4+768x^2+576}+\operatorname{RootSum}\left(17179869184t^4+216955879424t^2+4152675581883,\left(t\mapsto t\log\left(-\frac{31641829376t^3}{1549210136091}-\frac{455309168896t}{1549210136091}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*x
**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 + 41
52675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309168
896*_t/1549210136091 + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(180) = 360.
time = 3.21, size = 580, normalized size = 2.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
 + 18) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3
^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*
log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/165888*sqrt(2
)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 661*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1189
8*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 5*x + 1/64*(406*x^7 + 889*x^5 + 1272
*x^3 + 531*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B]
time = 0.99, size = 176, normalized size = 0.75

$$5x+\frac{\frac{203x^7}{32}+\frac{889x^5}{64}+\frac{159x^3}{8}+\frac{531x}{64}}{x^8+4x^6+10x^4+12x^2+9}+\frac{\operatorname{atan}\left(\frac{x\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}+\frac{1316761\sqrt{2}\,x\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}}{262144\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}-\frac{\operatorname{atan}\left(\frac{x\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}-\frac{1316761\sqrt{2}\,x\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}}{262144\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)

[Out] 5*x + (atan((x*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(1/2)*1497157257i)/131072 - 3725116869/131072)) + (1316761*2^(1/2)*x*(- 2^(1/2)*2633522i - 1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 - 3725116869/131072)))*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(1/2)*1497157257i)/131072 + 3725116869/131072)) - (1316761*2^(1/2)*x*(2^(1/2)*2633522i - 1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 + 3725116869/131072)))*(2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 + ((531*x)/64 + (159*x^3)/8 + (889*x^5)/64 + (203*x^7)/32)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)

3.120 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=238

$$-\frac{25x(3+x^2)}{16\left(3+2x^2+x^4\right)^2}+\frac{x(238-59x^2)}{64\left(3+2x^2+x^4\right)}-\frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] -25/16*x*(x^2+3)/(x^4+2*x^2+3)^2+1/64*x*(-59*x^2+238)/(x^4+2*x^2+3)-1/256*a
rctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-146505+98481*3^(1/
2))^(1/2)+1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-14
6505+98481*3^(1/2))^(1/2)+1/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(146
505+98481*3^(1/2))^(1/2)-1/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(1465
05+98481*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1682, 1692, 1183, 648, 632, 210, 642}

$$-\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{256}\sqrt{3\left(32827\sqrt{3}-48835\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{x(238-59x^2)}{64\left(x^4+2x^2+3\right)}-\frac{25x(x^2+3)}{16\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(64*(3 + 2*
x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(-48835 + 32827*Sqrt[3])]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3
*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/51
2 - (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
 + x^2])/512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{450-750x^2-672x^4+480x^6}{(3+2x^2+x^4)^2}\,dx$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-9936+18792x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{\int \frac{-9936\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-993\right.}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}}}{9216\sqrt{6\left(-1+\sqrt{3}\right.}}$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{1}{256}\left(261-46\sqrt{3}\right)\int \frac{}{\sqrt{3}-}$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} + \frac{1}{512}\sqrt{146505+98481\sqrt{3}}\,\log$$

$$= -\frac{25x(3+x^2)}{16(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64(3+2x^2+x^4)} - \frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 129, normalized size = 0.54

$$\frac{1}{256}\left(\frac{4x(414+199x^2+120x^4-59x^6)}{(3+2x^2+x^4)^2} + \frac{3\left(174+133i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3\left(174-133i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 + (
133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(17
```

```
4 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/25
6
```

Maple [A]
time = 0.04, size = 287, normalized size = 1.21

method	result
risch	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(87\_R^2-46)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{\left(307\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+399\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{1024}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+1/1024*(307*(-2+2
*3^(1/2))^(1/2)*3^(1/2)+399*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^
(1/2))^(1/2))+1/256*(-184*3^(1/2)+1/2*(307*(-2+2*3^(1/2))^(1/2)*3^(1/2)+399
*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*
x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1/1024*(-307*(-2+2*3^(1/2))^(1
/2)*3^(1/2)-399*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2)
)+1/256*(-184*3^(1/2)-1/2*(-307*(-2+2*3^(1/2))^(1/2)*3^(1/2)-399*(-2+2*3^(1
/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(
1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(177) = 354$.
time = 0.37, size = 548, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/2076490005504*(1914264223824*x^7 - 3893418760320*x^5 + 164728*2909552208
3^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3
) + 3232835787)*arctan(1/1214880276996365518761363*29095522083^(3/4)*sqrt(2
0591)*sqrt(199701965070351*x^2 + 98481*29095522083^(1/4)*(87*sqrt(3)*sqrt(2
)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 1997019650703
51*sqrt(3))*(46*sqrt(3) + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/59
9105895211053*29095522083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqr
t(3) + 3232835787) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 164728*2909552208
3^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3
) + 3232835787)*arctan(1/1214880276996365518761363*29095522083^(3/4)*sqrt(2
0591)*sqrt(199701965070351*x^2 - 98481*29095522083^(1/4)*(87*sqrt(3)*sqrt(2
)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 1997019650703
51*sqrt(3))*(46*sqrt(3) + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/59
9105895211053*29095522083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqr
t(3) + 3232835787) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 6456586110864*x^3
 + 29095522083^(1/4)*(48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) + 98481*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*
sqrt(3) + 3232835787)*log(9698507361*x^2 + 98481/20591*29095522083^(1/4)*(8
7*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787)
+ 9698507361*sqrt(3)) - 29095522083^(1/4)*(48835*sqrt(3)*sqrt(2)*(x^8 + 4*x
^6 + 10*x^4 + 12*x^2 + 9) + 98481*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
9))*sqrt(-1603106545*sqrt(3) + 3232835787)*log(9698507361*x^2 - 98481/20591
*29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x)*sqrt(-1603106545*s
qrt(3) + 3232835787) + 9698507361*sqrt(3)) - 13432294723104*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(201) = 402$.
time = 0.70, size = 1198, normalized size = 5.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-59*x**7 + 120*x**5 + 199*x**3 + 414*x)/(64*x**8 + 256*x**6 + 640*x**4 + 7
68*x**2 + 576) - sqrt(146505/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*(-
307*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884650
6)/675940757 + 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 122
8*sqrt(48835 + 32827*sqrt(3))/20591) - 941929306825573*sqrt(2)*sqrt(1603106
545*sqrt(3) + 2808846506)/456895906973733049 - 47771215762*sqrt(6)*sqrt(160
3106545*sqrt(3) + 2808846506)/41754888382161 + 9747794966790882353/4568959
```

```
06973733049 + 5200450130596150*sqrt(3)/41754888382161) + sqrt(146505/262144
 + 98481*sqrt(3)/262144)*log(x**2 + x*(-1228*sqrt(48835 + 32827*sqrt(3))/20
591 - 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 307*sqrt(6)*
sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/675940757
) - 941929306825573*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/456895906
973733049 - 47771215762*sqrt(6)*sqrt(1603106545*sqrt(3) + 2808846506)/41754
888382161 + 974779496667908823 53/456895906973733049 + 5200450130596150*sqrt
(3)/41754888382161) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 280884650
6)/131072 + 146505/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*
x/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2
*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) -
40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sq
rt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3)
 + 2808846506) + 48835 + 98481*sqrt(3))) - 31879062*sqrt(48835 + 32827*sqrt
(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 4883
5 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)))
 + 921*sqrt(2)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884
6506)/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 488
35 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqr
t(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
)) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/131072 + 14650
5/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*x/(-1894372*sqrt(
-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) +
 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(160
3106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) - 921*sqrt(2)*sqrt(
48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/(-1894372*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
+ 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(16
03106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) + 31879062*sqrt(48
835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 28
08846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) +
2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3))) + 40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*s
qrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3
)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(177) = 354.

time = 4.62, size = 577, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/
4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/1843
2*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(2)*
sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt
(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x -
 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/36864*sqrt(
2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sq
rt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(
3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(
3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/
6*sqrt(3) + 1/2) + sqrt(3)) + 1/36864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) +
29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sq
rt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/
64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B]
time = 0.15, size = 173, normalized size = 0.73

$$\frac{-\frac{59\,x^7}{64}+\frac{15\,x^5}{8}+\frac{199\,x^3}{64}+\frac{207\,x}{32}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\,\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}+\frac{61773\,\sqrt{2}\,x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}}{262144\,\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\,\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}-\frac{61773\,\sqrt{2}\,x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}}{262144\,\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] ((207*x)/32 + (199*x^3)/64 + (15*x^5)/8 - (59*x^7)/64)/(12*x^2 + 10*x^4 + 4
*x^6 + x^8 + 9) + (atan((x*(293010 - 2^(1/2)*123546i)^(1/2)*61773i)/(131072
*((2^(1/2)*4262337i)/65536 + 56892933/131072)) + (61773*2^(1/2)*x*(293010 -
 2^(1/2)*123546i)^(1/2))/(262144*((2^(1/2)*4262337i)/65536 + 56892933/13107
2)))*(293010 - 2^(1/2)*123546i)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*123546i +
 293010)^(1/2)*61773i)/(131072*((2^(1/2)*4262337i)/65536 - 56892933/131072)
) - (61773*2^(1/2)*x*(2^(1/2)*123546i + 293010)^(1/2))/(262144*((2^(1/2)*42
62337i)/65536 - 56892933/131072)))*(2^(1/2)*123546i + 293010)^(1/2)*1i)/256
```

3.121 $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=246

$$\frac{25x(1+x^2)}{16\left(3+2x^2+x^4\right)^2}-\frac{x(353+88x^2)}{192\left(3+2x^2+x^4\right)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)$$

```
[Out] 25/16*x*(x^2+1)/(x^4+2*x^2+3)^2-1/192*x*(88*x^2+353)/(x^4+2*x^2+3)-11/2304*
arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5475+3267*3^(1/2)
)^(1/2)+11/2304*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-54
75+3267*3^(1/2))^(1/2)-11/4608*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5475
+3267*3^(1/2))^(1/2)+11/4608*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5475+3
267*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1682, 1692, 1183, 648, 632, 210, 642}

$$-\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{11}{768}\sqrt{\frac{1}{3}\left(1089\sqrt{3}-1825\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{25x(x^2+1)}{16\left(x^4+2x^2+3\right)^2}-\frac{x(88x^2+353)}{192\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(192*(3 + 2*
x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[
3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11*Sqrt[(-1825 + 1089*Sqrt[3])/3
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*S
qrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/1536 + (11*Sqrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/1536
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}+\frac{1}{96}\int \frac{-150+78x^2+480x^4}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192\,(3+2x^2+x^4)}+\frac{\int \frac{6072-2112x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192\,(3+2x^2+x^4)}+\frac{\int \frac{6072\sqrt{2\left(-1+\sqrt{3}\right)}-\left(6072+\right.}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}}}{9216\sqrt{6\left(-1+\sqrt{3}\right.}}$$

$$= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192\,(3+2x^2+x^4)}-\frac{\left(11\left(24-23\sqrt{3}\right)\right)\int \frac{}{\sqrt{3}-\sqrt{}}}{2304}$$

$$= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192\,(3+2x^2+x^4)}-\frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\,\log\left(\right.$$

$$= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192\,(3+2x^2+x^4)}-\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 133, normalized size = 0.54

$$\frac{1}{768}\left(-\frac{4x(759+670x^2+529x^4+88x^6)}{(3+2x^2+x^4)^2}-\frac{11i\left(-16i+31\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{11i\left(16i+31\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

[Out] ((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(-16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((11*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768

Maple [A]
time = 0.04, size = 287, normalized size = 1.17

method	result
risch	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(-8\_R^2+23)\ln(x-\_R)}{\_R^3+\_R}\right)}{768}$
default	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}-\frac{11\left(47\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+93\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{9216}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)

[Out] (-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2-11/9216*(47*(-2+2*3^(1/2))^(1/2)*3^(1/2)+93*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))-11/2304*(-92*3^(1/2)+1/2*(47*(-2+2*3^(1/2))^(1/2)*3^(1/2)+93*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+11/9216*(47*(-2+2*3^(1/2))^(1/2)*3^(1/2)+93*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))+11/2304*(92*3^(1/2)-1/2*(47*(-2+2*3^(1/2))^(1/2)*3^(1/2)+93*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] -1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(177) = 354$.
time = 0.39, size = 574, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/27952128*(12811392*x^7 + 77013936*x^5 + 1348*sqrt(6)*3^(3/4)*sqrt(2)*(x^
8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan(1/
2226179538*sqrt(337)*sqrt(11)*sqrt(6)*3^(3/4)*sqrt(sqrt(6)*3^(1/4)*(8*sqrt(
3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*
(23*sqrt(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/2001
78*sqrt(6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt
(3) + 3557763) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 1348*sqrt(6)*3^(3/4)*
sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763
)*arctan(1/4848619033764*sqrt(337)*sqrt(6)*3^(3/4)*sqrt(-52180524*sqrt(6)*3
^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 174089882221
2*x^2 + 1740898822212*sqrt(3))*(23*sqrt(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987
425*sqrt(3) + 3557763) - 1/200178*sqrt(6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 2
4*sqrt(2)*x)*sqrt(-1987425*sqrt(3) + 3557763) + 1/2*sqrt(3)*sqrt(2) - 1/2*s
qrt(2)) - sqrt(6)*3^(1/4)*(3267*x^8 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1
825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt
(3) + 3557763)*log(52180524/337*sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-
1987425*sqrt(3) + 3557763) + 5165871876*x^2 + 5165871876*sqrt(3)) + sqrt(6)
*3^(1/4)*(3267*x^8 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1825*sqrt(3)*(x^8
+ 4*x^6 + 10*x^4 + 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt(3) + 3557763)*lo
g(-52180524/337*sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3)
+ 3557763) + 5165871876*x^2 + 5165871876*sqrt(3)) + 97541280*x^3 + 11049825
6*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(207) = 414$.
time = 0.72, size = 1200, normalized size = 4.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-88*x**7 - 529*x**5 - 670*x**3 - 759*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
 2304*x**2 + 1728) - sqrt(220825/7077888 + 14641*sqrt(3)/786432)*log(x**2 +
 x*(-47*sqrt(6)*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/3
66993 + 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 188*sqrt(1825 + 10
89*sqrt(3))/337) - 24765218375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)/1346
83862049 - 38128468*sqrt(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923 + 904
13874433403/134683862049 + 144251139148*sqrt(3)/371029923) + sqrt(220825/70
77888 + 14641*sqrt(3)/786432)*log(x**2 + x*(-188*sqrt(1825 + 1089*sqrt(3))/
337 - 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 47*sqrt(6)*sqrt(1825
 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/366993) - 24765218375*sqrt
```

```
(2)*sqrt(1987425*sqrt(3) + 3444194)/134683862049 - 38128468*sqrt(6)*sqrt(19
87425*sqrt(3) + 3444194)/371029923 + 90413874433403/134683862049 + 14425113
9148*sqrt(3)/371029923) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqrt(3) + 344419
4)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(733986*sqrt(3)*x/(
15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)
) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425
*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 204732*sqrt(3)*sqrt(1825 + 10
89*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 +
 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)
*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 156048*sqrt(1825
 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1
825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sq
rt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) + 141*sqrt(2)
*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/(15502*sqrt(-2*s
qrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*
sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444
194) + 1825 + 3267*sqrt(3)))) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqrt(3) +
3444194)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(733986*sqrt(
3)*x/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*s
qrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1
987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 141*sqrt(2)*sqrt(1825 +
 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/(15502*sqrt(-2*sqrt(2)*sqrt(
1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425
*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825
+ 3267*sqrt(3))) + 156048*sqrt(1825 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*
sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(19
87425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) +
1825 + 3267*sqrt(3))) + 204732*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/(15502*sqr
t(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sq
rt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3)
+ 3444194) + 1825 + 3267*sqrt(3))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(177) = 354$.
time = 4.57, size = 577, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 11/124416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6
*sqrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
```

```
(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/124416
*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(2)*sqr
t(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sqrt(6*sqrt
(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1
/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/248832*sqrt(2)*
(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)
*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18
) + 207*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(
3) + 1/2) + sqrt(3)) - 11/248832*sqrt(2)*(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/
4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) +
 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 207*3^(1/4)*sqrt(6*sqrt(3) + 1
8))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/192*(88*x
^7 + 529*x^5 + 670*x^3 + 759*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B]
time = 1.01, size = 174, normalized size = 0.71

$$-\frac{\frac{11\,x^7}{24}+\frac{529\,x^5}{192}+\frac{335\,x^3}{96}+\frac{253\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950-\sqrt{2}\,2022{}\mathrm{i}}\,448547{}\mathrm{i}}{31850496\,\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581{}\mathrm{i}}{10616832}\right)}-\frac{448547\,\sqrt{2}\,x\,\sqrt{10950-\sqrt{2}\,2022{}\mathrm{i}}}{63700992\,\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581{}\mathrm{i}}{10616832}\right)}\right)\,\sqrt{10950-\sqrt{2}\,2022{}\mathrm{i}}\,11{}\mathrm{i}}{2304}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950+\sqrt{2}\,2022{}\mathrm{i}}\,448547{}\mathrm{i}}{31850496\,\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581{}\mathrm{i}}{10616832}\right)}+\frac{448547\,\sqrt{2}\,x\,\sqrt{10950+\sqrt{2}\,2022{}\mathrm{i}}}{63700992\,\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581{}\mathrm{i}}{10616832}\right)}\right)\,\sqrt{10950+\sqrt{2}\,2022{}\mathrm{i}}\,11{}\mathrm{i}}{2304}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (atan((x*(10950 - 2^(1/2)*2022i)^(1/2)*448547i)/(31850496*((2^(1/2)*1031658
1i)/10616832 - 21081709/10616832)) - (448547*2^(1/2)*x*(10950 - 2^(1/2)*202
2i)^(1/2))/(63700992*((2^(1/2)*10316581i)/10616832 - 21081709/10616832)))*(
10950 - 2^(1/2)*2022i)^(1/2)*11i)/2304 - ((253*x)/64 + (335*x^3)/96 + (529*
x^5)/192 + (11*x^7)/24)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - (atan((x*(2^(
1/2)*2022i + 10950)^(1/2)*448547i)/(31850496*((2^(1/2)*10316581i)/10616832
+ 21081709/10616832)) + (448547*2^(1/2)*x*(2^(1/2)*2022i + 10950)^(1/2))/(6
3700992*((2^(1/2)*10316581i)/10616832 + 21081709/10616832)))*(2^(1/2)*2022i
 + 10950)^(1/2)*11i)/2304
```

3.122 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3} \, dx$

Optimal. Leaf size=248

$$\frac{25x(1-x^2)}{48(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192(3+2x^2+x^4)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+$$

```
[Out] 25/48*x*(-x^2+1)/(x^4+2*x^2+3)^2+1/192*x*(51*x^2+64)/(x^4+2*x^2+3)-1/768*ar
ctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3057*3^(1/2))^
(1/2)+1/768*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3
057*3^(1/2))^(1/2)+1/1536*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(3873+3057
*3^(1/2))^(1/2)-1/1536*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(3873+3057*3^
(1/2))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1692, 1192, 1183, 648, 632, 210, 642}

$$-\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{256}\sqrt{\frac{1}{3}\left(1019\sqrt{3}-1291\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}+\frac{x(51x^2+64)}{192(x^4+2x^2+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(192*(3 + 2*x
^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
 - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTa
n[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291
+ 1019*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sq
rt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{1}{96}\int \frac{78+230x^2}{(3+2x^2+x^4)^2}\,dx$$

$$= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}+\frac{\int \frac{-288+1224x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}+\frac{\int \frac{-288\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-288-1224\sqrt{\ldots}\right.}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2 \ldots}}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}+\frac{1}{768}\left(51-4\sqrt{3}\right)\int \frac{\ldots}{\sqrt{3}-\sqrt{2\left(-1\ldots\right.}}$$

$$= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\,\log\left(\sqrt{\ldots}\right.$$

$$= \frac{25x(1-x^2)}{48\,(3+2x^2+x^4)^2}+\frac{x(64+51x^2)}{192\,(3+2x^2+x^4)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\,\tan^{-1}\ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 129, normalized size = 0.52

$$\frac{1}{768}\left(\frac{4x(292+181x^2+166x^4+51x^6)}{(3+2x^2+x^4)^2}+\frac{3\left(34+21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{3\left(34-21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (2
1*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34 -
 (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

Maple [A]
time = 0.05, size = 287, normalized size = 1.16

method	result
risch	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(17\_R^2-4)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(55\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+63\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{3072}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+1/3072*(55*(-2+2*
3^(1/2))^(1/2)*3^(1/2)+63*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1
/2))^(1/2))+1/768*(-16*3^(1/2)+1/2*(55*(-2+2*3^(1/2))^(1/2)*3^(1/2)+63*(-2+
2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1/3072*(-55*(-2+2*3^(1/2))^(1/2)*3^
(1/2)-63*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))+1/768
*(-16*3^(1/2)-1/2*(-55*(-2+2*3^(1/2))^(1/2)*3^(1/2)-63*(-2+2*3^(1/2))^(1/2)
)*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2
))/(2+2*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(177) = 354$.
time = 0.38, size = 580, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/7991829504*(2122829712*x^7 + 6909602592*x^5 - 3404*3115083^(1/4)*sqrt(6)*
sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) +
```

```
 3115083)*arctan(1/41378565634793586*3115083^(3/4)*sqrt(3057)*sqrt(851)*sqr
t(6)*sqrt(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3)
+ 3115083) + 2601507*x^2 + 2601507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2)
)*sqrt(-1315529*sqrt(3) + 3115083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4
*sqrt(3)*sqrt(2)*x + 51*sqrt(2)*x)*sqrt(-1315529*sqrt(3) + 3115083) - 1/2*s
qrt(3)*sqrt(2) + 1/2*sqrt(2)) - 3404*3115083^(1/4)*sqrt(6)*sqrt(3)*sqrt(2)*
(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) + 3115083)*arctan
(1/82757131269587172*3115083^(3/4)*sqrt(851)*sqrt(6)*sqrt(-12228*3115083^(1
/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 3181122
7596*x^2 + 31811227596*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(-1315
529*sqrt(3) + 3115083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3)*sqr
t(2)*x + 51*sqrt(2)*x)*sqrt(-1315529*sqrt(3) + 3115083) + 1/2*sqrt(3)*sqrt(
2) - 1/2*sqrt(2)) - 3115083^(1/4)*sqrt(6)*(3057*x^8 + 12228*x^6 + 30570*x^4
 + 36684*x^2 + 1291*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 27513)*sq
rt(-1315529*sqrt(3) + 3115083)*log(12228/851*3115083^(1/4)*sqrt(6)*(17*sqrt
(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 37380996*x^2 + 37380996*sqr
t(3)) + 3115083^(1/4)*sqrt(6)*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2
 + 1291*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*
sqrt(3) + 3115083)*log(-12228/851*3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x
)*sqrt(-1315529*sqrt(3) + 3115083) + 37380996*x^2 + 37380996*sqrt(3)) + 753
3964272*x^3 + 12154240704*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(201) = 402$.
time = 0.69, size = 1195, normalized size = 4.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
2304*x**2 + 1728) - sqrt(1291/786432 + 1019*sqrt(3)/786432)*log(x**2 + x*(-
55*sqrt(6)*sqrt(1291 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169
 + 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 220*sqrt(1291 + 1019*sq
rt(3))/851) - 26628761029*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/751982074
561 - 40176070*sqrt(6)*sqrt(1315529*sqrt(3) + 2390882)/2213882457 + 7609499
4709709/751982074561 + 133967471914*sqrt(3)/2213882457) + sqrt(1291/786432
+ 1019*sqrt(3)/786432)*log(x**2 + x*(-220*sqrt(1291 + 1019*sqrt(3))/851 - 4
9606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 55*sqrt(6)*sqrt(1291 + 1019
*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169) - 26628761029*sqrt(2)*sqr
t(1315529*sqrt(3) + 2390882)/751982074561 - 40176070*sqrt(6)*sqrt(1315529*s
qrt(3) + 2390882)/2213882457 + 76094994709709/751982074561 + 133967471914*s
qrt(3)/2213882457) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/393216
 + 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/(-6808*sqrt(-2
```

```
*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2
)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 23
90882) + 1291 + 3057*sqrt(3))) - 224180*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/(
-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)
) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529
*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 148818*sqrt(1291 + 1019*sqrt(
3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sq
rt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(13
15529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) + 165*sqrt(2)*sqrt(1291 +
1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/(-6808*sqrt(-2*sqrt(2)*sqrt(1
315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*
sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 +
 3057*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/393216 +
 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/(-6808*sqrt(-2*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*
sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390
882) + 1291 + 3057*sqrt(3))) - 165*sqrt(2)*sqrt(1291 + 1019*sqrt(3))*sqrt(1
315529*sqrt(3) + 2390882)/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 239
0882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*s
qrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) + 14
8818*sqrt(1291 + 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3)
+ 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 23908
82)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)))
 + 224180*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(131
5529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sq
rt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3
057*sqrt(3))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(177) = 354.
time = 3.99, size = 577, normalized size = 2.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/165
888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2)*sqrt
```

```
(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
 - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/331776*sq
rt(2)*(306*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)
*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sq
rt(3) + 18) + 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/331776*sqrt(2)*(306*3^(3/4)*sqrt(2)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2
) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(6*sqrt(3) + 18)*(s
qrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 144*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B]
time = 1.01, size = 173, normalized size = 0.70

$$\frac{\frac{17\,x^7}{64}+\frac{83\,x^5}{96}+\frac{181\,x^3}{192}+\frac{73\,x}{48}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746-\sqrt{2}\,5106{}\mathrm{i}}\,851{}\mathrm{i}}{1179648\,\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851{}\mathrm{i}}{98304}\right)}+\frac{851\,\sqrt{2}\,x\,\sqrt{7746-\sqrt{2}\,5106{}\mathrm{i}}}{2359296\,\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851{}\mathrm{i}}{98304}\right)}\right)\,\sqrt{7746-\sqrt{2}\,5106{}\mathrm{i}}\,1{}\mathrm{i}}{768}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746+\sqrt{2}\,5106{}\mathrm{i}}\,851{}\mathrm{i}}{1179648\,\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851{}\mathrm{i}}{98304}\right)}-\frac{851\,\sqrt{2}\,x\,\sqrt{7746+\sqrt{2}\,5106{}\mathrm{i}}}{2359296\,\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851{}\mathrm{i}}{98304}\right)}\right)\,\sqrt{7746+\sqrt{2}\,5106{}\mathrm{i}}\,1{}\mathrm{i}}{768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] ((73*x)/48 + (181*x^3)/192 + (83*x^5)/96 + (17*x^7)/64)/(12*x^2 + 10*x^4 +
4*x^6 + x^8 + 9) + (atan((x*(7746 - 2^(1/2)*5106i)^(1/2)*851i)/(1179648*((2
^(1/2)*851i)/98304 + 46805/393216)) + (851*2^(1/2)*x*(7746 - 2^(1/2)*5106i)
^(1/2))/(2359296*((2^(1/2)*851i)/98304 + 46805/393216)))*(7746 - 2^(1/2)*51
06i)^(1/2)*1i)/768 - (atan((x*(2^(1/2)*5106i + 7746)^(1/2)*851i)/(1179648*(
(2^(1/2)*851i)/98304 - 46805/393216)) - (851*2^(1/2)*x*(2^(1/2)*5106i + 774
6)^(1/2))/(2359296*((2^(1/2)*851i)/98304 - 46805/393216)))*(2^(1/2)*5106i +
 7746)^(1/2)*1i)/768
```

3.123 $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=253

$$-\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.}{2304}$$

```
[Out] -4/27/x-25/144*x*(x^2+5)/(x^4+2*x^2+3)^2-1/1728*x*(242*x^2+325)/(x^4+2*x^2+
3)-1/13824*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2))^
(1/2)+1/13824*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2
))^(1/2)+1/6912*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(17
9133+165483*3^(1/2))^(1/2)-1/6912*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^
(1/2))^(1/2))*(179133+165483*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}-\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304}-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}-\frac{25x(x^2+5)}{144\left(x^4+2x^2+3\right)^2}-\frac{x(242x^2+325)}{1728\left(x^4+2x^2+3\right)}-\frac{4}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325 + 242*x^2)
)/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[
2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(59711 + 5516
1*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/2304 - (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/4608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2
*(-1 + Sqrt[3])]*x + x^2])/4608
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{128+30x^2-\frac{250x^4}{3}}{x^2\left(3+2x^2+x^4\right)^2}\,dx$$

$$= -\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}+\frac{\int \frac{2048-\frac{56x^2}{3}-\frac{1936x^4}{3}}{x^2(3+2x^2+x^4)}\,dx}{4608}$$

$$= -\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}+\frac{\int \left(\frac{2048}{3x^2}-\frac{8(173+166x^2)}{3+2x^2+x^4}\right)dx}{4608}$$

$$= -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}-\frac{1}{576}\int \frac{173+166x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}-\frac{\int \frac{173\sqrt{2\left(-1+\sqrt{3}\right)}\cdots}{\sqrt{3}-\sqrt{2\left(-1+\cdots\right.}}}{1152\sqrt{6\left(-1\cdots\right.}}$$

$$= -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\cdots\right.}}{\cdots}$$

$$= -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\cdots\right.}}{\cdots}$$

$$= -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\cdots\right.}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 140, normalized size = 0.55

$$\frac{-\frac{12\left(768+1849x^2+1412x^4+611x^6+166x^8\right)}{x\left(3+2x^2+x^4\right)^2}+\frac{3i\left(332i+7\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{3i\left(-332i+7\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]

[Out] ((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4)^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/6912

Maple [A]
time = 0.05, size = 293, normalized size = 1.16

method	result
risch	$\frac{-\frac{83}{288}x^8-\frac{611}{576}x^6-\frac{353}{144}x^4-\frac{1849}{576}x^2-\frac{4}{3}}{x(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^4+238844\_Z^2+3042735921\right)}\_R\ln\left(-1950\_R^3-37653769\_R+290913\right.\right.}{2304}$
default	$-\frac{\frac{121}{32}x^7+\frac{809}{64}x^5+\frac{419}{16}x^3+\frac{2475}{64}x}{27(x^4+2x^2+3)^2}-\frac{\left(325\sqrt{-2+2\sqrt{3}}\sqrt{3}-21\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right)}{27648}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)

[Out] -1/27*(121/32*x^7+809/64*x^5+419/16*x^3+2475/64*x)/(x^4+2*x^2+3)^2-1/27648*(325*(-2+2*3^(1/2))^(1/2)*3^(1/2)-21*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))-1/6912*(692*3^(1/2)+1/2*(325*(-2+2*3^(1/2))^(1/2)*3^(1/2)-21*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-1/27648*(-325*(-2+2*3^(1/2))^(1/2)*3^(1/2)+21*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))-1/6912*(692*3^(1/2)-1/2*(-325*(-2+2*3^(1/2))^(1/2)*3^(1/2)+21*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-4/27/x

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] -1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(182) = 364$.

time = 0.40, size = 633, normalized size = 2.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/2978955242496*(858518351136*x^8 + 3159968147856*x^6 + 210956*1391283^(1/
4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*
sqrt(59711*sqrt(3) + 165483)*arctan(1/158116656523365388898*1391283^(3/4)*sq
rt(52739)*sqrt(681)*sqrt(227)*sqrt(6)*sqrt(1391283^(1/4)*sqrt(681)*sqrt(6)*
(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 107745777*x^2 + 1077
45777*sqrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(59711*sqrt(3) + 165
483) - 1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(173*sqrt(3)*sqrt(2)*
x - 498*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) + 1/2*sqrt(3)*sqrt(2) - 1/2
*sqrt(2)) + 210956*1391283^(1/4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4
*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(59711*sqrt(3) + 165483)*arctan(1/7684469
507035557904428*1391283^(3/4)*sqrt(52739)*sqrt(681)*sqrt(6)*sqrt(-53616492*
1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3)
+ 165483) + 5776950590554284*x^2 + 5776950590554284*sqrt(3))*(173*sqrt(3)*s
qrt(2) - 498*sqrt(2))*sqrt(59711*sqrt(3) + 165483) - 1/440249244822*1391283
^(3/4)*sqrt(681)*sqrt(6)*(173*sqrt(3)*sqrt(2)*x - 498*sqrt(2)*x)*sqrt(59711
*sqrt(3) + 165483) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 7302577781952*x^4
 - 1391283^(1/4)*sqrt(681)*sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 +
 1985796*x^3 - 59711*sqrt(3)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 148934
7*x)*sqrt(59711*sqrt(3) + 165483)*log(53616492/52739*1391283^(1/4)*sqrt(681
)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 1095384931
56*x^2 + 109538493156*sqrt(3)) + 1391283^(1/4)*sqrt(681)*sqrt(6)*(165483*x^
9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 59711*sqrt(3)*(x^9 + 4*x^7 + 1
0*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*sqrt(3) + 165483)*log(-536164
92/52739*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711
*sqrt(3) + 165483) + 109538493156*x^2 + 109538493156*sqrt(3)) + 95626532003
04*x^2 + 3971940323328)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)
```

Sympy [A]
time = 0.36, size = 75, normalized size = 0.30

$$\frac{-166x^8-611x^6-1412x^4-1849x^2-768}{576x^9+2304x^7+5760x^5+6912x^3+5184x}+\operatorname{RootSum}\left(4174708211712t^4+15652880384t^2+37564641,\left(t\mapsto t\log\left(-\frac{98146713600t^3}{11971753}-\frac{9639364864t}{323237331}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-166*x**8 - 611*x**6 - 1412*x**4 - 1849*x**2 - 768)/(576*x**9 + 2304*x**7
+ 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(4174708211712*_t**4 + 156528803
84*_t**2 + 37564641, Lambda(_t, _t*log(-98146713600*_t**3/11971753 - 963936
4864*_t/323237331 + x)))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(182) = 364.
time = 4.18, size = 582, normalized size = 2.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/14
92992*sqrt(2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 8
3*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 1494*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)
*sqrt(-6*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(
1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1492992*sqrt(2)*(1494*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 83*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sqrt(6*s
qrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 3
114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
 1/2) + sqrt(3)) - 1/1728*(242*x^7 + 809*x^5 + 1676*x^3 + 2475*x)/(x^4 + 2*
x^2 + 3)^2 - 4/27/x
```

Mupad [**B**]
time = 0.99, size = 179, normalized size = 0.71

$$-\frac{\frac{83x^8}{288}+\frac{611x^6}{576}+\frac{353x^4}{144}+\frac{1849x^2}{576}+\frac{4}{3}}{x^9+4x^7+10x^5+12x^3+9x}+\frac{\operatorname{atan}\left(\frac{x\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}+\frac{52739\sqrt{2}\,x\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}}{1719926784\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,1\mathrm{i}}{6912}-\frac{\operatorname{atan}\left(\frac{x\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}-\frac{52739\sqrt{2}\,x\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}}{1719926784\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\sqrt{-358266+\sqrt{2}\,316434\mathrm{i}}\,1\mathrm{i}}{6912}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^3),x)
```

```
[Out] (atan((x*(- 2^(1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*91
23847i)/286654464 - 17140175/286654464)) + (52739*2^(1/2)*x*(- 2^(1/2)*3164
34i - 358266)^(1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 - 17140175/2
86654464)))*(- 2^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - (atan((x*(2^(1/2)
*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9123847i)/286654464 +
 17140175/286654464)) - (52739*2^(1/2)*x*(2^(1/2)*316434i - 358266)^(1/2))/
```

(1719926784*((2^(1/2)*9123847i)/286654464 + 17140175/286654464)))*(2^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - ((1849*x^2)/576 + (353*x^4)/144 + (611*x^6)/576 + (83*x^8)/288 + 4/3)/(9*x + 12*x^3 + 10*x^5 + 4*x^7 + x^9)

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=262

$$-\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2}+\frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(-\right.}}{\sqrt{2}}\right.}{20736}$$

```
[Out] -4/81/x^3+7/27/x+25/432*x*(5*x^2+7)/(x^4+2*x^2+3)^2+1/5184*x*(1025*x^2+1474
)/(x^4+2*x^2+3)+1/124416*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-30014223+
33721353*3^(1/2))^(1/2)-1/124416*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-3
0014223+33721353*3^(1/2))^(1/2)-1/62208*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/
(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2)+1/62208*arctan((2*x+
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2
)
```

Rubi [A]

time = 0.25, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.226$, Rules used = {1683, 1678, 1183, 648, 632, 210, 642}

$$-\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}+\frac{\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{20736}-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)}{41472}-\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\,\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)}{41472}+\frac{25x(5x^2+7)}{432\left(x^4+2x^2+3\right)^2}+\frac{x(1025x^2+1474)}{5184\left(x^4+2x^2+3\right)}+\frac{7}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + (x*
(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10004741 + 11240451*Sq
rt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/207
36 + (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741
 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4147
2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{128-\frac{160x^2}{3}+50x^4+\frac{1250x^6}{9}}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \frac{2048-\frac{6656x^2}{3}+\frac{2576x^4}{9}+\frac{8200x^6}{9}}{x^4(3+2x^2+x^4)}\,dx}{4608}$$

$$= \frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \left(\frac{2048}{3x^4}-\frac{3584}{3x^2}+\frac{8(2242+2369x^2)}{9(3+2x^2+x^4)}\right) dx}{4608}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \frac{2242+2369x^2}{3+2x^2+x^4}\,dx}{5184}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\int \frac{2242\sqrt{2\left(-1+\cdots\right.}}{\sqrt{3}-\sqrt{2\cdots}}}{10368\sqrt{\cdots}}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\left(2242-2369\sqrt{3}\cdots\right.}{20\cdots}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} + \frac{\sqrt{-\frac{10004741}{12}+\cdots}}{\cdots}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432\left(3+2x^2+x^4\right)^2} + \frac{x(1474+1025x^2)}{5184\left(3+2x^2+x^4\right)} - \frac{\sqrt{\frac{1}{3}\left(10004741+\cdots\right.}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 139, normalized size = 0.53

$$\frac{\frac{4(-2304+9024x^2+20090x^4+19939x^6+8644x^8+2369x^{10})}{x^3(3+2x^2+x^4)^2}+\frac{\left(4738+127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(4738-127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^3
*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[
2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*S
qrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736
```

Maple [A]
time = 0.05, size = 298, normalized size = 1.14

method	result
risch	$\frac{\frac{2369}{5184}x^{10}+\frac{2161}{1296}x^{8}+\frac{19939}{5184}x^{6}+\frac{10045}{2592}x^{4}+\frac{47}{27}x^{2}-\frac{4}{9}}{x^{3}\left(x^{4}+2x^{2}+3\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^{4}+40018964\_Z^{2}+126347738683401\right)}\_R\ln\left(29190\_R^{3}+101\right.\right.}{20736}$
default	$\frac{\frac{1025}{192}x^{7}+\frac{881}{48}x^{5}+\frac{7523}{192}x^{3}+\frac{1087}{32}x}{27\left(x^{4}+2x^{2}+3\right)^{2}}+\frac{\left(4865\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+381\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^{2}+\sqrt{3}-x\sqrt{-2+2\sqrt{3}}\right.}{248832}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(1025/192*x^7+881/48*x^5+7523/192*x^3+1087/32*x)/(x^4+2*x^2+3)^2+1/248
832*(4865*(-2+2*3^(1/2))^(1/2)*3^(1/2)+381*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(
1/2)-x*(-2+2*3^(1/2))^(1/2))+1/62208*(8968*3^(1/2)+1/2*(4865*(-2+2*3^(1/2))
^(1/2)*3^(1/2)+381*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2)
)^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+1/248832*(-4
865*(-2+2*3^(1/2))^(1/2)*3^(1/2)-381*(-2+2*3^(1/2))^(1/2))*ln(x^2+3^(1/2)+x
*(-2+2*3^(1/2))^(1/2))+1/62208*(8968*3^(1/2)-1/2*(-4865*(-2+2*3^(1/2))^(1/2
)*3^(1/2)-381*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/
2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-4/81/x^3+7/27/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x^
11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242)/
(x^4 + 2*x^2 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(189) = 378$.
time = 0.40, size = 652, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/135934787413472256*(62119890312985296*x^10 + 226662866975704896*x^8 + 522
840224968600176*x^6 + 47239676*713236683^(1/4)*sqrt(15419)*sqrt(6)*sqrt(3)*
sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(10004741*sqrt(3) + 33
721353)*arctan(1/276093525919725583675206533346*713236683^(3/4)*sqrt(1180991
9)*sqrt(46257)*sqrt(15419)*sqrt(6)*sqrt(713236683^(1/4)*sqrt(15419)*sqrt(6)
*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 546291423183
*x^2 + 546291423183*sqrt(3))*(2242*sqrt(3)*sqrt(2) - 7107*sqrt(2))*sqrt(100
04741*sqrt(3) + 33721353) - 1/50539604724352062*713236683^(3/4)*sqrt(15419)
*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*x)*sqrt(10004741*sqrt(3) +
33721353) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 47239676*713236683^(1/4)*s
qrt(15419)*sqrt(6)*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)
*sqrt(10004741*sqrt(3) + 33721353)*arctan(1/402544360790959900998451125784
68*713236683^(3/4)*sqrt(11809919)*sqrt(15419)*sqrt(6)*sqrt(-98331465348*7132
36683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqr
t(3) + 33721353) + 537176361486287683626 84*x^2 + 53717636148628768362684*sq
rt(3))*(2242*sqrt(3)*sqrt(2) - 7107*sqrt(2))*sqrt(10004741*sqrt(3) + 337213
53) - 1/50539604724352062*713236683^(3/4)*sqrt(15419)*sqrt(6)*(2242*sqrt(3)
*sqrt(2)*x - 7107*sqrt(2)*x)*sqrt(10004741*sqrt(3) + 33721353) - 1/2*sqrt(3
)*sqrt(2) + 1/2*sqrt(2)) + 526799745203830560*x^4 - 713236683^(1/4)*sqrt(15
419)*sqrt(6)*(33721353*x^11 + 134885412*x^9 + 337213530*x^7 + 404656236*x^5
 + 303492177*x^3 - 10004741*sqrt(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3
))*sqrt(10004741*sqrt(3) + 33721353)*log(98331465348/11809919*713236683^(1/
4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33
721353) + 4548518592602436*x^2 + 4548518592602436*sqrt(3)) + 713236683^(1/4
)*sqrt(15419)*sqrt(6)*(33721353*x^11 + 134885412*x^9 + 337213530*x^7 + 4046
56236*x^5 + 303492177*x^3 - 10004741*sqrt(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^
5 + 9*x^3))*sqrt(10004741*sqrt(3) + 33721353)*log(-98331465348/11809919*713
236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sq
rt(3) + 33721353) + 4548518592602436*x^2 + 4548518592602436*sqrt(3)) + 2366
27222534562816*x^2 - 60415461072654336)/(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9
*x^3)
```

Sympy [A]
time = 0.37, size = 80, normalized size = 0.31

$$\operatorname{RootSum}\left(338151365148672t^4 + 2622682824704t^2 + 19257390441, \left(t \mapsto t\log\left(\frac{357010935644160t^3}{182097141061} + \frac{26016957890816t}{1638874269549} + x\right)\right)\right) + \frac{2369x^{10} + 8644x^8 + 19939x^6 + 20090x^4 + 9024x^2 - 2304}{5184x^{11} + 20736x^9 + 51840x^7 + 62208x^5 + 46656x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

```
[Out] RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(_
t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/1638874269
549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x**2
- 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(189) = 378.
time = 7.55, size = 589, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3^
(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3) +
 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712*3^
(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18))*a
rctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
 1/2)) - 1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) -
80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3)
+ 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*s
qrt(3) + 1/2)) - 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sq
rt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3
^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3)
 - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*
sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) +
1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 1
8) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3^(3/4)*(6*sqrt(3)
 + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 80712*3^(
1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/5184*(1025*x^7
+ 3524*x^5 + 7523*x^3 + 6522*x)/(x^4 + 2*x^2 + 3)^2 + 1/81*(21*x^2 - 4)/x^3
```

Mupad [B]
time = 1.02, size = 185, normalized size = 0.71

$$\frac{\frac{2369\,x^{10}}{5184}+\frac{2161\,x^8}{1296}+\frac{19939\,x^6}{5184}+\frac{10045\,x^4}{2592}+\frac{47\,x^2}{27}-\frac{4}{9}}{x^{11}+4\,x^9+10\,x^7+12\,x^5+9\,x^3}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-60028446-\sqrt{2}\,70859514{}\mathrm{i}}\,11809919{}\mathrm{i}}{626913312768\,\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199{}\mathrm{i}}{104485552128}\right)}+\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60028446-\sqrt{2}\,70859514{}\mathrm{i}}}{1253826625536\,\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199{}\mathrm{i}}{104485552128}\right)}\right)\,\sqrt{-60028446-\sqrt{2}\,70859514{}\mathrm{i}}\,1{}\mathrm{i}}{62208}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-60028446+\sqrt{2}\,70859514{}\mathrm{i}}\,11809919{}\mathrm{i}}{626913312768\,\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199{}\mathrm{i}}{104485552128}\right)}-\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60028446+\sqrt{2}\,70859514{}\mathrm{i}}}{1253826625536\,\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199{}\mathrm{i}}{104485552128}\right)}\right)\,\sqrt{-60028446+\sqrt{2}\,70859514{}\mathrm{i}}\,1{}\mathrm{i}}{62208}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^3),x)
```

```
[Out] ((47*x^2)/27 + (10045*x^4)/2592 + (19939*x^6)/5184 + (2161*x^8)/1296 + (236
9*x^10)/5184 - 4/9)/(9*x^3 + 12*x^5 + 10*x^7 + 4*x^9 + x^11) - (atan((x*(-
2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*1323
8919199i)/104485552128 - 57455255935/208971104256)) + (11809919*2^(1/2)*x*(
- 2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*13238919199
i)/104485552128 - 57455255935/208971104256)))*(- 2^(1/2)*70859514i - 600284
46)^(1/2)*1i)/62208 + (atan((x*(2^(1/2)*70859514i - 60028446)^(1/2)*1180991
9i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971
104256)) - (11809919*2^(1/2)*x*(2^(1/2)*70859514i - 60028446)^(1/2))/(12538
26625536*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971104256)))
*(2^(1/2)*70859514i - 60028446)^(1/2)*1i)/62208
```

3.125 $\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=149

$$\frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(c^2e+b^2g-c(bf+\dots$$

[Out] 1/2*(-b*g+c*f)*x^2/c^2+1/4*g*x^4/c+1/4*(c^2*e+b^2*g-c*(a*g+b*f))*ln(c*x^4+b*x^2+a)/c^3-1/2*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*c*(3*a*g+b*f))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)

Rubi [A]
time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1677, 1671, 648, 632, 212, 642}

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(cf-bg)}{2c^2}+\frac{gx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2+gx^3}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{cf-bg}{c^2}+\frac{gx}{c}+\frac{c^2d-acf+abg+(c^2e+b^2g-c(bf+ag))x}{c^2(a+bx+cx^2)}\right.\right.\\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\text{Subst}\left(\int \frac{c^2d-acf+abg+(c^2e+b^2g-c(bf+ag))x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}\\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{(c^2e+b^2g-c(bf+ag))\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}\\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{(c^2e+b^2g-c(bf+ag))\log(a+bx^2+cx^4)}{4c^3}-\frac{(2c^3d\\
&= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\tanh^{-1}\left(\right.}{2c^3\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 142, normalized size = 0.95

$$\frac{2c(cf-bg)x^2+c^2gx^4+\frac{2(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2e+b^2g-c(bf+ag))\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g
+ b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A]
time = 0.11, size = 151, normalized size = 1.01

method	result
default	$-\frac{-\frac{1}{2} g\,x^{4} c+b g\,x^{2}-c\,x^{2} f}{2 c^{2}}+\frac{\frac{\left(-a c g+b^{2} g-f b c+c^{2} e\right) \ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2 c}+\frac{2 \left(a b g-a c f+c^{2} d-\frac{\left(-a c g+b^{2} g-f b c+c^{2} e\right) b}{2 c}\right) \arctan\left(\frac{2 c\,x^{2}+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{2 c^{2}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*(-1/2*g*x^4*c+b*g*x^2-c*x^2*f)+1/2/c^2*(1/2*(-a*c*g+b^2*g-b*c*f+c^
2*e)/c*ln(c*x^4+b*x^2+a)+2*(a*b*g-a*c*f+c^2*d-1/2*(-a*c*g+b^2*g-b*c*f+c^2*e
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.47, size = 486, normalized size = 3.26

$$\left[\frac{(b^{2} c^{2} - 4\,a c^{3}) g x^{4} + 2\,((b^{2} c^{2} - 4\,a c^{3}) f - (b^{3} c - 4\,a b c^{2}) g) x^{2} + (2\,c^{3} d - b c^{2} e + (b^{2} c - 2\,a c^{2}) f - (b^{3} - 3\,a b c) g) \sqrt{b^{2} - 4\,a c} \log\left(\frac{2\,c^{2} x^{4} + 2\,b c x^{2} + b^{2} - 2\,a c - (2\,c x^{2} + b) \sqrt{b^{2} - 4\,a c}}{c x^{4} + b x^{2} + a}\right) + ((b^{2} c^{2} - 4\,a c^{3}) e - (b^{3} c - 4\,a b c^{2}) f + (b^{4} - 5\,a b^{2} c + 4\,a^{2} c^{2}) g) \log\left(c x^{4} + b x^{2} + a\right)}{4\,(b^{2} c^{3} - 4\,a c^{4})}, \frac{(b^{2} c^{2} - 4\,a c^{3}) g x^{4} + 2\,((b^{2} c^{2} - 4\,a c^{3}) f - (b^{3} c - 4\,a b c^{2}) g) x^{2} - 2\,(2\,c^{3} d - b c^{2} e + (b^{2} c - 2\,a c^{2}) f - (b^{3} - 3\,a b c) g) \sqrt{-b^{2} + 4\,a c} \arctan\left(-\frac{(2\,c x^{2} + b) \sqrt{-b^{2} + 4\,a c}}{b^{2} - 4\,a c}\right) + ((b^{2} c^{2} - 4\,a c^{3}) e - (b^{3} c - 4\,a b c^{2}) f + (b^{4} - 5\,a b^{2} c + 4\,a^{2} c^{2}) g) \log\left(c x^{4} + b x^{2} + a\right)}{4\,(b^{2} c^{3} - 4\,a c^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b
*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c
 - 4*a*b*c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/
(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3
)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)
*f - (b^3 - 3*a*b*c)*g)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f +
(b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4
)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.65, size = 146, normalized size = 0.98

$$\frac{cgx^4+2\,cfx^2-2\,bgx^2}{4\,c^2}-\frac{(bcf-b^2g+acg-c^2e)\log\left(cx^4+bx^2+a\right)}{4\,c^3}+\frac{(2\,c^3d+b^2cf-2\,ac^2f-b^3g+3\,abcg-bc^2e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 - 1/4*(b*c*f - b^2*g + a*c*g - c^
2*e)*log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d + b^2*c*f - 2*a*c^2*f - b^3*
g + 3*a*b*c*g - b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*c^3)
```

Mupad [B]
time = 1.68, size = 1834, normalized size = 12.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(f/(2*c) - (b*g)/(2*c^2)) + (g*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 1
```

0*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*
(((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a*b*c
^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b
^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*d - b^
3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^2)^(1/2
)) - (b*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)*(2*b^
4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*
a*b^2*c*g))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((((4*
c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a*b*c^4*g)/c
^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f
+ 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*g + 2*b^2*c^2
*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(2*
(16*a*c^4 - 4*b^2*c^3)) - (b^5*g^2 + b*c^4*e^2 + b^3*c^2*f^2 - c^5*d*e + 2*
a^2*b*c^2*g^2 + a*c^4*d*g + a*c^4*e*f + b*c^4*d*f - 2*b^4*c*f*g - a*b*c^3*f
^2 - 3*a*b^3*c*g^2 - b^2*c^3*d*g - 2*b^2*c^3*e*f - a^2*c^3*f*g + 2*b^3*c^2*
e*g + 4*a*b^2*c^2*f*g - 3*a*b*c^3*e*g)/c^4 + (b*(2*c^3*d - b^3*g - 2*a*c^2*
f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c -
b^2)^(1/2))) + ((((8*a^2*c^4*g - 8*a*c^5*e + 8*a*b*c^4*f - 8*a*b^2*c^3*g)/
c^4 - (8*a*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f
+ 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*d - b^3*g -
2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^2)^(1/2)) - (
a*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)*(2*b^4*g +
2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*
c*g))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((((8*a^2*c^4*
g - 8*a*c^5*e + 8*a*b*c^4*f - 8*a*b^2*c^3*g)/c^4 - (8*a*c^2*(2*b^4*g + 2*b^
2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g)
)/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*
c^4*e^2 + a*b^4*g^2 + a^3*c^2*g^2 + a*b^2*c^2*f^2 - 2*a^2*b^2*c*g^2 - 2*a^2
*c^3*e*g + 2*a*b^2*c^2*e*g + 2*a^2*b*c^2*f*g - 2*a*b*c^3*e*f - 2*a*b^3*c*f*
g)/c^4 + (a*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2
)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(4*c^6*d^2 + b^6*g^2 +
4*a^2*c^4*f^2 + b^2*c^4*e^2 + b^4*c^2*f^2 - 4*a*b^2*c^3*f^2 - 8*a*c^5*d*f -
4*b*c^5*d*e - 2*b^5*c*f*g + 9*a^2*b^2*c^2*g^2 - 6*a*b^4*c*g^2 + 4*b^2*c^4*
d*f - 4*b^3*c^3*d*g - 2*b^3*c^3*e*f + 2*b^4*c^2*e*g - 6*a*b^2*c^3*e*g + 10*
a*b^3*c^2*f*g - 12*a^2*b*c^3*f*g + 12*a*b*c^4*d*g + 4*a*b*c^4*e*f))*(2*c^3*
d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*c^3*(4*a*c - b^2
)^(1/2))

3.126 $\int \frac{x^4\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=594

$$\frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce -}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (-2*b*g+c*f)*x/c^3+1/3*g*x^3/c^2+1/2*x*(a*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*
c*(3*a*g+b*f))+(b^3*c*f+b*c^2*(-3*a*f+c*d)-b^4*g-b^2*c*(-4*a*g+c*e)+2*a*c^2
*(-a*g+c*e))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*c^2*(13*a*f+c*d)-5*b^4*g-b^
2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(-3*b^4*c*f+4*a*c^3*(-5*a*f+c*d)+b
^2*c^2*(19*a*f+c*d)+5*b^5*g+b^3*c*(-34*a*g+c*e)-4*a*b*c^2*(-13*a*g+2*c*e))/
(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*
c^2*(13*a*f+c*d)-5*b^4*g-b^2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(3*b^4*
c*f-4*a*c^3*(-5*a*f+c*d)-b^2*c^2*(19*a*f+c*d)-5*b^5*g-b^3*c*(-34*a*g+c*e)+4
*a*b*c^2*(-13*a*g+2*c*e))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 11.26, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1682, 1690, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c(ce-24ag)-\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd)+2ac^2(3ce-7ag)-5b^4g+3b^3cf\right)}{2\sqrt{2}c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-b^2c(ce-24ag)+\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd)+2ac^2(3ce-7ag)-5b^4g+3b^3cf\right)}{2\sqrt{2}c^{7/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(a(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d)+x^2(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(-g)+b^3cf))}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x(cf-2bg)}{c^3}+\frac{gx^3}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b*e + 2*a*f
) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d - 3*a*f) - b^4*g - b
^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e
- 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) -
b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e
- 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*
c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 1
9*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[
```

```
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b}{2c^3\,(b^2-}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b}{2c^3\,(b^2-}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b}{2c^3\,(b^2-}$$

Mathematica [A]
time = 1.64, size = 721, normalized size = 1.21

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(c^3*d - b*
c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-(b^3*g
) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*Sqrt[
b^2 - 4*a*c]*f - 34*a*g) + b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c^2*(-
2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*g) - b
^2*c*(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a*c]*g)
+ b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f - 4*a*
g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*g + b^3*c*(c*e - 3
*Sqrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + b^2
*c*(c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 - 4*a*c]*g) +
2*a*c^2*(2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*
c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f
+ 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

Maple [A]

time = 0.10, size = 760, normalized size = 1.28

method	result
risch	$\frac{g\,x^3}{3c^2}-\frac{2bgx}{c^3}+\frac{fx}{c^2}+\frac{\frac{\left(2a^2c^2g-4a\,b^2cg+3ab\,c^2f-2a\,c^3e+b^4g-b^3cf+b^2c^2e-b\,c^3d\right)x^3}{8ac-2b^2}-\frac{a\left(3abcg-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x}{2\left(4ac-b^2\right)}}{c^3\left(c\,x^4+b\,x^2+a\right)}+$
default	$-\frac{-\frac{1}{3}cg\,x^3+2bgx-cfx}{c^3}+\frac{\frac{\left(2a^2c^2g-4a\,b^2cg+3ab\,c^2f-2a\,c^3e+b^4g-b^3cf+b^2c^2e-b\,c^3d\right)x^3}{8ac-2b^2}-\frac{a\left(3abcg-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*c*g*x^3+2*b*g*x-c*f*x)+1/c^3*((1/2*(2*a^2*c^2*g-4*a*b^2*c*g+3*
a*b*c^2*f-2*a*c^3*e+b^4*g-b^3*c*f+b^2*c^2*e-b*c^3*d)/(4*a*c-b^2)*x^3-1/2*a*
(3*a*b*c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2)*x)/(c*x^4+b
*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(-14*a^2*c^2*g*(-4*a*c+b^2)^(1/2)+24*a*b^2*c*
g*(-4*a*c+b^2)^(1/2)-13*a*b*c^2*f*(-4*a*c+b^2)^(1/2)+6*a*c^3*e*(-4*a*c+b^2)
^(1/2)-5*b^4*g*(-4*a*c+b^2)^(1/2)+3*b^3*c*f*(-4*a*c+b^2)^(1/2)-b^2*c^2*e*(-
4*a*c+b^2)^(1/2)-b*c^3*d*(-4*a*c+b^2)^(1/2)+52*a^2*b*c^2*g-20*a^2*c^3*f-34*
a*b^3*c*g+19*a*b^2*c^2*f-8*a*b*c^3*e+4*a*c^4*d+5*b^5*g-3*b^4*c*f+b^3*c^2*e+
b^2*c^3*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-14*a^2*c^2*g*(-
4*a*c+b^2)^(1/2)+24*a*b^2*c*g*(-4*a*c+b^2)^(1/2)-13*a*b*c^2*f*(-4*a*c+b^2)^
(1/2)+6*a*c^3*e*(-4*a*c+b^2)^(1/2)-5*b^4*g*(-4*a*c+b^2)^(1/2)+3*b^3*c*f*(-4
*a*c+b^2)^(1/2)-b^2*c^2*e*(-4*a*c+b^2)^(1/2)-b*c^3*d*(-4*a*c+b^2)^(1/2)-52*
a^2*b*c^2*g+20*a^2*c^3*f+34*a*b^3*c*g-19*a*b^2*c^2*f+8*a*b*c^3*e-4*a*c^4*d-
5*b^5*g+3*b^4*c*f-b^3*c^2*e-b^2*c^3*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((b*c^3*d - b^2*c^2*e + 2*a*c^3*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2)*
f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*
x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*e -
(b*c^3*d + b^2*c^2*e - 6*a*c^3*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 - 24*a
*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 - 19*a^
2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3 + 3*(c
*f - 2*b*g)*x)/c^3
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10761 vs. $2(561) = 1122$.
time = 7.93, size = 10761, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^3*d*x^3 + b^3*c*f*x^3 - 3*a*b*c^2*f*x^3 - b^4*g*x^3 + 4*a*b^2*c*g*
x^3 - 2*a^2*c^2*g*x^3 - b^2*c^2*x^3*e + 2*a*c^3*x^3*e + 2*a*c^3*d*x + a*b^2
*c*f*x - 2*a^2*c^2*f*x - a*b^3*g*x + 3*a^2*b*c*g*x - a*b*c^2*x*e)/((b^2*c^3
 - 4*a*c^4)*(c*x^4 + b*x^2 + a)) - 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b
```

^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(b^2*c^3 - 4*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*f + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 - 4*a*c^4)^2*g + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*e + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 - 2*a*b^4*c^8 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^9 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^9 + 16*a^2*b^2*c^9 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^10 - 32*a^3*c^10 + 2*(b^2 - 4*a*c)*a*b^2*c^8 - 8*(b^2 - 4*a*c)*a^2*c^9)*d*abs(b^2*c^3 - 4*a*c^4) + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 - 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^

```
2*c^8 - 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*b^2*c^7
+ 80*(b^2 - 4*a*c)*a^3*c^8)*f*abs(b^2*c^3 - 4*a*c^4) - 2*(5*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 1
0*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 78*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 118*a^2*b^5*c^6 - 304*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 464
*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 608*a
^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2 - 4*a*c)*a^2*b^3*c^6 + 152*
(b^2 - 4*a*c)*a^3*b*c^7)*g*abs(b^2*c^3 - 4*a*c^4) - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 2*a*b^5*
c^7 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + sqrt(...
```

Mupad [B]
time = 4.73, size = 2500, normalized size = 4.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(b^4*g + b^2*c^2*e + 2*a^2*c^2*g - 2*a*c^3*e - b*c^3*d - b^3*c*f + 3*
a*b*c^2*f - 4*a*b^2*c*g))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2*f - 2*a*c^3*d +
 a*b^3*g + a*b*c^2*e - a*b^2*c*f - 3*a^2*b*c*g))/(2*(4*a*c - b^2)))/(a*c^3
+ c^4*x^4 + b*c^3*x^2) + x*(f/c^2 - (2*b*g)/c^3) + atan(((((2048*a^4*c^10*d
 - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c
^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a
^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*
g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 8
0*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6
- 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b
^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9
)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*
c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2
 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c
^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2
+ 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44
800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 +
 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 -
```

49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)
^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c
^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*
e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5
*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 2
58*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g
*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) +
192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^
4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d
*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 26
88*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^
4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g +
44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119
616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*
b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^
(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c -
b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*
f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/
(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*
b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192
*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 -
8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(
1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640
*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 +
288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a
^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*
b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*
a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*
c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g -
15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35
840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1
536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^1
1*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*
c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*
b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*
(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512

```
*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b
^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^
7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^...
```

3.127 $\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=471

$$\frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2c^3d-c^2(b\right.}{}$$

[Out] g*x/c^2-1/2*x*(b*c*(a*f+c*d)-a*b^2*g-2*a*c*(-a*g+c*e)+(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*c*(3*a*g+b*f))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-b*c*(13*a*g+b*f)+(b^3*c*f-4*b*c^2*(2*a*f+c*d)-3*b^4*g+4*a*c^2*(-5*a*g+c*e)+b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-b*c*(13*a*g+b*f)+(-b^3*c*f+4*b*c^2*(2*a*f+c*d)+3*b^4*g-4*a*c^2*(-5*a*g+c*e)-b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A]
time = 4.35, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1682, 1690, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{2\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{gx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d -
c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2*c^2*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f +
13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g
) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g
) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^
2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1682

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [**A**]
time = 1.19, size = 575, normalized size = 1.22

$$\frac{4\sqrt{c}\,gx-\frac{2\sqrt{c}\,x\left(-b^3gx^2+b^2\left(-ag+cfx^2\right)+2c\left(a^2g+c^2dx^2-ac\left(e+fx^2\right)\right)+bc\left(c\left(d-ex^2\right)+a\left(f+3gx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\left(-3b^4g+b^2c\left(ce-\sqrt{b^2-4ac}\,f+19ag\right)+2c^2\left(c\sqrt{b^2-4ac}\,d+2ace+3a\sqrt{b^2-4ac}\,f-10a^2g\right)+b^3\left(cf+3\sqrt{b^2-4ac}\,g\right)-bc\left(4c^2d+c\sqrt{b^2-4ac}\,e+8acf+13a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(3b^4g-b^2c\left(ce+\sqrt{b^2-4ac}\,f+19ag\right)+2c^2\left(c\sqrt{b^2-4ac}\,d-2ace+3a\sqrt{b^2-4ac}\,f+10a^2g\right)+b^3\left(-cf+3\sqrt{b^2-4ac}\,g\right)+bc\left(4c^2d-c\sqrt{b^2-4ac}\,e+8acf-13a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c*
(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x^2
))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c*e
- Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e +
3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g) - b
*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c]*g))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e + Sqrt[b^2
 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3*a*Sqrt[b
^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g) + b*c*(4*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [**A**]
time = 0.08, size = 584, normalized size = 1.24

method	result
risch	$\frac{gx}{c^2}+\frac{\frac{(3abcg-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d)x^3}{8ac-2b^2}+\frac{(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\left(-\frac{(13abcg-6a\,c\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{gx}{c^2}-\frac{-\frac{(3abcg-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d)x^3}{2(4ac-b^2)}-\frac{(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d)x}{2(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{(13abcg\sqrt{-4ac+b^2}-6a\,c^2f\sqrt{-4a\cdots}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] g*x/c^2-1/c^2*((-1/2*(3*a*b*c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4
*a*c-b^2)*x^3-1/2*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)
*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(13*a*b*c*g*(-4*a*c+b^2)^(1/2)-6*
a*c^2*f*(-4*a*c+b^2)^(1/2)-3*b^3*g*(-4*a*c+b^2)^(1/2)+b^2*c*f*(-4*a*c+b^2)^
(1/2)+b*c^2*e*(-4*a*c+b^2)^(1/2)-2*c^3*d*(-4*a*c+b^2)^(1/2)+20*a^2*c^2*g-19
*a*b^2*c*g+8*a*b*c^2*f-4*a*c^3*e+3*b^4*g-b^3*c*f-b^2*c^2*e+4*b*c^3*d)/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(13*a*b*c*g*(-4*a*c+b^2)^(1/2)-6*
a*c^2*f*(-4*a*c+b^2)^(1/2)-3*b^3*g*(-4*a*c+b^2)^(1/2)+b^2*c*f*(-4*a*c+b^2)^
(1/2)+b*c^2*e*(-4*a*c+b^2)^(1/2)-2*c^3*d*(-4*a*c+b^2)^(1/2)-20*a^2*c^2*g+19
*a*b^2*c*g-8*a*b*c^2*f+4*a*c^3*e-3*b^4*g+b^3*c*f+b^2*c^2*e-4*b*c^3*d)/c/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*x^3 + (
b*c^2*d + a*b*c*f - 2*a*c^2*e - (a*b^2 - 2*a^2*c)*g)*x)/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + g*x/c^2 + 1/2*
integrate((b*c^2*d + a*b*c*f - 2*a*c^2*e - (2*c^3*d - b*c^2*e - (b^2*c - 6*
```

```
a*c^2)*f + (3*b^3 - 13*a*b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4 + b*x
^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23774 vs. $2(430) = 860$.
time = 176.56, size = 23774, normalized size = 50.48

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*g*x^5 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f
 - (3*b^3 - 11*a*b*c)*g)*x^3 + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3
- 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^5 + 12*a*b*c^6)*d
^2 - 4*(3*a*b^2*c^5 + 4*a^2*c^6)*d*e + (a*b^3*c^4 + 12*a^2*b*c^5)*e^2 + (a*
b^5*c^2 - 15*a^2*b^3*c^3 + 60*a^3*b*c^4)*f^2 + (9*a*b^7 - 105*a^2*b^5*c + 3
85*a^3*b^3*c^2 - 420*a^4*b*c^3)*g^2 - 2*((3*a*b^3*c^4 - 28*a^2*b*c^5)*d - (
a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*e)*f + 2*((9*a*b^4*c^3 - 54*a^2*b^2
*c^4 + 40*a^3*c^5)*d - (3*a*b^5*c^2 - 13*a^2*b^3*c^3 - 12*a^3*b*c^4)*e - (3
*a*b^6*c - 40*a^2*b^4*c^2 + 150*a^3*b^2*c^3 - 120*a^4*c^4)*f)*g + (a*b^6*c^
5 - 12*a^2*b^4*c^6 + 48*a^3*b^2*c^7 - 64*a^4*c^8)*sqrt((c^10*d^4 - 2*a*c^9*
d^2*e^2 + a^2*c^8*e^4 + (a^2*b^4*c^4 - 18*a^3*b^2*c^5 + 81*a^4*c^6)*f^4 + (
81*a^2*b^8 - 918*a^3*b^6*c + 3051*a^4*b^4*c^2 - 2550*a^5*b^2*c^3 + 625*a^6*
c^4)*g^4 - 4*(3*(a^2*b^2*c^6 - 9*a^3*c^7)*d - (a^2*b^3*c^5 - 9*a^3*b*c^6)*e
)*f^3 + 4*(9*(9*a^2*b^5*c ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 9170 vs. $2(440) = 880$.
time = 7.38, size = 9170, normalized size = 19.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] g*x/c^2 - 1/2*(2*c^3*d*x^3 + b^2*c*f*x^3 - 2*a*c^2*f*x^3 - b^3*g*x^3 + 3*a*
b*c*g*x^3 - b*c^2*x^3*e + b*c^2*d*x + a*b*c*f*x - a*b^2*g*x + 2*a^2*c*g*x -
 2*a*c^2*x*e)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/16*(2*(2*b^2*c^
5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2
*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b^2 - 4*a*c)*c^
5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*
a*c^4)*(b^2*c^2 - 4*a*c^3)^2*f + (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b
^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*g - (2*b^3*c^4 - 8*a*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c
^2 - 4*a*c^3)^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c^6 + 2*b^5*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 +
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^7 - 16*a*b^3*c^7 - 4*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^8 + 32*a^2*b*c^8 - 2*(b^2 - 4*a*c)*b^
3*c^6 + 8*(b^2 - 4*a*c)*a*b*c^7)*d*abs(-b^2*c^2 + 4*a*c^3) - 2*(sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 +
 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 -
 4*a*c)*a^2*b*c^6)*f*abs(-b^2*c^2 + 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^
4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sq
```

```
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 2
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 4
0*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 -
4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)
*g*abs(-b^2*c^2 + 4*a*c^3) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 16*a^2*b
^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^8 + 32*a^3*c^8 - 2
*(b^2 - 4*a*c)*a*b^2*c^6 + 8*(b^2 - 4*a*c)*a^2*c^7)*abs(-b^2*c^2 + 4*a*c^3)
*e - 4*(2*b^6*c^9 - 16*a*b^4*c^10 + 32*a^2*b^2*c^11 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^10 - 2*(b^2 - 4*a*c)*b^4*c^9 + 8*(b^2 - 4*a*c)*a*b^2*
c^10)*d + (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b...
```

Mupad [B]
time = 4.22, size = 2500, normalized size = 5.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*(4*
a*c - b^2)) + (x*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*
(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7*g -
16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 153
6*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g
 + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*
c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g
)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^5*d^2
*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2
- a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2)
+ 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b
^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2
- 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2
```

*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^
3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e
*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^
8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10
*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 38
4*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^
3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g
 - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752
*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*
c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*
a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^
4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*
f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c -
b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280
*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^
13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*
a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^
7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3
840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9
*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^
5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a
^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*
b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1
8*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g
 - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^
3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4
*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2
*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6
*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 154
8*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^
6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 5
1*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9
)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*
g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c
^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c

```
^10)))^(1/2) - (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2
 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c
^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*
c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^
5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6
*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 18
4*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 4
72*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*...
```

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=449

$$\frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(c\right.}{}$$

```
[Out] 1/2*x*(c*(b^2*d-2*a*(-a*f+c*d)-a*b*(a*g+c*e)/c)+(b*c*(a*f+c*d)-a*b^2*g-2*a*
c*(-a*g+c*e))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^2*g/c-2*a*(3*a*g+c*e)+
(b^2*c*(-a*f+c*d)-4*a*c^2*(a*f+3*c*d)-a*b^3*g+4*a*b*c*(2*a*g+c*e))/c/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1
/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^
2*g/c-2*a*(3*a*g+c*e)+(-b^2*c*(-a*f+c*d)+4*a*c^2*(a*f+3*c*d)+a*b^3*g-4*a*b*
c*(2*a*g+c*e))/c/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.96, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1692, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}\,a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2g}{c}-\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}\,a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a*f)
- 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*
a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a
*b^2*g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f)
- a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A]
time = 1.02, size = 512, normalized size = 1.14

$$\frac{\frac{2\sqrt{c}\,x\left(b\left(-ace-a^2g+c^2dx^2+acfx^2\right)+b^2\left(cd-agx^2\right)+2ac\left(-c\left(d+ex^2\right)+a\left(f+gx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-ab^3g+bc\left(c\sqrt{b^2-4ac}\,d+4ace+a\sqrt{b^2-4ac}\,f+8a^2g\right)+b^2\left(c^2d-acf+a\sqrt{b^2-4ac}\,g\right)-2ac\left(6c^2d+c\sqrt{b^2-4ac}\,e+2acf+3a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(ab^3g+bc\left(c\sqrt{b^2-4ac}\,d-4ace+a\sqrt{b^2-4ac}\,f-8a^2g\right)+2ac\left(6c^2d-c\sqrt{b^2-4ac}\,e+2acf-3a\sqrt{b^2-4ac}\,g\right)+b^2\left(-c^2d+acf+a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a*
g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e
+ a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 - 4*a*
c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^2 - 4
*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*(c*Sqrt
[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*c*(6*c^2
*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) + b^2*(-(c^
2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
a*c^(3/2))
```

Maple [A]
time = 0.08, size = 543, normalized size = 1.21

method	result
risch	$\frac{-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\left(\frac{\left(6a^2cg-a\,b^2g-abcf+2a\,c^2\right.}{4ac-b^2}\right.}{}$
default	$\frac{-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{-\frac{\left(6a^2cg\sqrt{-4ac+b^2}-a\,b^2g\sqrt{-4ac+b^2}-\sqrt{\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/2
*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+b*
x^2+a)+2/a/(4*a*c-b^2)*(-1/8*(6*a^2*c*g*(-4*a*c+b^2)^(1/2)-a*b^2*g*(-4*a*c+
b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*a*b*c*f+2*a*c^2*e*(-4*a*c+b^2)^(1/2)-b*c^2*d*
(-4*a*c+b^2)^(1/2)-8*a^2*b*c*g+4*a^2*c^2*f+a*b^3*g+a*b^2*c*f-4*a*b*c^2*e+12
*c^3*a*d-b^2*c^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*a^2*c
*g*(-4*a*c+b^2)^(1/2)-a*b^2*g*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*a*b*c*f
+2*a*c^2*e*(-4*a*c+b^2)^(1/2)-b*c^2*d*(-4*a*c+b^2)^(1/2)+8*a^2*b*c*g-4*a^2*
c^2*f-a*b^3*g-a*b^2*c*f+4*a*b*c^2*e-12*c^3*a*d+b^2*c^2*d)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d + a*b*c*f - 2*a*c^2*e - (a*b^2 - 2*a^2*c)*g)*x^3 + (2*a^2*c*f
 - a^2*b*g - a*b*c*e + (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*
b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate((2*
a^2*c*f - a^2*b*g - a*b*c*e - (b*c^2*d + a*b*c*f - 2*a*c^2*e + (a*b^2 - 6*a
^2*c)*g)*x^2 - (b^2*c - 6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^
2*c^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19375 vs. $2(408) = 816$.
time = 141.88, size = 19375, normalized size = 43.15

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 - sqrt(1/2
)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b
*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^2 + 2*(a*b^4*c^
3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d*e + (a^2*b^3*c^3 + 12*a^3*b*c^4)*e^2 + (a
^3*b^3*c^2 + 12*a^4*b*c^3)*f^2 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*g^
2 - 2*((3*a^2*b^3*c^3 - 28*a^3*b*c^4)*d + 2*(3*a^3*b^2*c^3 + 4*a^4*c^4)*e)*
f + 2*(2*(5*a^3*b^2*c^3 - 36*a^4*c^4)*d - (3*a^3*b^3*c^2 - 28*a^4*b*c^3)*e
+ (a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*f)*g + (a^3*b^6*c^3 - 12*a^4*b^4
*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt((4*a^3*b*c^6*d*e^3 + a^4*c^6*e^4 +
 12*a^5*c^5*d*f^3 + a^6*c^4*f^4 + (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^4
 + 4*(a*b^3*c^6 - 9*a^2*b*c^7)*d^3*e + 6*(a^2*b^2*c^6 - 3*a^3*c^7)*d^2*e^2
+ (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*g^4 - 4*(3*(a^6*b^2*c^2 - 9*a^7*c^3
)*e - (a^6*b^3*c - 9*a^7*b*c^2)*f)*g^3 - 2*(2*a^4*b*c^5*d*e + a^5*c^5*e^2 +
 (a^3*b^2*c^5 - 27*a^4*c^ ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 8913 vs. $2(414) = 828$.
time = 6.92, size = 8913, normalized size = 19.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*(b*c^2*d*x^3 + a*b*c*f*x^3 - a*b^2*g*x^3 + 2*a^2*c*g*x^3 - 2*a*c^2*x^3*e + b^2*c*d*x - 2*a*c^2*d*x + 2*a^2*c*f*x - a^2*b*g*x - a*b*c*x*e)/((c*x^4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*f + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*g - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 192*a^4*c^7 + 2*(b^2 - 4*a*c)*a*b^4*c^4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^

```
5 + 48*(b^2 - 4*a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) - 4*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4
 - 2*a^3*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 -
 4*a*c)*a^4*c^5)*f*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*a^3*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^4 + 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b*c^5 - 32*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a^3*b^3*c^3 - 8*(b^2 - 4*a*
c)*a^4*b*c^4)*g*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*
c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 - 32*a^4*b*c^6 + 2*(b^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*
a^3*b*c^5)*abs(a*b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 2
24*a^4*b^3*c^8 - 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5...
```

Mupad [B]
time = 5.82, size = 2500, normalized size = 5.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c^2*d - b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a*c*(4*a*c -
b^2)) - (x^3*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*a*c*
(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((6144*a^5*c^7*d + 2048*a^6*c^
```

6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*
b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*
a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g +
768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/
(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*
b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^
2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4
*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 28
8*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c
^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c -
b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96
*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/
2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*
b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g -
1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f
 + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6
*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f -
 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c
^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f +
 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5
*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e
*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*
f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3
*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^
4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a
^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 76
8*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^
4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c
*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*
b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2
 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9
)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b
^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a
^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^
3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a
^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 358
4*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d
*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a
^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f
*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a
^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c
^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 9
60*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9

```
)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g +
1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f
*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10
*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2
*c^8)))^(1/2) + (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*
g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2
+ 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2
*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e
 - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g +
 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2
*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c
 - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840
*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 +
 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*
a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^
2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(
1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2
*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(...
```

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=460

$$-\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^2cd-\right.}{}$$

```
[Out] -d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e)-2*a^2*g)+(b^2*c*d-2*
a*c*(-a*f+c*d)-a*b*(a*g+c*e))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arc
tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^2*c*d-2*a*c*(-a*f+
5*c*d)-a*b*(a*g+c*e)+(3*b^3*c*d-4*a*b*c*(a*f+4*c*d)-a*b^2*(-a*g+c*e)+4*a^2*
c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(3*b^2*c*d-2*a*c*(-a*f+5*c*d)-a*b*(a*g+c*e)+(-3*b^3*c*d+4*a*b*c*(a*f+
4*c*d)+a*b^2*(-a*g+c*e)-4*a^2*c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.81, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1683, 1678, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{2\sqrt{2}a^2\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{d}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2
*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(2*a^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e +
 a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3
*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d - 4
*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(c\right.\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af)-ab(c\right.\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af\right.\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af\right.\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{a^2x}-\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)-2a^2g\right)+\left(b^2cd-2ac(cd-af\right.\right.}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A]
time = 1.51, size = 529, normalized size = 1.15

$$-\frac{\frac{4d}{x}-\frac{2x\left(-b^3d+b^2\left(ae-cdx^2\right)+ab\left(3cd-af+cex^2+agx^2\right)+2a\left(a^2g+c^2dx^2-ac\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(3b^3cd+b^2\left(3c\sqrt{b^2-4ac}\,d-ace+a^2g\right)+2ac\left(-5c\sqrt{b^2-4ac}\,d+6ace+a\sqrt{b^2-4ac}\,f+2a^2g\right)-ab\left(16c^2d+c\sqrt{b^2-4ac}\,e+4acf+a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-3b^3cd+b^2\left(3c\sqrt{b^2-4ac}\,d+ace-a^2g\right)-2ac\left(5c\sqrt{b^2-4ac}\,d+6ace-a\sqrt{b^2-4ac}\,f+2a^2g\right)+ab\left(16c^2d-c\sqrt{b^2-4ac}\,e+4acf-a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f + c
*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]
*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^
2 - 4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f +
 a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*c*(5*c*S
qrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) + a*b*(16*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^2
```

Maple [A]
time = 0.14, size = 550, normalized size = 1.20

method	result
default	$\frac{-\frac{\left(a^{2}bg-2a^{2}cf+abce+2a\,c^{2}d-b^{2}cd\right)x^{3}}{2\left(4ac-b^{2}\right)}-\frac{\left(2a^{3}g-a^{2}bf-2a^{2}ce+a\,b^{2}e+3abcd-b^{3}d\right)x}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+2c\left(-\frac{\left(-a^{2}bg\sqrt{-4ac+b^{2}}+2a^{2}cf\sqrt{-4ac+\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*((-1/2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/(4*a*c-b^2)*x^3-
1/2*(2*a^3*g-a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b^2)*x)/(c*x
^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2*a^2*c*f*(-
4*a*c+b^2)^(1/2)-a*b*c*e*(-4*a*c+b^2)^(1/2)-10*a*c^2*d*(-4*a*c+b^2)^(1/2)+3
*b^2*c*d*(-4*a*c+b^2)^(1/2)+4*a^3*g*c+a^2*b^2*g-4*a^2*b*c*f+12*a^2*c^2*e-a*
b^2*c*e-16*a*b*c^2*d+3*b^3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+
1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2*a^2*c*f*(-4*a*c+b^2)^(1/2)-a*b*c*e*(-4*a
*c+b^2)^(1/2)-10*a*c^2*d*(-4*a*c+b^2)^(1/2)+3*b^2*c*d*(-4*a*c+b^2)^(1/2)-4*
a^3*g*c-a^2*b^2*g+4*a^2*b*c*f-12*a^2*c^2*e+a*b^2*c*e+16*a*b*c^2*d-3*b^3*c*d
)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-d/a^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*((2*a^2*c*f - a^2*b*g - a*b*c*e + (3*b^2*c - 10*a*c^2)*d)*x^4 + (a^2*b
*f - 2*a^3*g - a*b^2*e + 2*a^2*c*e + (3*b^3 - 11*a*b*c)*d)*x^2 + 2*(a*b^2 -
 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^
3*b^2 - 4*a^4*c)*x) + 1/2*integrate((a^2*b*f - 2*a^3*g + a*b^2*e - 6*a^2*c*
e - (2*a^2*c*f - a^2*b*g - a*b*c*e + (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 -
 13*a*b*c)*d)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23991 vs. $2(418) = 836$.

time = 128.53, size = 23991, normalized size = 52.15

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*(a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4 - 2*(a^
2*b*f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-((9*b^7*c - 105*a*b^5*c^2 + 385*a^2*b^3*c^3 - 420*a^3*b*c^4
)*d^2 - 2*(3*a*b^6*c - 40*a^2*b^4*c^2 + 150*a^3*b^2*c^3 - 120*a^4*c^4)*d*e
+ (a^2*b^5*c - 15*a^3*b^3*c^2 + 60*a^4*b*c^3)*e^2 + (a^4*b^3*c + 12*a^5*b*c
^2)*f^2 + (a^5*b^3 + 12*a^6*b*c)*g^2 - 2*((3*a^2*b^5*c - 13*a^3*b^3*c^2 - 1
2*a^4*b*c^3)*d - (a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*e)*f + 2*((9*a^3*
b^4*c - 54*a^4*b^2*c^2 + 40*a^5*c^3)*d - (3*a^4*b^3*c - 28*a^5*b*c^2)*e - 2
*(3*a^5*b^2*c + 4*a^6*c^2)*f)*g + (a^5*b^6*c - 12*a^6*b^4*c^2 + 48*a^7*b^2*
c^3 - 64*a^8*c^4)*sqrt((a^8*c^2*f^4 + a^10*g^4 + (81*b^8*c^2 - 918*a*b^6*c^
3 + 3051*a^2*b^4*c^4 - 2550*a^3*b^2*c^5 + 625*a^4*c^6)*d^4 - 4*(27*a*b^7*c^
2 - 351*a^2*b^5*c^3 + 1197*a^3*b^3*c^4 - 550*a^4*b*c^5)*d^3*e + 6*(9*a^2*b^
6*c^2 - 132*a^3*b^4*c^3 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9176 vs. $2(425) = 850$.
time = 9.08, size = 9176, normalized size = 19.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a^2*b*g*x^4 - a*b*c*
x^4*e + 3*b^3*d*x^2 - 11*a*b*c*d*x^2 + a^2*b*f*x^2 - 2*a^3*g*x^2 - a*b^2*x^
2*e + 2*a^2*c*x^2*e + 2*a*b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^
```

2 - 4*a^3*c)) + 1/16*((6*b^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*c^4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*
c^4)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2
 - 4*a^3*c)^2*f - (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)
^2*g - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a^2*b^2 - 4*a^3*c)^2*e - 2*(3*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 37*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^6*c^2 - 6*a^2*b^7*c^2 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^3*c^3 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 74*a^3*b^5*c^3 - 208*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 104*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^3*c^4 - 304*a^4*b^3*c^4 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^5 + 416*a^5*b*c^5 + 6*(b^2 - 4*a*c)*a^2*b^5*c^2 - 50*(b^2 - 4*a*c)*a^
3*b^3*c^3 + 104*(b^2 - 4*a*c)*a^4*b*c^4)*d*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^5*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^4*c^2 - 2*a^4*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*
c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 16*a^5*b^3*c^3 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 32*a^6*b*c^4 + 2*(b^2 - 4*a*c)*a^4*b^3
c^2 - 8(b^2 - 4*a*c)*a^5*b*c^3)*f*abs(a^2*b^2 - 4*a^3*c) - 4*(sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^6*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2
 - 2*a^5*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^3 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*a^6*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^

```
2 - 4*a*c)*c)*a^6*c^4 - 32*a^7*c^4 + 2*(b^2 - 4*a*c)*a^5*b^2*c^2 - 8*(b^2 -
 4*a*c)*a^6*c^3)*g*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*
c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 2*a^3*b^6*c^2
 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 20*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^4*c^3 + 28*a^4*b^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 128*a^5*b^2*c^4 + 24*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 192*a^6*c^5 + 2*(b^2 - 4*a
*c)*a^3*b^4*c^2 - 20*(b^2 - 4*a*c)*a^4*b^2*c^3 + 48*(b^2 - 4*a*c)*a^5*c^4)*
abs(a^2*b^2 - 4*a^3*c)*e + (6*a^4*b^8*c^3 - 80*a^5*b^6*c^4 + 352*a^6*b^4*c^
5 - 512*a^7*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b^8*c + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^6*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^7*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*b^4*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^5*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^6*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)...
```

Mupad [B]
time = 7.76, size = 2500, normalized size = 5.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2
) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^
2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^
2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c
^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*
d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^
4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a
^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g
 - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*
d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c
^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*
b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*
c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f
- 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d
*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 107
```

52*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 131072*a^16*c^7*g - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g - 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 163840*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a

```
^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*
a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^
6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*
c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^
6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3...
```

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=542

$$-\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-ab(3cd-a\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^2
*(b*e+4*c*d)/a+b^2*f-a*(b*g+2*c*f))+c*(b^3*d-a*b^2*e-a*b*(-a*f+3*c*d)+2*a^2
*(-a*g+c*e))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*
c*d)+2*a^2*(-a*g+5*c*e)+(5*b^4*d-3*a*b^3*e+4*a^2*c*(-3*a*f+7*c*d)-a*b^2*(-a
*f+29*c*d)+4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*c*d)+2*a^2*(-a*g+5*c
*e)+(-5*b^4*d+3*a*b^3*e-4*a^2*c*(-3*a*f+7*c*d)+a*b^2*(-a*f+29*c*d)-4*a^2*b*
(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

Rubi [A]
time = 4.86, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1683, 1678, 1180, 211}

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3d\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3d\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/3*d/(a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d +
 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g)) + c*(b^3*d - a*
b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) +
2*a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^
2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a
*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f)
- a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*
c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1683

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x
*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-a\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c(b^3d-ab^2e-a\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+b\right.\right.}{2a^3\left(b^2-4ac\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+b\right.\right.}{2a^3\left(b^2-4ac\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+b\right.\right.}{2a^3\left(b^2-4ac\right)}$$

Mathematica [A]
time = 1.32, size = 612, normalized size = 1.13

$$\frac{-\frac{4ad}{x^3}+\frac{24bd-12ae}{x}+\frac{6x\left(b^4d+b^3\left(-ae+cdx^2\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+ab\left(-a^2g-3c^2dx^2+ac\left(3e+fx^2\right)\right)+2a^2c\left(c\left(d+ex^2\right)-a\left(f+gx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4d+b^3\left(5\sqrt{b^2-4ac}\,d-3ae\right)+ab^2\left(-29cd-3\sqrt{b^2-4ac}\,e+af\right)+ab\left(-19c\sqrt{b^2-4ac}\,d+16ace+a\sqrt{b^2-4ac}\,f+4a^2g\right)-2a^2\left(-14c^2d-5c\sqrt{b^2-4ac}\,e+6acf+a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(5b^4d-b^3\left(5\sqrt{b^2-4ac}\,d+3ae\right)+ab^2\left(-29cd+3\sqrt{b^2-4ac}\,e+af\right)+ab\left(19c\sqrt{b^2-4ac}\,d+16ace-a\sqrt{b^2-4ac}\,f+4a^2g\right)+2a^2\left(14c^2d-5c\sqrt{b^2-4ac}\,e-6acf+a\sqrt{b^2-4ac}\,g\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2*d*x^2 + a*c*(3*e +
f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3
*a*e) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2
 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-14*c^2*
d - 5*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*
c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(19*c*S
qrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) + 2*a^2*(1
4*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [A]
time = 0.15, size = 629, normalized size = 1.16

method	result
default	$\frac{\frac{c\left(2a^{3}g-a^{2}bf-2a^{2}ce+a\,b^{2}e+3abcd-b^{3}d\right)x^{3}}{8ac-2b^{2}}+\frac{\left(a^{3}bg+2a^{3}cf-a^{2}b^{2}f-3a^{2}bce-2a^{2}c^{2}d+a\,b^{3}e+4a\,b^{2}cd-b^{4}d\right)x}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(-\frac{\left(2a^{3}g\sqrt{-4ac+b^{2}}-a\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((1/2*c*(2*a^3*g-a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b^
2)*x^3+1/2*(a^3*b*g+2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a
*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(2*a^3
*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1
/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*(-4*a*c+b^2)^(1/2)*a*b*c*d-5*(-4*a*c+b^
2)^(1/2)*b^3*d-4*a^3*b*g+12*a^3*c*f-a^2*b^2*f-16*a^2*b*c*e-28*a^2*c^2*d+3*a
*b^3*e+29*a*b^2*c*d-5*b^4*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(
2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^
2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*(-4*a*c+b^2)^(1/2)*a*b*c*d-5*(-4*a
*c+b^2)^(1/2)*b^3*d+4*a^3*b*g-12*a^3*c*f+a^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*
d-3*a*b^3*e-29*a*b^2*c*d+5*b^4*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1
/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/6*(3*(a^2*b*c*f - 2*a^3*c*g - 3*a*b^2*c*e + 10*a^2*c^2*e + (5*b^3*c - 19*
a*b*c^2)*d)*x^6 - (3*a^3*b*g + 9*a*b^3*e - 33*a^2*b*c*e - (15*b^4 - 62*a*b^
2*c + 14*a^2*c^2)*d - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 - 2*(3*a^2*b^2*e - 12*a^
3*c*e - 5*(a*b^3 - 4*a^2*b*c)*d)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c
 - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) -
1/2*integrate(-(a^3*b*g - 3*a*b^3*e + 13*a^2*b*c*e + (a^2*b*c*f - 2*a^3*c*g
```

```
 - 3*a*b^2*c*e + 10*a^2*c^2*e + (5*b^3*c - 19*a*b*c^2)*d)*x^2 + (5*b^4 - 24
*a*b^2*c + 14*a^2*c^2)*d + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/(
a^3*b^2 - 4*a^4*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10422 vs. $2(509) = 1018$.
time = 6.39, size = 10422, normalized size = 19.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 + a^2*b*c*f*x^3 - 2*a^3*c*g*x^3 - a*b^2*
c*x^3*e + 2*a^2*c^2*x^3*e + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x + a^2*b
^2*f*x - 2*a^3*c*f*x - a^3*b*g*x - a*b^3*x*e + 3*a^2*b*c*x*e)/((a^3*b^2 - 4
*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b
*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2
+ 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^
```

3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(2*a^3*b^2*c^2 - 8*a^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 2*(b^2 - 4*a*c)*a^3*c^2)*(a^3*b^2 - 4*a^4*c)^2*g - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 28*a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b^2 - 4*a*c)*a^6*b^2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*a^6*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 16*a^7*b^3*c^2 - 4*sqrt(2)*sqrt

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 32*a^8*b*c^3 + 2*(b^2 - 4*a*c)*a^6*
b^3*c - 8*(b^2 - 4*a*c)*a^7*b*c^2)*g*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c -
 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 3
04*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2...
```

Mupad [B]
time = 8.47, size = 2500, normalized size = 4.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan((((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2
) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a
^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2
- 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^
4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116
928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a
^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 -
 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 +
 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^
13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^
12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10
*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*
b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g
+ 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*
(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b
^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 1
19616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 1
0*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b
^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g
*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4
*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2
*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g -
 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a
```

```
^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^
3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1
536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*
c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) +
184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b
^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4
096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c
^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1
/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^
2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*
c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1
16928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9
*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(
1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2
 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^
2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2
 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*
b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*
b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^
10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^
2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*
g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*
g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6
*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e -
 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e +
 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5
*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d
*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a
^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c
^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g
 - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6
*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*
c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g -
 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*
b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c -
 b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^...
```

3.131 $\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x\right.$

Optimal. Leaf size=20

$$x^3\left(a+bx^2+cx^4\right)^{1+p}$$

[Out] x^3*(c*x^4+b*x^2+a)^(1+p)

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {1602}

$$x^3\left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]

[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

Mathematica [A]
time = 0.38, size = 20, normalized size = 1.00

$$x^3\left(a+bx^2+cx^4\right)^{1+p}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^
4),x]

[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)

Maple [A]
time = 0.03, size = 21, normalized size = 1.05

method	result	size
gosper	$x^{3}\left(c\,x^{4}+b\,x^{2}+a\right)^{1+p}$	21
risch	$\left(c\,x^{4}+b\,x^{2}+a\right)^{p} x^{3}\left(c\,x^{4}+b\,x^{2}+a\right)$	31
norman	$a\,x^{3}{\mathrm e}^{p \ln\left(c\,x^{4}+b\,x^{2}+a\right)}+b\,x^{5}{\mathrm e}^{p \ln\left(c\,x^{4}+b\,x^{2}+a\right)}+c\,x^{7}{\mathrm e}^{p \ln\left(c\,x^{4}+b\,x^{2}+a\right)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x,method=_RETUR
NVERBOSE)
```

```
[Out] x^3*(c*x^4+b*x^2+a)^(1+p)
```

Maxima [A]
time = 0.32, size = 31, normalized size = 1.55

$$\left(cx^{7}+bx^{5}+ax^{3}\right)\left(cx^{4}+bx^{2}+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="maxima")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Fricas [A]
time = 0.40, size = 31, normalized size = 1.55

$$\left(cx^{7}+bx^{5}+ax^{3}\right)\left(cx^{4}+bx^{2}+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="fricas")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 174.57, size = 54, normalized size = 2.70

$$ax^{3}\left(a+bx^{2}+cx^{4}\right)^{p}+bx^{5}\left(a+bx^{2}+cx^{4}\right)^{p}+cx^{7}\left(a+bx^{2}+cx^{4}\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
```

```
[Out] a*x**3*(a + b*x**2 + c*x**4)**p + b*x**5*(a + b*x**2 + c*x**4)**p + c*x**7*
(a + b*x**2 + c*x**4)**p
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.
time = 5.21, size = 58, normalized size = 2.90

$$(cx^4 + bx^2 + a)^p cx^7 + (cx^4 + bx^2 + a)^p bx^5 + (cx^4 + bx^2 + a)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="giac")
```

```
[Out] (c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2
+ a)^p*a*x^3
```

Mupad [B]
time = 1.10, size = 31, normalized size = 1.55

$$\left(c\,x^7+b\,x^5+a\,x^3\right)\,{\left(c\,x^4+b\,x^2+a\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*a + b*x^2*(2*p + 5) + c*x^4*(4*p + 7))*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] (a*x^3 + b*x^5 + c*x^7)*(a + b*x^2 + c*x^4)^p
```

3.132 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\,\sqrt{d+ex}}\,dx$

Optimal. Leaf size=210

$$-\frac{d^4(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\,\sqrt{d+ex}}{e^{10}}+\frac{d^2(4cd^4+3bd^2e^2+2ae^4)\,(d-ex)^{3/2}(d+ex)^{3/2}}{3e^{10}}-\frac{(6cd^4+3bd^2e\ldots}{}$$

```
[Out] 1/3*d^2*(2*a*e^4+3*b*d^2*e^2+4*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^10-1/5
*(a*e^4+3*b*d^2*e^2+6*c*d^4)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^10+1/7*(b*e^2+4
*c*d^2)*(-e*x+d)^(7/2)*(e*x+d)^(7/2)/e^10-1/9*c*(-e*x+d)^(9/2)*(e*x+d)^(9/2
)/e^10-d^4*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^10
```

Rubi [A]
time = 0.21, antiderivative size = 278, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {534, 1265, 911, 1167}

$$-\frac{(d^2-e^2x^2)^3(ae^4+3bd^2e^2+6cd^4)}{5e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}+\frac{d^2(d^2-e^2x^2)^2(2ae^4+3bd^2e^2+4cd^4)}{3e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{d^4(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}+\frac{(d^2-e^2x^2)^4(be^2+4cd^2)}{7e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{c(d^2-e^2x^2)^5}{9e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^10*Sqrt[d - e*x]*Sqr
t[d + e*x])) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^2)/(3
*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2
- e^2*x^2)^3)/(5*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((4*c*d^2 + b*e^2)*(d^
2 - e^2*x^2)^4)/(7*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^5
)/(9*e^10*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\,\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\,\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{x^2(a+bx+cx^2)}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2 \left(\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}\right)\,dx, x, \sqrt{}\right.}{e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^8+bd^6e^2+ad^4e^4}{e^8}-\frac{d^2(4cd^4+3bd^2e^2+2ae^4)x^2}{e^8}+\frac{(6cd^4+3bd^2e^2+ae^4}{e^8}\right.\right.}{e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{d^4(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}+\frac{d^2(4cd^4+3bd^2e^2+2ae^4)(d^2-e^2x^2)^2}{3e^{10}\sqrt{d-ex}\,\sqrt{d+ex}}-$$

Mathematica [A]
time = 0.32, size = 149, normalized size = 0.71

$$-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,(21ae^4(8d^4+4d^2e^2x^2+3e^4x^4)+9b(16d^6e^2+8d^4e^4x^2+6d^2e^6x^4+5e^8x^6)+c(128d^8+64d^6e^2x^2+48d^4e^4x^4+40d^2e^6x^6+35e^8x^8))}{315e^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^
4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c*(
128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)))/
e^10
```

Maple [A]
time = 0.17, size = 145, normalized size = 0.69

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6}{315e^{10}}$
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6}{315e^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+72b\,d^4e^4x^2+64c\,d^6}{315e^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^6
*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b*d
^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^10
```

Maxima [A]
time = 0.50, size = 271, normalized size = 1.29

$$-\frac{1}{9}\sqrt{-x^2e^2+d^2}\,cx^8e^{(-2)}-\frac{8}{63}\sqrt{-x^2e^2+d^2}\,cd^2x^6e^{(-4)}-\frac{16}{105}\sqrt{-x^2e^2+d^2}\,cd^4x^4e^{(-6)}-\frac{64}{315}\sqrt{-x^2e^2+d^2}\,cd^6x^2e^{(-8)}-\frac{128}{315}\sqrt{-x^2e^2+d^2}\,cd^8e^{(-10)}-\frac{1}{7}\sqrt{-x^2e^2+d^2}\,bx^6e^{(-2)}-\frac{6}{35}\sqrt{-x^2e^2+d^2}\,bd^2x^4e^{(-4)}-\frac{8}{35}\sqrt{-x^2e^2+d^2}\,bd^4x^2e^{(-6)}-\frac{16}{35}\sqrt{-x^2e^2+d^2}\,bd^6e^{(-8)}-\frac{1}{5}\sqrt{-x^2e^2+d^2}\,ax^4e^{(-2)}-\frac{4}{15}\sqrt{-x^2e^2+d^2}\,ad^2x^2e^{(-4)}-\frac{8}{15}\sqrt{-x^2e^2+d^2}\,ad^4e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/9*sqrt(-x^2*e^2 + d^2)*c*x^8*e^(-2) - 8/63*sqrt(-x^2*e^2 + d^2)*c*d^2*x^
6*e^(-4) - 16/105*sqrt(-x^2*e^2 + d^2)*c*d^4*x^4*e^(-6) - 64/315*sqrt(-x^2*
e^2 + d^2)*c*d^6*x^2*e^(-8) - 128/315*sqrt(-x^2*e^2 + d^2)*c*d^8*e^(-10) -
1/7*sqrt(-x^2*e^2 + d^2)*b*x^6*e^(-2) - 6/35*sqrt(-x^2*e^2 + d^2)*b*d^2*x^4
*e^(-4) - 8/35*sqrt(-x^2*e^2 + d^2)*b*d^4*x^2*e^(-6) - 16/35*sqrt(-x^2*e^2
+ d^2)*b*d^6*e^(-8) - 1/5*sqrt(-x^2*e^2 + d^2)*a*x^4*e^(-2) - 4/15*sqrt(-x^
2*e^2 + d^2)*a*d^2*x^2*e^(-4) - 8/15*sqrt(-x^2*e^2 + d^2)*a*d^4*e^(-6)
```

Fricas [A]
time = 0.38, size = 138, normalized size = 0.66

$$-\frac{(35\,ce^8x^8+128\,cd^8+144\,bd^6e^2+168\,ad^4e^4+5\,(8\,cd^2e^6+9\,be^8)x^6+3\,(16\,cd^4e^4+18\,bd^2e^6+21\,ae^8)x^4+4\,(16\,cd^6e^2+18\,bd^4e^4+21\,ad^2e^6)x^2)\sqrt{ex+d}\,\sqrt{-ex+d}}{315\,e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*d
^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 + 4*
(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d)/e^10
```

Sympy [C] Result contains complex when optimal does not.
time = 81.99, size = 367, normalized size = 1.75

$$-\frac{iad^5G_{6,6}^{6,2}\left(\begin{matrix}-\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1\\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}-\frac{ad^5G_{6,6}^{2,6}\left(\begin{matrix}-3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}-\frac{ibd^7G_{6,6}^{6,2}\left(\begin{matrix}-\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1\\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}-\frac{bd^7G_{6,6}^{2,6}\left(\begin{matrix}-4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}-\frac{icd^9G_{6,6}^{6,2}\left(\begin{matrix}-\frac{17}{4},-\frac{15}{4} & -4,-4,-\frac{7}{2},1\\ -\frac{9}{2},-\frac{17}{4},-4,-\frac{15}{4},-\frac{7}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}}-\frac{cd^9G_{6,6}^{2,6}\left(\begin{matrix}-5,-\frac{19}{4},-\frac{9}{2},-\frac{17}{4},-4,1 & \\ -\frac{19}{4},-\frac{17}{4} & -5,-\frac{9}{2},-\frac{9}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4
, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - a*d**5*meijerg(((-3
, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**
2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*b*d**7*meijerg(((-
13/4, -11/4), (-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()),
d**2/(e**2*x**2))/(4*pi**(3/2)*e**8) - b*d**7*meijerg(((-4, -15/4, -7/2, -1
3/4, -3, 1), ()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*
I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**8) - I*c*d**9*meijerg(((-17/4, -15/4), (
-4, -4, -7/2, 1)), ((-9/2, -17/4, -4, -15/4, -7/2, 0), ()), d**2/(e**2*x**2
))/(4*pi**(3/2)*e**10) - c*d**9*meijerg(((-5, -19/4, -9/2, -17/4, -4, 1), (
)), ((-19/4, -17/4), (-5, -9/2, -9/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x*
*2))/(4*pi**(3/2)*e**10)
```

Giac [A]
time = 3.55, size = 225, normalized size = 1.07

$$-\frac{1}{315}\left(315\,cd^8+315\,bd^6e^2+315\,ad^4e^4-\left(840\,cd^7+630\,bd^5e^2+420\,ad^3e^4-\left(1932\,cd^6+1071\,bd^4e^2+462\,ad^2e^4-\left(2952\,cd^5+1116\,bd^3e^2+252\,ade^4-\left(3098\,cd^4+729\,bd^2e^2-5\left(440\,cd^3+54\,bde^2-\left(204\,cd^2+7\left(\left((xe+d)c-8\,cd\right)(xe+d)+9\,be^2\right)(xe+d)\right)(xe+d)+63\,ae^4\right)(xe+d)\right)(xe+d)\right)(xe+d)\right)(xe+d)\right)\sqrt{xe+d}\,\sqrt{-xe+d}\,e^{(-10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/315*(315*c*d^8 + 315*b*d^6*e^2 + 315*a*d^4*e^4 - (840*c*d^7 + 630*b*d^5*
e^2 + 420*a*d^3*e^4 - (1932*c*d^6 + 1071*b*d^4*e^2 + 462*a*d^2*e^4 - (2952*
c*d^5 + 1116*b*d^3*e^2 + 252*a*d*e^4 - (3098*c*d^4 + 729*b*d^2*e^2 - 5*(440
*c*d^3 + 54*b*d*e^2 - (204*c*d^2 + 7*((x*e + d)*c - 8*c*d)*(x*e + d) + 9*b*
e^2)*(x*e + d))*(x*e + d) + 63*a*e^4)*(x*e + d))*(x*e + d))*(x*e + d))*(x*e
 + d))*sqrt(x*e + d)*sqrt(-x*e + d)*e^(-10)
```

Mupad [B]
time = 1.65, size = 287, normalized size = 1.37

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{128\,c\,d^9+144\,b\,d^7\,e^2+168\,a\,d^5\,e^4}{315\,e^{10}}+\frac{x^7\,\left(40\,c\,d^2\,e^7+45\,b\,e^9\right)}{315\,e^{10}}+\frac{x^2\,\left(64\,c\,d^7\,e^2+72\,b\,d^5\,e^4+84\,a\,d^3\,e^6\right)}{315\,e^{10}}+\frac{x^3\,\left(64\,c\,d^6\,e^3+72\,b\,d^4\,e^5+84\,a\,d^2\,e^7\right)}{315\,e^{10}}+\frac{c\,x^9}{9\,e}+\frac{x^5\,\left(48\,c\,d^4\,e^5+54\,b\,d^2\,e^7+63\,a\,e^9\right)}{315\,e^{10}}+\frac{x\,\left(128\,c\,d^8\,e+144\,b\,d^6\,e^3+168\,a\,d^4\,e^5\right)}{315\,e^{10}}+\frac{x^6\,\left(40\,c\,d^3\,e^6+45\,b\,d\,e^8\right)}{315\,e^{10}}+\frac{x^4\,\left(48\,c\,d^5\,e^4+54\,b\,d^3\,e^6+63\,a\,d\,e^8\right)}{315\,e^{10}}+\frac{c\,d\,x^8}{9\,e^2}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((128*c*d^9 + 168*a*d^5*e^4 + 144*b*d^7*e^2)/(315*e^10) +
 (x^7*(45*b*e^9 + 40*c*d^2*e^7))/(315*e^10) + (x^2*(84*a*d^3*e^6 + 72*b*d^5
*e^4 + 64*c*d^7*e^2))/(315*e^10) + (x^3*(84*a*d^2*e^7 + 72*b*d^4*e^5 + 64*c
*d^6*e^3))/(315*e^10) + (c*x^9)/(9*e) + (x^5*(63*a*e^9 + 54*b*d^2*e^7 + 48*
c*d^4*e^5))/(315*e^10) + (x*(168*a*d^4*e^5 + 144*b*d^6*e^3 + 128*c*d^8*e))/
(315*e^10) + (x^6*(40*c*d^3*e^6 + 45*b*d*e^8))/(315*e^10) + (x^4*(54*b*d^3*
e^6 + 48*c*d^5*e^4 + 63*a*d*e^8))/(315*e^10) + (c*d*x^8)/(9*e^2)))/(d + e*x
)^(1/2)
```

3.133 $\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=159

$$-\frac{d^2(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^8}+\frac{(3cd^4+2bd^2e^2+ae^4)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^8}-\frac{(3cd^2+be^2)(d$$

```
[Out] 1/3*(a*e^4+2*b*d^2*e^2+3*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^8-1/5*(b*e^2
+3*c*d^2)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^8+1/7*c*(-e*x+d)^(7/2)*(e*x+d)^(7/
2)/e^8-d^2*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^8
```

Rubi [A]
time = 0.13, antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {534, 1265, 785}

$$\frac{(d^2-e^2x^2)^2(ae^4+2bd^2e^2+3cd^4)}{3e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^2(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)^3(be^2+3cd^2)}{5e^8\sqrt{d-ex}\sqrt{d+ex}}+\frac{c(d^2-e^2x^2)^4}{7e^8\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^8*Sqrt[d - e*x]*Sqrt
[d + e*x])) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^2)/(3*e^8*Sq
rt[d - e*x]*Sqrt[d + e*x]) - ((3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^3)/(5*e^8*S
qrt[d - e*x]*Sqrt[d + e*x]) + (c*(d^2 - e^2*x^2)^4)/(7*e^8*Sqrt[d - e*x]*Sq
rt[d + e*x])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{x(a+bx+cx^2)}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^6+bd^4e^2+ad^2e^4}{e^6\sqrt{d^2-e^2x}} + \frac{(-3cd^4-2bd^2e^2-ae^4)\sqrt{d^2-e^2x}}{e^6} + \frac{(3cd^2+be^2)}{\cdots}\right.\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{d^2(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^8\sqrt{d-ex}\sqrt{d+ex}} + \frac{(3cd^4+2bd^2e^2+ae^4)(d^2-e^2x^2)^2}{3e^8\sqrt{d-ex}\sqrt{d+ex}} - \frac{(3c\cdots}{5e^{\cdots}}$$

Mathematica [A]
time = 0.24, size = 116, normalized size = 0.73

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(35ae^4(2d^2+e^2x^2)+7b(8d^4e^2+4d^2e^4x^2+3e^6x^4)+3c(16d^6+8d^4e^2x^2+6d^2e^4x^4+5e^6x^6))}{105e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d^
4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*e^
4*x^4 + 5*e^6*x^6)))/e^8
```

Maple [A]
time = 0.17, size = 109, normalized size = 0.69

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\,d^4e^2+48c\,d^6)}{105e^8}$
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\,d^4e^2+48c\,d^6)}{105e^8}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\,d^4e^2+48c\,d^6)}{105e^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^4
*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4*e
^2+48*c*d^6)/e^8
```

Maxima [A]
time = 0.49, size = 199, normalized size = 1.25

$$-\frac{1}{7}\sqrt{-x^2e^2+d^2}\,cx^6e^{(-2)}-\frac{6}{35}\sqrt{-x^2e^2+d^2}\,cd^2x^4e^{(-4)}-\frac{8}{35}\sqrt{-x^2e^2+d^2}\,cd^4x^2e^{(-6)}-\frac{16}{35}\sqrt{-x^2e^2+d^2}\,cd^6e^{(-8)}-\frac{1}{5}\sqrt{-x^2e^2+d^2}\,bx^4e^{(-2)}-\frac{4}{15}\sqrt{-x^2e^2+d^2}\,bd^2x^2e^{(-4)}-\frac{8}{15}\sqrt{-x^2e^2+d^2}\,bd^4e^{(-6)}-\frac{1}{3}\sqrt{-x^2e^2+d^2}\,ax^2e^{(-2)}-\frac{2}{3}\sqrt{-x^2e^2+d^2}\,ad^2e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/7*sqrt(-x^2*e^2 + d^2)*c*x^6*e^(-2) - 6/35*sqrt(-x^2*e^2 + d^2)*c*d^2*x^
4*e^(-4) - 8/35*sqrt(-x^2*e^2 + d^2)*c*d^4*x^2*e^(-6) - 16/35*sqrt(-x^2*e^2
 + d^2)*c*d^6*e^(-8) - 1/5*sqrt(-x^2*e^2 + d^2)*b*x^4*e^(-2) - 4/15*sqrt(-x
^2*e^2 + d^2)*b*d^2*x^2*e^(-4) - 8/15*sqrt(-x^2*e^2 + d^2)*b*d^4*e^(-6) - 1
/3*sqrt(-x^2*e^2 + d^2)*a*x^2*e^(-2) - 2/3*sqrt(-x^2*e^2 + d^2)*a*d^2*e^(-4
)
```

Fricas [A]
time = 0.39, size = 104, normalized size = 0.65

$$-\frac{\left(15\,ce^6x^6+48\,cd^6+56\,bd^4e^2+70\,ad^2e^4+3\,(6\,cd^2e^4+7\,be^6)x^4+(24\,cd^4e^2+28\,bd^2e^4+35\,ae^6)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{105\,e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2*
e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*sqrt(e*x
 + d)*sqrt(-e*x + d)/e^8
```

Sympy [C] Result contains complex when optimal does not.
time = 56.86, size = 367, normalized size = 2.31

$$-\frac{iad^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{ad^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{ibd^5G_{6,6}^{6,2}\left(\begin{matrix}-\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1\\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}-\frac{bd^5G_{6,6}^{2,6}\left(\begin{matrix}-3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}-\frac{icd^7G_{6,6}^{6,2}\left(\begin{matrix}-\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1\\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}-\frac{cd^7G_{6,6}^{2,6}\left(\begin{matrix}-4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4
, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - a*d**3*meijerg(((-2
, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4) - I*b*d**5*meijerg(((-9/
4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**6) - b*d**5*meijerg(((-3, -11/4, -5/2, -9/4, -2
, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e
**2*x**2))/(4*pi**(3/2)*e**6) - I*c*d**7*meijerg(((-13/4, -11/4), (-3, -3,
-5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()), d**2/(e**2*x**2))/(4*pi
**(3/2)*e**8) - c*d**7*meijerg(((-4, -15/4, -7/2, -13/4, -3, 1), ()), ((-15
/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*p
i**(3/2)*e**8)
```

Giac [A]
time = 4.54, size = 163, normalized size = 1.03

$$-\frac{1}{105}\left(105\,cd^6 + 105\,bd^4e^2 + 105\,ad^2e^4 - \left(210\,cd^5 + 140\,bd^3e^2 + 70\,ade^4 - \left(357\,cd^4 + 154\,bd^2e^2 - 3\left(124\,cd^3 + 28\,bde^2 - \left(81\,cd^2 + 5\left((xe+d)c - 6\,cd\right)(xe+d) + 7\,be^2\right)(xe+d)\right)(xe+d) + 35\,ae^4\right)(xe+d)\right)(xe+d)\right)\sqrt{xe+d}\,\sqrt{-xe+d}\,e^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/105*(105*c*d^6 + 105*b*d^4*e^2 + 105*a*d^2*e^4 - (210*c*d^5 + 140*b*d^3*
e^2 + 70*a*d*e^4 - (357*c*d^4 + 154*b*d^2*e^2 - 3*(124*c*d^3 + 28*b*d*e^2 -
 (81*c*d^2 + 5*((x*e + d)*c - 6*c*d)*(x*e + d) + 7*b*e^2)*(x*e + d))*(x*e +
 d) + 35*a*e^4)*(x*e + d))*(x*e + d))*sqrt(x*e + d)*sqrt(-x*e + d)*e^(-8)
```

Mupad [B]
time = 1.49, size = 215, normalized size = 1.35

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{48\,c\,d^7+56\,b\,d^5\,e^2+70\,a\,d^3\,e^4}{105\,e^8}+\frac{x^5\,\left(18\,c\,d^2\,e^5+21\,b\,e^7\right)}{105\,e^8}+\frac{c\,x^7}{7\,e}+\frac{x^3\,\left(24\,c\,d^4\,e^3+28\,b\,d^2\,e^5+35\,a\,e^7\right)}{105\,e^8}+\frac{x\,\left(48\,c\,d^6\,e+56\,b\,d^4\,e^3+70\,a\,d^2\,e^5\right)}{105\,e^8}+\frac{x^4\,\left(18\,c\,d^3\,e^4+21\,b\,d\,e^6\right)}{105\,e^8}+\frac{x^2\,\left(24\,c\,d^5\,e^2+28\,b\,d^3\,e^4+35\,a\,d\,e^6\right)}{105\,e^8}+\frac{c\,d\,x^6}{7\,e^2}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((48*c*d^7 + 70*a*d^3*e^4 + 56*b*d^5*e^2)/(105*e^8) + (x^
5*(21*b*e^7 + 18*c*d^2*e^5))/(105*e^8) + (c*x^7)/(7*e) + (x^3*(35*a*e^7 + 2
8*b*d^2*e^5 + 24*c*d^4*e^3))/(105*e^8) + (x*(70*a*d^2*e^5 + 56*b*d^4*e^3 +
48*c*d^6*e))/(105*e^8) + (x^4*(18*c*d^3*e^4 + 21*b*d*e^6))/(105*e^8) + (x^2
*(28*b*d^3*e^4 + 24*c*d^5*e^2 + 35*a*d*e^6))/(105*e^8) + (c*d*x^6)/(7*e^2))
)/(d + e*x)^(1/2)
```

3.134 $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=109

$$-\frac{(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^6}+\frac{(2cd^2+be^2)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^6}-\frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

```
[Out] 1/3*(b*e^2+2*c*d^2)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^6-1/5*c*(-e*x+d)^(5/2)*(
e*x+d)^(5/2)/e^6-(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6
```

Rubi [A]
time = 0.08, antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {534, 1261, 712}

$$-\frac{(d^2-e^2x^2)(ae^4+bd^2e^2+cd^4)}{e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{(d^2-e^2x^2)^2(be^2+2cd^2)}{3e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{c(d^2-e^2x^2)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e*x]*Sqrt[d +
 e*x])) + ((2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^2)/(3*e^6*Sqrt[d - e*x]*Sqrt[d
 + e*x]) - (c*(d^2 - e^2*x^2)^3)/(5*e^6*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\,\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^4+bd^2e^2+ae^4}{e^4\sqrt{d^2-e^2x}} + \frac{(-2cd^2-be^2)\sqrt{d^2-e^2x}}{e^4} + \frac{c(d^2-e^2x)^{3/2}}{e^4}\right) dx$$

$$= -\frac{(cd^4+bd^2e^2+ae^4)(d^2-e^2x^2)}{e^6\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{(2cd^2+be^2)(d^2-e^2x^2)^2}{3e^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{c(d^2-e^2x^2}{5e^6\sqrt{d-ex}\,\sqrt{}}$$

Mathematica [A]
time = 0.16, size = 80, normalized size = 0.73

$$-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,(5(2bd^2e^2+3ae^4+be^4x^2)+c(8d^4+4d^2e^2x^2+3e^4x^4))}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2) +
 c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)))/e^6
```

Maple [A]
time = 0.14, size = 73, normalized size = 0.67

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10d^2e^2b+8d^4c)}{15e^6}$	73
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10d^2e^2b+8d^4c)}{15e^6}$	73
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3c\,x^4e^4+5b\,e^4x^2+4c\,d^2e^2x^2+15e^4a+10d^2e^2b+8d^4c)}{15e^6}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^2
+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

Maxima [**A**]
time = 0.49, size = 127, normalized size = 1.17

$$-\frac{1}{5}\sqrt{-x^2e^2+d^2}\,cx^4e^{(-2)}-\frac{4}{15}\sqrt{-x^2e^2+d^2}\,cd^2x^2e^{(-4)}-\frac{8}{15}\sqrt{-x^2e^2+d^2}\,cd^4e^{(-6)}-\frac{1}{3}\sqrt{-x^2e^2+d^2}\,bx^2e^{(-2)}-\frac{2}{3}\sqrt{-x^2e^2+d^2}\,bd^2e^{(-4)}-\sqrt{-x^2e^2+d^2}\,ae^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*c*x^4*e^(-2) - 4/15*sqrt(-x^2*e^2 + d^2)*c*d^2*x^
2*e^(-4) - 8/15*sqrt(-x^2*e^2 + d^2)*c*d^4*e^(-6) - 1/3*sqrt(-x^2*e^2 + d^2
)*b*x^2*e^(-2) - 2/3*sqrt(-x^2*e^2 + d^2)*b*d^2*e^(-4) - sqrt(-x^2*e^2 + d^
2)*a*e^(-2)
```

Fricas [**A**]
time = 0.41, size = 71, normalized size = 0.65

$$-\frac{\left(3\,ce^4x^4+8\,cd^4+10\,bd^2e^2+15\,ae^4+\left(4\,cd^2e^2+5\,be^4\right)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{15\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*b
*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

Sympy [**C**] Result contains complex when optimal does not.
time = 37.09, size = 350, normalized size = 3.21

$$-\frac{iadG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{adG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{ibd^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{bd^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{icd^5G_{6,6}^{6,2}\left(\begin{matrix}-\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1\\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}-\frac{cd^5G_{6,6}^{2,6}\left(\begin{matrix}-3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0)
, ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg(((-1, -3/4, -1/2,
 -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg(((-5/4, -3/4), (-1,
 -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4
*pi**(3/2)*e**4) - b*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7
/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e**4) - I*c*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2,
```

```
 -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - c*d*
*5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2,
 -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6)
```

Giac [A]
time = 5.20, size = 103, normalized size = 0.94

$$-\frac{1}{15}\left(15\,cd^4+15\,bd^2e^2-\left(20\,cd^3+10\,bde^2-\left(22\,cd^2+3\left((xe+d)c-4\,cd\right)(xe+d)+5\,be^2\right)(xe+d)\right)(xe+d)+15\,ae^4\right)\sqrt{xe+d}\,\sqrt{-xe+d}\,e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/15*(15*c*d^4 + 15*b*d^2*e^2 - (20*c*d^3 + 10*b*d*e^2 - (22*c*d^2 + 3*((x
*e + d)*c - 4*c*d)*(x*e + d) + 5*b*e^2)*(x*e + d))*(x*e + d) + 15*a*e^4)*sq
rt(x*e + d)*sqrt(-x*e + d)*e^(-6)
```

Mupad [B]
time = 1.38, size = 143, normalized size = 1.31

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{8\,c\,d^5+10\,b\,d^3\,e^2+15\,a\,d\,e^4}{15\,e^6}+\frac{x^3\,\left(4\,c\,d^2\,e^3+5\,b\,e^5\right)}{15\,e^6}+\frac{c\,x^5}{5\,e}+\frac{x^2\,\left(4\,c\,d^3\,e^2+5\,b\,d\,e^4\right)}{15\,e^6}+\frac{x\,\left(8\,c\,d^4\,e+10\,b\,d^2\,e^3+15\,a\,e^5\right)}{15\,e^6}+\frac{c\,d\,x^4}{5\,e^2}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((8*c*d^5 + 10*b*d^3*e^2 + 15*a*d*e^4)/(15*e^6) + (x^3*(5
*b*e^5 + 4*c*d^2*e^3))/(15*e^6) + (c*x^5)/(5*e) + (x^2*(4*c*d^3*e^2 + 5*b*d
*e^4))/(15*e^6) + (x*(15*a*e^5 + 10*b*d^2*e^3 + 8*c*d^4*e))/(15*e^6) + (c*d
*x^4)/(5*e^2)))/(d + e*x)^(1/2)
```

3.135 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=93

$$-\frac{(cd^2+be^2)\sqrt{d-ex}\sqrt{d+ex}}{e^4}+\frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4}-\frac{a\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d}$$

```
[Out] 1/3*c*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^4-a*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/
2)/d)/d-(b*e^2+c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {534, 1265, 911, 1167, 214}

$$-\frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(be^2+cd^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^2 + b*e^2)*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])) + (c
*(d^2 - e^2*x^2)^2)/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (a*Sqrt[d^2 - e^2
*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\
&= \frac{\sqrt{d^2-e^2x^2}\,\mathrm{Subst}\left(\int \frac{a+bx+cx^2}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\sqrt{d^2-e^2x^2}\,\mathrm{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{\left(2cd^2+be^2\right)x^2}{e^4}+\frac{cx^4}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\sqrt{d^2-e^2x^2}\,\mathrm{Subst}\left(\int \left(b+\frac{cd^2}{e^2}-\frac{cx^2}{e^2}+\frac{a}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right)dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}\\
&= -\frac{\left(cd^2+be^2\right)\left(d^2-e^2x^2\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\left(d^2-e^2x^2\right)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(a\sqrt{d^2-e^2x^2}\right)\mathrm{Subst}\left(\cdots\right.}{e^2\sqrt{d-\cdots}}\\
&= -\frac{\left(cd^2+be^2\right)\left(d^2-e^2x^2\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\left(d^2-e^2x^2\right)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\cdots\right.}{d\sqrt{d-ex}\sqrt{d+\cdots}}
\end{aligned}
$$

Mathematica [A]

time = 0.22, size = 106, normalized size = 1.14

$$-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,(2cd^2+3be^2+ce^2x^2)}{3e^4}+\frac{a\log\left(-1+\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d}-\frac{a\log\left(d+\frac{d\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/e^4 + (a
*Log[-1 + Sqrt[d + e*x]/Sqrt[d - e*x]])/d - (a*Log[d + (d*Sqrt[d + e*x])/Sq
rt[d - e*x]])/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 143, normalized size = 1.54

method	result
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(\mathrm{csgn}(d)cd\,e^2x^2\sqrt{-e^2x^2+d^2}+3\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^4+3\,\mathrm{csgn}(d)\sqrt{-}\right.}{3d\sqrt{-e^2x^2+d^2}\,e^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*c*d*e^2*x^2*(-e^2*x^2+d^2)^(1/
2)+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4+3*csgn(d)*(-e^2*x^2+d
^2)^(1/2)*b*d*e^2+2*csgn(d)*(-e^2*x^2+d^2)^(1/2)*c*d^3)*csgn(d)/(-e^2*x^2+d
^2)^(1/2)/e^4
```

Maxima [A]
time = 0.50, size = 98, normalized size = 1.05

$$-\frac{1}{3}\sqrt{-x^2e^2+d^2}\,cx^2e^{(-2)}-\frac{2}{3}\sqrt{-x^2e^2+d^2}\,cd^2e^{(-4)}-\sqrt{-x^2e^2+d^2}\,be^{(-2)}-\frac{a\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/3*sqrt(-x^2*e^2 + d^2)*c*x^2*e^(-2) - 2/3*sqrt(-x^2*e^2 + d^2)*c*d^2*e^(
-4) - sqrt(-x^2*e^2 + d^2)*b*e^(-2) - a*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2
+ d^2)*d/abs(x))/d
```

Fricas [A]
time = 0.39, size = 80, normalized size = 0.86

$$\frac{3\,ae^4\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)-\left(cde^2x^2+2\,cd^3+3\,bde^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{3\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*c
*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 45.40, size = 304, normalized size = 3.27

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibdG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{bdG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2}-\frac{icd^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{cd^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()
), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*
pi**(3/2)*d) - I*b*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0
, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijerg(((-5/
4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1,
 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**
2*x**2))/(4*pi**(3/2)*e**4)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 7.37652e-11Francis algorithm not precise enough for[1.0,-772.7947352
```

```
08,124419.104743,-7478366.70813,154801136.25]schur row 1 1.06106e-10Francis
 algorithm not p
```

Mupad [B]
time = 2.95, size = 161, normalized size = 1.73

$$\frac{a\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d}-\frac{\sqrt{d-e\,x}\,\left(\frac{2\,c\,d^3}{3\,e^4}+\frac{c\,x^3}{3\,e}+\frac{c\,d\,x^2}{3\,e^2}+\frac{2\,c\,d^2\,x}{3\,e^3}\right)}{\sqrt{d+e\,x}}-\frac{\left(\frac{b\,d}{e^2}+\frac{b\,x}{e}\right)\,\sqrt{d-e\,x}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] (a*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - ((d - e*
x)^(1/2)*((2*c*d^3)/(3*e^4) + (c*x^3)/(3*e) + (c*d*x^2)/(3*e^2) + (2*c*d^2*
x)/(3*e^3)))/(d + e*x)^(1/2) - (((b*d)/e^2 + (b*x)/e)*(d - e*x)^(1/2))/(d +
 e*x)^(1/2)
```

3.136 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=99

$$-\frac{c\sqrt{d-ex}\,\sqrt{d+ex}}{e^2}-\frac{a\sqrt{d-ex}\,\sqrt{d+ex}}{2d^2x^2}-\frac{(2bd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{d-ex}\,\sqrt{d+ex}}{d}\right)}{2d^3}$$

```
[Out] -1/2*(a*e^2+2*b*d^2)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^3-c*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/e^2-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {534, 1265, 911, 1171, 396, 214}

$$-\frac{\sqrt{d^2-e^2x^2}\,(ae^2+2bd^2)\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{c(d^2-e^2x^2)}{e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((c*(d^2 - e^2*x^2))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x])) - (a*(d^2 - e^2*x^
2))/(2*d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((2*b*d^2 + a*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3*Sqrt[d - e*x]*Sqrt[d + e*x
])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
```

```
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^3\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^2\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-a-\frac{2(cd^4+bd^2e^2)}{e^4}+\frac{2cd^2x^2}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x \cdots\right.}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{c(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(e^2\left(\frac{2cd^4}{e^6}+\frac{-a-\frac{2(cd^4+bd\cdots}{e^4}}{e^2}\cdots\right.\right.}{\cdots}$$

$$= -\frac{c(d^2-e^2x^2)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{a(d^2-e^2x^2)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(2bd^2+ae^2)\sqrt{d^2-e^2x\cdots}}{2d^3\sqrt{d-\cdots}}$$

Mathematica [A]
time = 0.25, size = 85, normalized size = 0.86

$$-\frac{\frac{\sqrt{d-ex}\sqrt{d+ex}\,(ade^2+2cd^3x^2)}{e^2x^2}+2(2bd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/2*((Sqrt[d - e*x]*Sqrt[d + e*x]*(a*d*e^2 + 2*c*d^3*x^2))/(e^2*x^2) + 2*(
2*b*d^2 + a*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 163, normalized size = 1.65

method	result

default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(2\,\mathrm{csgn}(d)c\,d^3x^2\sqrt{-e^2x^2+d^2}+\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^4x^2+2\ln\left(\frac{2d\left(\sqrt{\right.}\right.}{}\right)\right.}{2d^3\sqrt{-e^2x^2+d^2}\,e^2x^2}$
risch	$-\frac{a\sqrt{-ex+d}\,\sqrt{ex+d}}{2d^2x^2}+\frac{\left(-\frac{c\sqrt{-(ex-d)(ex+d)}}{e^2}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)a\,e^2}{2d^2\sqrt{d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{}}{}\right.}{}\right.}{\sqrt{ex+d}\,\sqrt{-ex+d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^3*(2*csgn(d)*c*d^3*x^2*(-e^2*x^2+d^2)^(
1/2)+ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4*x^2+2*ln(2*d*((-e^2*x
^2+d^2)^(1/2)*csgn(d)+d)/x)*b*d^2*e^2*x^2+csgn(d)*a*d*e^2*(-e^2*x^2+d^2)^(1
/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/e^2/x^2
```

Maxima [A]
time = 0.49, size = 117, normalized size = 1.18

$$-\sqrt{-x^2e^2+d^2}\,ce^{(-2)}-\frac{b\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d}-\frac{ae^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{2\,d^3}-\frac{\sqrt{-x^2e^2+d^2}\,a}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -sqrt(-x^2*e^2 + d^2)*c*e^(-2) - b*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2
)*d/abs(x))/d - 1/2*a*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x
))/d^3 - 1/2*sqrt(-x^2*e^2 + d^2)*a/(d^2*x^2)
```

Fricas [A]
time = 0.41, size = 98, normalized size = 0.99

$$-\frac{2\,cd^4x^2-(2\,bd^2e^2+ae^4)x^2\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)+(2\,cd^3x^2+ade^2)\sqrt{ex+d}\,\sqrt{-ex+d}}{2\,d^3e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x
+ d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3*e
^2*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 7.37652e-11Francis algorithm not precise enough for[1.0,-772.7947352
08,124419.104743,-7478366.70813,154801136.25]schur row 1 1.06106e-10Francis
 algorithm not p
```

Mupad [**B**]
time = 5.15, size = 422, normalized size = 4.26

$$\frac{b\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d}-\frac{\left(\frac{c\,d}{e^2}+\frac{c\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}}-\frac{\frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{a\,e^2}{2}+\frac{15\,a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}}{\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}}-\frac{a\,e^2\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,d^3}+\frac{a\,e^2\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{2\,d^3}+\frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{32\,d^3\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] (b*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (((c*d)/
e^2 + (c*x)/e)*(d - e*x)^(1/2))/(d + e*x)^(1/2) - ((a*e^2*((d + e*x)^(1/2)
- d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (a*e^2)/2 + (15*a*e^2*((d + e
*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e
*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^
(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2
) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) - (a*e^2*log(((d + e*x)^(1/2
) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (a*e^2*log(((d + e*x)^
(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) + (a*e^2*((d
 + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

3.137 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=126

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}-\frac{(4bd^2+3ae^2)\sqrt{d-ex}\sqrt{d+ex}}{8d^4x^2}-\frac{(8cd^4+4bd^2e^2+3ae^4)\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d-}}{d}\right.}{8d^5}$$

[Out] -1/8*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^5-1/4*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^4-1/8*(3*a*e^2+4*b*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^4/x^2

Rubi [A]
time = 0.19, antiderivative size = 182, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {534, 1265, 911, 1171, 393, 214}

$$-\frac{\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)(3ae^4+4bd^2e^2+8cd^4)}{8d^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(3ae^2+4bd^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]

[Out] -1/4*(a*(d^2 - e^2*x^2))/(d^2*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((4*b*d^2 + 3*a*e^2)*(d^2 - e^2*x^2))/(8*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^5*Sqrt[d - e*x]*Sqrt[d + e*x])

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
```

```
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^3\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-3a-\frac{4(cd^4+bd^2e^2)}{e^4}+\frac{4cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx\right.}{4d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{(4bd^2+3ae^2)(d^2-e^2x^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(4b+\frac{8cd^2}{e^2}+\frac{3ae}{d^2}\right.\right.}{}$$

$$= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{(4bd^2+3ae^2)(d^2-e^2x^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(8cd^4+4bd^2e^2+}{}$$

Mathematica [A]
time = 0.26, size = 102, normalized size = 0.81

$$-\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}(2ad^2+4bd^2x^2+3ae^2x^2)}{x^4}+2(8cd^4+4bd^2e^2+3ae^4)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/8*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2))
/x^4 + 2*(8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e
*x]])/d^5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 222, normalized size = 1.76

method	result

risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3a\,e^2x^2+4b\,d^2x^2+2a\,d^2)}{8d^4x^4}+\frac{\left(-\frac{3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)e^4a}{8d^4\sqrt{d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-\cdots}}{x}\right.}{2d^2\sqrt{\cdots}}\right.}{\sqrt{\cdots}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(3\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^4x^4+4\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)b\,d^2e^2x\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^5*(3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(
d)+d)/x)*a*e^4*x^4+4*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*b*d^2*e^2*x
^4+8*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*c*d^4*x^4+3*csgn(d)*a*d*e^2
*x^2*(-e^2*x^2+d^2)^(1/2)+4*csgn(d)*b*d^3*x^2*(-e^2*x^2+d^2)^(1/2)+2*csgn(d
)*a*d^3*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^4
```

Maxima [A]

time = 0.49, size = 184, normalized size = 1.46

$$-\frac{c\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d}-\frac{be^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{2\,d^3}-\frac{3\,ae^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{8\,d^5}-\frac{\sqrt{-x^2e^2+d^2}\,b}{2\,d^2x^2}-\frac{3\,\sqrt{-x^2e^2+d^2}\,ae^2}{8\,d^4x^2}-\frac{\sqrt{-x^2e^2+d^2}\,a}{4\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -c*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d - 1/2*b*e^2*log(2*
d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 - 3/8*a*e^4*log(2*d^2/abs
(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^5 - 1/2*sqrt(-x^2*e^2 + d^2)*b/(d^
2*x^2) - 3/8*sqrt(-x^2*e^2 + d^2)*a*e^2/(d^4*x^2) - 1/4*sqrt(-x^2*e^2 + d^2
)*a/(d^2*x^4)
```

Fricas [A]

time = 0.38, size = 102, normalized size = 0.81

$$\frac{(8\,cd^4+4\,bd^2e^2+3\,ae^4)x^4\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)-(2\,ad^3+(4\,bd^3+3\,ade^2)x^2)\sqrt{ex+d}\,\sqrt{-ex+d}}{8\,d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*log((sqrt(e*x + d)*sqrt(-e*x + d
) - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d))/(d^5*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 7.37652e-11Francis algorithm not precise enough for[1.0,-772.7947352
08,124419.104743,-7478366.70813,154801136.25]schur row 1 1.06106e-10Francis
 algorithm not p
```

Mupad [**B**]
time = 10.82, size = 932, normalized size = 7.40

$$\frac{\frac{a e^4}{4}+\frac{6 a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e x}-\sqrt{d}\right)^2}-\frac{53 a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^4}{2 \left(\sqrt{d-e x}-\sqrt{d}\right)^4}-\frac{87 a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e x}-\sqrt{d}\right)^6}+\frac{657 a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^8}{4 \left(\sqrt{d-e x}-\sqrt{d}\right)^8}-\frac{121 a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-e x}-\sqrt{d}\right)^{10}}}{\frac{256 d^5 \left(\sqrt{d+e x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e x}-\sqrt{d}\right)^4}-\frac{1024 d^5 \left(\sqrt{d+e x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e x}-\sqrt{d}\right)^6}+\frac{1536 d^5 \left(\sqrt{d+e x}-\sqrt{d}\right)^8}{\left(\sqrt{d-e x}-\sqrt{d}\right)^8}-\frac{1024 d^5 \left(\sqrt{d+e x}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-e x}-\sqrt{d}\right)^{10}}+\frac{256 d^5 \left(\sqrt{d+e x}-\sqrt{d}\right)^{12}}{\left(\sqrt{d-e x}-\sqrt{d}\right)^{12}}}-\frac{\frac{b e^2 \left(\sqrt{d+e x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e x}-\sqrt{d}\right)^2}-\frac{b e^2}{2}+\frac{15 b e^2 \left(\sqrt{d+e x}-\sqrt{d}\right)^4}{2 \left(\sqrt{d-e x}-\sqrt{d}\right)^4}}{\frac{16 d^3 \left(\sqrt{d+e x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e x}-\sqrt{d}\right)^2}-\frac{32 d^3 \left(\sqrt{d+e x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e x}-\sqrt{d}\right)^4}+\frac{16 d^3 \left(\sqrt{d+e x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e x}-\sqrt{d}\right)^6}}+\frac{c \left(\ln\left(\frac{\left(\sqrt{d+e x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e x}-\sqrt{d}}{\sqrt{d-e x}-\sqrt{d}}\right)\right)}{d}-\frac{3 a e^4 \ln\left(\frac{\sqrt{d+e x}-\sqrt{d}}{\sqrt{d-e x}-\sqrt{d}}\right)}{8 d^5}-\frac{b e^2 \ln\left(\frac{\sqrt{d+e x}-\sqrt{d}}{\sqrt{d-e x}-\sqrt{d}}\right)}{2 d^3}+\frac{3 a e^4 \ln\left(\frac{\left(\sqrt{d+e x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e x}-\sqrt{d}\right)^2}-1\right)}{8 d^5}+\frac{b e^2 \ln\left(\frac{\left(\sqrt{d+e x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e x}-\sqrt{d}\right)^2}-1\right)}{2 d^3}+\frac{7 a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^2}{256 d^5 \left(\sqrt{d-e x}-\sqrt{d}\right)^2}+\frac{a e^4 \left(\sqrt{d+e x}-\sqrt{d}\right)^4}{1024 d^5 \left(\sqrt{d-e x}-\sqrt{d}\right)^4}+\frac{b e^2 \left(\sqrt{d+e x}-\sqrt{d}\right)^2}{32 d^3 \left(\sqrt{d-e x}-\sqrt{d}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^5*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((a*e^4)/4 + (6*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(
1/2))^2 - (53*a*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^
(1/2))^4) - (87*a*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(
1/2))^6 + (657*a*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d
^(1/2))^8) - (121*a*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) -
d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^
(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1
/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2
))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2)
)^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))
```

```
^12) - ((b*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2
 - (b*e^2)/2 + (15*b*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2)
 - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1
/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))
^6) + (c*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 -
 1) - log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (3
*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5
) - (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2
*d^3) + (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/
2))^2 - 1))/(8*d^5) + (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(
1/2) - d^(1/2))^2 - 1))/(2*d^3) + (7*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/(
256*d^5*((d - e*x)^(1/2) - d^(1/2))^2) + (a*e^4*((d + e*x)^(1/2) - d^(1/2))
^4)/(1024*d^5*((d - e*x)^(1/2) - d^(1/2))^4) + (b*e^2*((d + e*x)^(1/2) - d^
(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

3.138 $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=212

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{6d^2x^6}-\frac{(6bd^2+5ae^2)\sqrt{d-ex}\sqrt{d+ex}}{24d^4x^4}-\frac{(8cd^4+6bd^2e^2+5ae^4)\sqrt{d-ex}\sqrt{d+ex}}{16d^6x^2}-\frac{e^2}{}$$

```
[Out] -1/6*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^6-1/24*(5*a*e^2+6*b*d^2)*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/d^4/x^4-1/16*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*(-e*x+d)^(1
/2)*(e*x+d)^(1/2)/d^6/x^2-1/16*e^2*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*arctanh((-
e^2*x^2+d^2)^(1/2)/d)*(-e^2*x^2+d^2)^(1/2)/d^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 248, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {534, 1265, 911, 1171, 393, 205, 214}

$$-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)(5ae^4+6bd^2e^2+8cd^4)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(5ae^4+6bd^2e^2+8cd^4)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(5ae^2+6bd^2)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/6*(a*(d^2 - e^2*x^2))/(d^2*x^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*b*d^2
+ 5*a*e^2)*(d^2 - e^2*x^2))/(24*d^4*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*
c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(d^2 - e^2*x^2))/(16*d^6*x^2*Sqrt[d - e*x]*S
qrt[d + e*x]) - (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*
ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\,\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^7\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^4\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^4}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-5a-\frac{6(cd^4+bd^2e^2)}{e^4}+\frac{6cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx\right.}{6d^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(6bd^2+5ae^2)(d^2-e^2x^2)}{24d^4x^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left(\left(6b+\frac{8cd^2}{e^2}+\frac{5a}{d}\right.\right.}{}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(6bd^2+5ae^2)(d^2-e^2x^2)}{24d^4x^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(8cd^4+6bd^2e^2+}{16d^6x^2\sqrt{d-}}$$

$$= -\frac{a(d^2-e^2x^2)}{6d^2x^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(6bd^2+5ae^2)(d^2-e^2x^2)}{24d^4x^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(8cd^4+6bd^2e^2+}{16d^6x^2\sqrt{d-}}$$

Mathematica [A]
time = 0.40, size = 141, normalized size = 0.67

$$-\frac{\frac{d\sqrt{d-ex}\,\sqrt{d+ex}\,(6(2bd^4x^2+4cd^4x^4+3bd^2e^2x^4)+a(8d^4+10d^2e^2x^2+15e^4x^4))}{x^6}+6e^2(8cd^4+6bd^2e^2+5ae^4)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{48d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/48*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*d
^2*e^2*x^4) + a*(8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4)))/x^6 + 6*e^2*(8*c*d^
4 + 6*b*d^2*e^2 + 5*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^7
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 306, normalized size = 1.44

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(15a\,e^4x^4+18b\,d^2e^2x^4+24c\,d^4x^4+10a\,d^2e^2x^2+12b\,d^4x^2+8d^4a)}{48d^6x^6}+\frac{\left(-\frac{5e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2 \ldots}}{x}\right.}{16d^6\sqrt{d^2}} \ldots\right.}{\ldots}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(15\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^6x^6+18\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)+d\right)}{x}\right)b\,d^2e \ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^7*(15*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csg
n(d)+d)/x)*a*e^6*x^6+18*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*b*d^2*e^
4*x^6+24*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*c*d^4*e^2*x^6+15*(-e^2*
x^2+d^2)^(1/2)*csgn(d)*d*a*e^4*x^4+18*(-e^2*x^2+d^2)^(1/2)*csgn(d)*d^3*b*e^
2*x^4+24*(-e^2*x^2+d^2)^(1/2)*csgn(d)*d^5*c*x^4+10*csgn(d)*a*d^3*e^2*x^2*(-
e^2*x^2+d^2)^(1/2)+12*csgn(d)*b*d^5*x^2*(-e^2*x^2+d^2)^(1/2)+8*csgn(d)*a*d^
5*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^6
```

Maxima [A]

time = 0.50, size = 256, normalized size = 1.21

$$-\frac{ce^2\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{2d^3}-\frac{3be^4\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{8d^5}-\frac{\sqrt{-x^2e^2+d^2}\,c}{2d^2x^2}-\frac{3\sqrt{-x^2e^2+d^2}\,be^2}{8d^4x^2}-\frac{5ae^6\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{16d^7}-\frac{\sqrt{-x^2e^2+d^2}\,b}{4d^2x^4}-\frac{5\sqrt{-x^2e^2+d^2}\,ae^4}{16d^6x^2}-\frac{5\sqrt{-x^2e^2+d^2}\,ae^2}{24d^4x^4}-\frac{\sqrt{-x^2e^2+d^2}\,a}{6d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*c*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 - 3/8*b*
e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^5 - 1/2*sqrt(-x^2
*e^2 + d^2)*c/(d^2*x^2) - 3/8*sqrt(-x^2*e^2 + d^2)*b*e^2/(d^4*x^2) - 5/16*a
*e^6*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^7 - 1/4*sqrt(-x^
2*e^2 + d^2)*b/(d^2*x^4) - 5/16*sqrt(-x^2*e^2 + d^2)*a*e^4/(d^6*x^2) - 5/24
*sqrt(-x^2*e^2 + d^2)*a*e^2/(d^4*x^4) - 1/6*sqrt(-x^2*e^2 + d^2)*a/(d^2*x^6
)
```

Fricas [A]

time = 0.39, size = 137, normalized size = 0.65

$$\frac{3\,(8\,cd^4e^2+6\,bd^2e^4+5\,ae^6)x^6\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)-(8\,ad^5+3\,(8\,cd^5+6\,bd^3e^2+5\,ade^4)x^4+2\,(6\,bd^5+5\,ad^3e^2)x^2)\sqrt{ex+d}\,\sqrt{-ex+d}}{48\,d^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*log((sqrt(e*x + d)*sqrt(-
e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 + 2
*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^7*x^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 7.37652e-11Francis algorithm not precise enough for[1.0,-772.7947352
08,124419.104743,-7478366.70813,154801136.25]schur row 1 1.06106e-10Francis
 algorithm not p
```

Mupad [**B**]
time = 20.05, size = 1621, normalized size = 7.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^7*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((b*e^4)/4 + (6*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(
1/2))^2 - (53*b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^
(1/2))^4) - (87*b*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(
1/2))^6 + (657*b*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d
^(1/2))^8) - (121*b*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) -
d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^
```

(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12) - ((c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (c*e^2)/2 + (15*c*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) + ((a*e^6)/6 + (4*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 + (71*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 - (1558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(3*((d - e*x)^(1/2) - d^(1/2))^6) - (540*a*e^6*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2))^8 + (4248*a*e^6*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10 - (7683*a*e^6*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12 + (5558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)^(1/2) - d^(1/2))^14 - (3643*a*e^6*((d + e*x)^(1/2) - d^(1/2))^16)/(2*((d - e*x)^(1/2) - d^(1/2))^16))/((4096*d^7*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2))^8 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2))^10 - (81920*d^7*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))^12 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)^(1/2) - d^(1/2))^14 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^16)/((d - e*x)^(1/2) - d^(1/2))^16 + (4096*d^7*((d + e*x)^(1/2) - d^(1/2))^18)/((d - e*x)^(1/2) - d^(1/2))^18) - (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(16*d^7) - (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5) - (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(16*d^7) + (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(8*d^5) + (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) + (197*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/(8192*d^7*((d - e*x)^(1/2) - d^(1/2))^2) + (5*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/(4096*d^7*((d - e*x)^(1/2) - d^(1/2))^4) + (a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(24576*d^7*((d - e*x)^(1/2) - d^(1/2))^6) + (7*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/(256*d^5*((d - e*x)^(1/2) - d^(1/2))^2) + (b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(1024*d^5*((d - e*x)^(1/2) - d^(1/2))^4) + (c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)

3.139 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=216

$$-\frac{(5cd^4+6bd^2e^2+8ae^4)\,x\sqrt{d-ex}\sqrt{d+ex}}{16e^6}-\frac{(5cd^2+6be^2)\,x^3\sqrt{d-ex}\sqrt{d+ex}}{24e^4}+\frac{cx^5(-d+ex)\sqrt{d+e}}{6e^2\sqrt{d-ex}}$$

```
[Out] 1/6*c*x^5*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/16*(8*a*e^4+6*b*d^2*e^
2+5*c*d^4)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6-1/24*(6*b*e^2+5*c*d^2)*x^3*(-
e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*c*d^4)*arcta
n(e*x/(-e^2*x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e^7/(-e*x+d)^(1/2)/(e*x+d)
^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {534, 1281, 470, 327, 223, 209}

$$\frac{d^2\sqrt{d^2-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+6bd^2e^2+5cd^4)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x(d^2-e^2x^2)(8ae^4+6bd^2e^2+5cd^4)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{x^3(d^2-e^2x^2)(6be^2+5cd^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/16*((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e
*x]*Sqrt[d + e*x]) - ((5*c*d^2 + 6*b*e^2)*x^3*(d^2 - e^2*x^2))/(24*e^4*Sqrt
[d - e*x]*Sqrt[d + e*x]) - (c*x^5*(d^2 - e^2*x^2))/(6*e^2*Sqrt[d - e*x]*Sqr
t[d + e*x]) + (d^2*(5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rubi steps

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2(-6ae^2-(5cd^2+6be^2)x^2)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{(5cd^2+6be^2)x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^5(d^2-e^2x^2)}{6e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\Big((5cd^4+6bd^2e^2+8}{8e^4}$$

$$= -\frac{(5cd^4+6bd^2e^2+8ae^4)x(d^2-e^2x^2)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5cd^2+6be^2)x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^5}{6e^2\sqrt{d}}$$

$$= -\frac{(5cd^4+6bd^2e^2+8ae^4)x(d^2-e^2x^2)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5cd^2+6be^2)x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^5}{6e^2\sqrt{d}}$$

$$= -\frac{(5cd^4+6bd^2e^2+8ae^4)x(d^2-e^2x^2)}{16e^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5cd^2+6be^2)x^3(d^2-e^2x^2)}{24e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^5}{6e^2\sqrt{d}}$$

Mathematica [A]
time = 0.29, size = 134, normalized size = 0.62

$$\frac{-ex\sqrt{d-ex}\sqrt{d+ex}\,(6(3bd^2e^2+4ae^4+2be^4x^2)+c(15d^4+10d^2e^2x^2+8e^4x^4))+6d^2(5cd^4+6bd^2e^2+8ae^4)\tan^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{48e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*x^2)
 + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4))) + 6*d^2*(5*c*d^4 + 6*b*d^2*e^2
 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(48*e^7)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 273, normalized size = 1.26

method	result
risch	$-\frac{x(8cx^4e^4+12be^4x^2+10cd^2e^2x^2+24e^4a+18d^2e^2b+15d^4c)\sqrt{-ex+d}\,\sqrt{ex+d}}{48e^6} + \frac{\left(\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)a}{2e^2\sqrt{e^2}}+\right.}{}$

default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(8\,\mathrm{csgn}(e)c\,e^{5}x^{5}\sqrt{-e^{2}x^{2}+d^{2}}+12\,\mathrm{csgn}(e)b\,e^{5}x^{3}\sqrt{-e^{2}x^{2}+d^{2}}+10\,\mathrm{csgn}(e)c\,d^{2}e^{3}x^{3}\sqrt{-e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(8*csgn(e)*c*e^5*x^5*(-e^2*x^2+d^2)^(1/2
)+12*csgn(e)*b*e^5*x^3*(-e^2*x^2+d^2)^(1/2)+10*csgn(e)*c*d^2*e^3*x^3*(-e^2*
x^2+d^2)^(1/2)+24*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^5*a*x+18*(-e^2*x^2+d^2)^(1
/2)*csgn(e)*e^3*b*d^2*x+15*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e*c*d^4*x-24*arctan
(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*a*d^2*e^4-18*arctan(csgn(e)*e*x/(-e^2*x^
2+d^2)^(1/2))*b*d^4*e^2-15*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*c*d^6)*
csgn(e)/e^7/(-e^2*x^2+d^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 178, normalized size = 0.82

$$\frac{5}{16}\,cd^{6}\arcsin\left(\frac{xe}{d}\right)e^{(-7)}-\frac{1}{6}\,\sqrt{-x^{2}e^{2}+d^{2}}\,cx^{5}e^{(-2)}-\frac{5}{24}\,\sqrt{-x^{2}e^{2}+d^{2}}\,cd^{2}x^{3}e^{(-4)}-\frac{5}{16}\,\sqrt{-x^{2}e^{2}+d^{2}}\,cd^{4}xe^{(-6)}+\frac{3}{8}\,bd^{4}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{1}{4}\,\sqrt{-x^{2}e^{2}+d^{2}}\,bx^{3}e^{(-2)}-\frac{3}{8}\,\sqrt{-x^{2}e^{2}+d^{2}}\,bd^{2}xe^{(-4)}+\frac{1}{2}\,ad^{2}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,axe^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 5/16*c*d^6*arcsin(x*e/d)*e^(-7) - 1/6*sqrt(-x^2*e^2 + d^2)*c*x^5*e^(-2) - 5
/24*sqrt(-x^2*e^2 + d^2)*c*d^2*x^3*e^(-4) - 5/16*sqrt(-x^2*e^2 + d^2)*c*d^4
*x*e^(-6) + 3/8*b*d^4*arcsin(x*e/d)*e^(-5) - 1/4*sqrt(-x^2*e^2 + d^2)*b*x^3
*e^(-2) - 3/8*sqrt(-x^2*e^2 + d^2)*b*d^2*x*e^(-4) + 1/2*a*d^2*arcsin(x*e/d)
*e^(-3) - 1/2*sqrt(-x^2*e^2 + d^2)*a*x*e^(-2)
```

Fricas [A]
time = 0.36, size = 134, normalized size = 0.62

$$-\frac{\left(8\,ce^{5}x^{5}+2\,\left(5\,cd^{2}e^{3}+6\,be^{5}\right)x^{3}+3\,\left(5\,cd^{4}e+6\,bd^{2}e^{3}+8\,ae^{5}\right)x\right)\sqrt{ex+d}\,\sqrt{-ex+d}\,+6\,\left(5\,cd^{6}+6\,bd^{4}e^{2}+8\,ad^{2}e^{4}\right)\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,-d}{ex}\right)}{48\,e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d^
2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2
 + 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.20, size = 176, normalized size = 0.81

$$\frac{1}{48}\left(\left(33\,cd^5+30\,bd^3e^2+24\,ade^4-\left(85\,cd^4+54\,bd^2e^2-2\left(55\,cd^3+18\,bde^2-\left(45\,cd^2+4\left((xe+d)c-5\,cd\right)(xe+d)+6\,be^2\right)(xe+d)\right)(xe+d)+24\,ae^4\right)(xe+d)\right)\sqrt{xe+d}\,\sqrt{-xe+d}+6\left(5\,cd^6+6\,bd^4e^2+8\,ad^2e^4\right)\arcsin\left(\frac{\sqrt{2}\,\sqrt{xe+d}}{2\,\sqrt{d}}\right)\right)e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*((33*c*d^5 + 30*b*d^3*e^2 + 24*a*d*e^4 - (85*c*d^4 + 54*b*d^2*e^2 - 2*
(55*c*d^3 + 18*b*d*e^2 - (45*c*d^2 + 4*((x*e + d)*c - 5*c*d)*(x*e + d) + 6*
b*e^2)*(x*e + d))*(x*e + d) + 24*a*e^4)*(x*e + d))*sqrt(x*e + d)*sqrt(-x*e
+ d) + 6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arcsin(1/2*sqrt(2)*sqrt(x*e
+ d)/sqrt(d)))*e^(-7)
```

Mupad [**B**]
time = 23.12, size = 1132, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
a*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*a*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - ((175*c*d^6*(
(d + e*x)^(1/2) - d^(1/2))^3)/(12*((d - e*x)^(1/2) - d^(1/2))^3) + (311*c*d
^6*((d + e*x)^(1/2) - d^(1/2))^5)/(4*((d - e*x)^(1/2) - d^(1/2))^5) - (8361
*c*d^6*((d + e*x)^(1/2) - d^(1/2))^7)/(4*((d - e*x)^(1/2) - d^(1/2))^7) + (
42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^9)/(6*((d - e*x)^(1/2) - d^(1/2))^9
) - (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1/2) - d^(1
/2))^11) + (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^13)/(2*((d - e*x)^(1/2)
 - d^(1/2))^13) - (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^15)/(6*((d - e*x
```

)^(1/2) - d^(1/2))^15) + (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2))^17)/(4*((d - e*x)^(1/2) - d^(1/2))^17) - (311*c*d^6*((d + e*x)^(1/2) - d^(1/2))^19)/(4*((d - e*x)^(1/2) - d^(1/2))^19) - (175*c*d^6*((d + e*x)^(1/2) - d^(1/2))^21)/(12*((d - e*x)^(1/2) - d^(1/2))^21) - (5*c*d^6*((d + e*x)^(1/2) - d^(1/2))^23)/(4*((d - e*x)^(1/2) - d^(1/2))^23) + (5*c*d^6*((d + e*x)^(1/2) - d^(1/2)))/(4*((d - e*x)^(1/2) - d^(1/2)))))/(e^7*(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^12) - ((23*b*d^4*((d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^(1/2) - d^(1/2))^3) - (333*b*d^4*((d + e*x)^(1/2) - d^(1/2))^5)/(2*((d - e*x)^(1/2) - d^(1/2))^5) + (671*b*d^4*((d + e*x)^(1/2) - d^(1/2))^7)/(2*((d - e*x)^(1/2) - d^(1/2))^7) - (671*b*d^4*((d + e*x)^(1/2) - d^(1/2))^9)/(2*((d - e*x)^(1/2) - d^(1/2))^9) + (333*b*d^4*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1/2) - d^(1/2))^11) - (23*b*d^4*((d + e*x)^(1/2) - d^(1/2))^13)/(2*((d - e*x)^(1/2) - d^(1/2))^13) - (3*b*d^4*((d + e*x)^(1/2) - d^(1/2))^15)/(2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*b*d^4*((d + e*x)^(1/2) - d^(1/2)))/(2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*a*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*b*d^4*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5) + (5*c*d^6*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(4*e^7)

3.140 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=128

$$-\frac{(3cd^2+4be^2)\,x\sqrt{d-ex}\,\sqrt{d+ex}}{8e^4}+\frac{cx^3(-d+ex)\sqrt{d+ex}}{4e^2\sqrt{d-ex}}-\frac{(3cd^4+4bd^2e^2+8ae^4)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{4e^5}$$

```
[Out] -1/4*(8*a*e^4+4*b*d^2*e^2+3*c*d^4)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^5
+1/4*c*x^3*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/8*(4*b*e^2+3*c*d^2)*x
*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {534, 1173, 396, 223, 209}

$$\frac{\sqrt{d^2-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+4bd^2e^2+3cd^4)}{8e^5\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{x(d^2-e^2x^2)\,(4be^2+3cd^2)}{8e^4\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/8*((3*c*d^2 + 4*b*e^2)*x*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*
x]) - (c*x^3*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((3*c*d
^4 + 4*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(8*e^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\,\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-4ae^2-(3cd^2+4be^2)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{(3cd^2+4be^2)\,x(d^2-e^2x^2)}{8e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left((-8ae^4+d^2(-3cd^2-\right.}{8e}$$

$$= -\frac{(3cd^2+4be^2)\,x(d^2-e^2x^2)}{8e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left((-8ae^4+d^2(-3cd^2-\right.}{}$$

$$= -\frac{(3cd^2+4be^2)\,x(d^2-e^2x^2)}{8e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^3(d^2-e^2x^2)}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{(3cd^4+4bd^2e^2+8ae^4)}{8e^5\sqrt{}}$$

Mathematica [A]
time = 0.20, size = 98, normalized size = 0.77

$$\frac{-ex\sqrt{d-ex}\,\sqrt{d+ex}\,(3cd^2+4be^2+2ce^2x^2)+2(3cd^4+4bd^2e^2+8ae^4)\tan^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2)) + 2*(
3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(8*e^
5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 191, normalized size = 1.49

method	result
risch	$-\frac{x(2ce^2x^2+4e^2b+3cd^2)\sqrt{-ex+d}\,\sqrt{ex+d}}{8e^4}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)a}{\sqrt{e^2}}+\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)d^2b}{2e^2\sqrt{e^2}}+\cdots\right.}{\sqrt{ex+d}\,\sqrt{-\cdots}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(2\,\mathrm{csgn}(e)ce^3x^3\sqrt{-e^2x^2+d^2}+4\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(e)e^3bx+3\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(\cdots\right.}{8e^5\sqrt{-\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(2*csgn(e)*c*e^3*x^3*(-e^2*x^2+d^2)^(1/2)
+4*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*b*x+3*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e*c*
d^2*x-8*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*a*e^4-4*arctan(csgn(e)*e*x
/(-e^2*x^2+d^2)^(1/2))*b*d^2*e^2-3*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))
*c*d^4)*csgn(e)/e^5/(-e^2*x^2+d^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 107, normalized size = 0.84

$$\frac{3}{8}\,cd^4\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{1}{4}\sqrt{-x^2e^2+d^2}\,cx^3e^{(-2)}-\frac{3}{8}\sqrt{-x^2e^2+d^2}\,cd^2xe^{(-4)}+\frac{1}{2}\,bd^2\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,bxe^{(-2)}+a\arcsin\left(\frac{xe}{d}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] 3/8*c*d^4*arcsin(x*e/d)*e^(-5) - 1/4*sqrt(-x^2*e^2 + d^2)*c*x^3*e^(-2) - 3/
8*sqrt(-x^2*e^2 + d^2)*c*d^2*x*e^(-4) + 1/2*b*d^2*arcsin(x*e/d)*e^(-3) - 1/
2*sqrt(-x^2*e^2 + d^2)*b*x*e^(-2) + a*arcsin(x*e/d)*e^(-1)
```

Fricas [A]
time = 0.37, size = 100, normalized size = 0.78

$$-\frac{(2\,ce^3x^3+(3\,cd^2e+4\,be^3)x)\sqrt{ex+d}\,\sqrt{-ex+d}\,+2\,(3\,cd^4+4\,bd^2e^2+8\,ae^4)\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,-d}{ex}\right)}{8\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d)
+ 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d)
- d)/(e*x)))/e^5
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.72, size = 116, normalized size = 0.91

$$\frac{1}{8}\left(\left(5\,cd^3+4\,bde^2-\left(9\,cd^2+2\left((xe+d)c-3\,cd\right)(xe+d)+4\,be^2\right)(xe+d)\right)\sqrt{xe+d}\,\sqrt{-xe+d}+2\left(3\,cd^4+4\,bd^2e^2+8\,ae^4\right)\arcsin\left(\frac{\sqrt{2}\,\sqrt{xe+d}}{2\,\sqrt{d}}\right)\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((5*c*d^3 + 4*b*d*e^2 - (9*c*d^2 + 2*((x*e + d)*c - 3*c*d)*(x*e + d) +
4*b*e^2)*(x*e + d))*sqrt(x*e + d)*sqrt(-x*e + d) + 2*(3*c*d^4 + 4*b*d^2*e^2
 + 8*a*e^4)*arcsin(1/2*sqrt(2)*sqrt(x*e + d)/sqrt(d)))*e^(-5)
```

Mupad [**B**]
time = 12.86, size = 651, normalized size = 5.09

$$\frac{\frac{14\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4}-\frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}-\frac{\frac{23\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{333\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{671\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{671\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^9}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^9}+\frac{333\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{11}}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{11}}-\frac{23\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{13}}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{13}}-\frac{3\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)^{15}}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{15}}+\frac{3\,c\,d^4\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)}}{e^5\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^8}+\frac{2\,b\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3}+\frac{3\,c\,d^4\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
b*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*b*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*a*atan((e*
((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(e
^2)^(1/2) - ((23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^(1/2) -
```

```
d^(1/2))^3) - (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^5)/(2*((d - e*x)^(1/2
) - d^(1/2))^5) + (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^7)/(2*((d - e*x)^(
1/2) - d^(1/2))^7) - (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^9)/(2*((d - e*x
)^(1/2) - d^(1/2))^9) + (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d -
 e*x)^(1/2) - d^(1/2))^11) - (23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^13)/(2*(
(d - e*x)^(1/2) - d^(1/2))^13) - (3*c*d^4*((d + e*x)^(1/2) - d^(1/2))^15)/(
2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*c*d^4*((d + e*x)^(1/2) - d^(1/2)))/(
2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d + e*x)^(1/2) - d^(1/2))^2/((d - e
*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*b*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/
((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*c*d^4*atan(((d + e*x)^(1/2) - d^(1/2
))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5)
```

3.141 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=102

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}+\frac{cx(-d+ex)\sqrt{d+ex}}{2e^2\sqrt{d-ex}}-\frac{(cd^2+2be^2)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3}$$

```
[Out] -(2*b*e^2+c*d^2)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^3+1/2*c*x*(e*x-d)*(
e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x
```

Rubi [A]
time = 0.08, antiderivative size = 155, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {534, 1279, 396, 223, 209}

$$-\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(2be^2+cd^2)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((a*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (c*x*(d^2 - e^
2*x^2))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((c*d^2 + 2*b*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^2\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-bd^2-cd^2x^2}{\sqrt{d^2-e^2x^2}}\,dx}{d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2 \ldots}\right.}{2\sqrt{d-ex \ldots}}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2 \ldots}\right.}{2 \ldots}$$

$$= -\frac{a(d^2-e^2x^2)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx(d^2-e^2x^2)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{(cd^2+2be^2)\sqrt{d^2-e^2 \ldots}}{2e^3\sqrt{d- \ldots}}$$

Mathematica [A]
time = 0.18, size = 86, normalized size = 0.84

$$\frac{-\frac{e\sqrt{d-ex}\sqrt{d+ex}\,(2ae^2+cd^2x^2)}{d^2x} + 2(cd^2+2be^2)\tan^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x)) + 2*(c*d^
2 + 2*b*e^2)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(2*e^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 148, normalized size = 1.45

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(c\,d^{2}x^{2}+2a\,e^{2}\right)}{2e^{2}d^{2}x}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)b}{\sqrt{e^{2}}}+\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)c\,d^{2}}{2e^{2}\sqrt{e^{2}}}\right)\sqrt{(ex+}}{\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(\mathrm{csgn}(e)c\,d^{2}e\,x^{2}\sqrt{-e^{2}x^{2}+d^{2}}-2\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)b\,d^{2}e^{2}x-\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^{2}x^{2}+}}\right.\right.}{2d^{2}e^{3}\sqrt{-e^{2}x^{2}+d^{2}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2*(csgn(e)*c*d^2*e*x^2*(-e^2*x^2+d^2)^(
1/2)-2*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*b*d^2*e^2*x-arctan(csgn(e)*
e*x/(-e^2*x^2+d^2)^(1/2))*c*d^4*x+2*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*a)*csg
n(e)/e^3/(-e^2*x^2+d^2)^(1/2)/x
```

Maxima [A]
time = 0.49, size = 70, normalized size = 0.69

$$\frac{1}{2}\,cd^{2}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,cxe^{(-2)}+b\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,a}{d^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 1/2*c*d^2*arcsin(x*e/d)*e^(-3) - 1/2*sqrt(-x^2*e^2 + d^2)*c*x*e^(-2) + b*ar
csin(x*e/d)*e^(-1) - sqrt(-x^2*e^2 + d^2)*a/(d^2*x)
```

Fricas [A]
time = 0.36, size = 90, normalized size = 0.88

$$-\frac{2\,(cd^{4}+2\,bd^{2}e^{2})x\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{ex}\right)+(cd^{2}ex^{2}+2\,ae^{3})\sqrt{ex+d}\,\sqrt{-ex+d}}{2\,d^{2}e^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(
e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(92) = 184.
time = 5.66, size = 251, normalized size = 2.46

$$\frac{1}{2}\left(\left(\left(\pi+2\arctan\left(\frac{\sqrt{xe+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)^2}{xe+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)}\right)\right)\left(cd^2+2be^2\right)-\left((xe+d)c-cd\right)\sqrt{xe+d}\sqrt{-xe+d}-\frac{8a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)e^4}{\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^2-4\right)d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((pi + 2*arctan(1/2*sqrt(x*e + d)*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))^2
/(x*e + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))))*(c*d^2 + 2*b*e^2) - ((
x*e + d)*c - c*d)*sqrt(x*e + d)*sqrt(-x*e + d) - 8*a*((sqrt(2)*sqrt(d) - sq
rt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e +
d)))*e^4/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)*d^2))*e^(-3)
```

Mupad [**B**]
time = 7.00, size = 306, normalized size = 3.00

$$\frac{\frac{14cd^2\left(\sqrt{d+ex}-\sqrt{d}\right)^3}{\left(\sqrt{d-ex}-\sqrt{d}\right)^3}-\frac{14cd^2\left(\sqrt{d+ex}-\sqrt{d}\right)^5}{\left(\sqrt{d-ex}-\sqrt{d}\right)^5}+\frac{2cd^2\left(\sqrt{d+ex}-\sqrt{d}\right)^7}{\left(\sqrt{d-ex}-\sqrt{d}\right)^7}-\frac{2cd^2\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{d-ex}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{\left(\sqrt{d-ex}-\sqrt{d}\right)^2}+1\right)^4}-\frac{4b\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-ex}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}+\frac{2cd^2\,\mathrm{atan}\left(\frac{\sqrt{d+ex}-\sqrt{d}}{\sqrt{d-ex}-\sqrt{d}}\right)}{e^3}-\frac{\left(\frac{a}{d}+\frac{aex}{d^2}\right)\sqrt{d-ex}}{x\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
c*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*c*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*b*atan((e*
((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(e
^2)^(1/2) + (2*c*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^
(1/2))))/e^3 - ((a/d + (a*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*x)^(1/2))
```

3.142 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=157

$$-\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/3*a*(-e^2*x^2+d^2)/d^2/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/3*(2*a*e^2+3*b
*d^2)*(-e^2*x^2+d^2)/d^4/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+c*arctan(e*x/(-e^2*
x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {534, 1279, 462, 223, 209}

$$-\frac{(d^2-e^2x^2)(2ae^2+3bd^2)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(a*(d^2 - e^2*x^2))/(d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*b*d^2
+ 2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*Sqrt
[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(e*Sqrt[d - e*x]*Sqrt[d
+ e*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
```

```
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\,\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\,\int \frac{a+bx^2+cx^4}{x^4\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\,\int \frac{-3bd^2-2ae^2-3cd^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^2\sqrt{d-ex}\,\sqrt{d+ex}}\\
&= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\int}{\sqrt{d-ex}\,\sqrt{}}\\
&= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\mathrm{Su}}{\sqrt{}}\\
&= -\frac{a(d^2-e^2x^2)}{3d^2x^3\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(3bd^2+2ae^2)(d^2-e^2x^2)}{3d^4x\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\,\tan^{-1}}{e\sqrt{d-ex}}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 81, normalized size = 0.52

$$-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,(3bd^2x^2+a(d^2+2e^2x^2))}{3d^4x^3}+\frac{2c\tan^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(d^4
*x^3) + (2*c*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 146, normalized size = 0.93

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(2a\,e^2x^2+3b\,d^2x^2+a\,d^2)}{3d^4x^3}+\frac{c\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)\sqrt{(ex+d)(-ex+d)}}{\sqrt{e^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(-3\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^2x^2+d^2}}\right)c\,d^4x^3+2\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(e)e^3a\,x^2+3\sqrt{-e^2x^2+d^2}\right.}{3d^4\sqrt{-e^2x^2+d^2}\,e\,x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^4*(-3*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)
^(1/2))*c*d^4*x^3+2*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*a*x^2+3*(-e^2*x^2+d^2)
^(1/2)*csgn(e)*e*b*d^2*x^2+a*(-e^2*x^2+d^2)^(1/2)*d^2*csgn(e)*e)*csgn(e)/(-
e^2*x^2+d^2)^(1/2)/e/x^3
```

Maxima [A]
time = 0.50, size = 81, normalized size = 0.52

$$c\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\frac{\sqrt{-x^2e^2+d^2}\,b}{d^2x}-\frac{2\sqrt{-x^2e^2+d^2}\,ae^2}{3\,d^4x}-\frac{\sqrt{-x^2e^2+d^2}\,a}{3\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] c*arcsin(x*e/d)*e^(-1) - sqrt(-x^2*e^2 + d^2)*b/(d^2*x) - 2/3*sqrt(-x^2*e^2
 + d^2)*a*e^2/(d^4*x) - 1/3*sqrt(-x^2*e^2 + d^2)*a/(d^2*x^3)
```

Fricas [A]
time = 0.37, size = 90, normalized size = 0.57

$$-\frac{6\,cd^4x^3\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{ex}\right)+\left(ad^2e+\left(3\,bd^2e+2\,ae^3\right)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{3\,d^4ex^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (a*d^2
*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*e*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 55.95, size = 257, normalized size = 1.64

$$\frac{iae^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ae^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ibeG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{beG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
(0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*meijerg(((3/2, 7/4, 2,
 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*exp_polar(-2*I*pi)/(
e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2
)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + b
*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((1/4
, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2)
)/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1
/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(140) = 280.
time = 3.48, size = 555, normalized size = 3.54

$$\frac{1}{3}\left(3\left(\pi+2\arctan\left(\frac{\sqrt{xe+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)^2}{xe+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)}\right)\right)c-\frac{4\left(3\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^5e^2+3\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^5e^4-24\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^3e^2-8\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^3e^4+48\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)e^2+48\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)e^4\right)}{\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^2-4\right)^3d^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(pi + 2*arctan(1/2*sqrt(x*e + d)*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
^2/(x*e + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))))*c - 4*(3*b*d^2*((sqr
```

```
t(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(
d) - sqrt(-x*e + d)))^5*e^2 + 3*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(
x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 24*b*d
^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^3*e^2 - 8*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d
))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4
+ 48*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^2 + 48*a*((sqrt(2)*sqrt(d) - sqrt(-
x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))
*e^4)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(
sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^3*d^4))*e^(-1)
```

Mupad [B]
time = 2.27, size = 138, normalized size = 0.88

$$-\frac{4\,c\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}-\frac{\left(\frac{b}{d}+\frac{b\,e\,x}{d^2}\right)\,\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}}-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{3\,d}+\frac{2\,a\,e^2\,x^2}{3\,d^3}+\frac{2\,a\,e^3\,x^3}{3\,d^4}+\frac{a\,e\,x}{3\,d^2}\right)}{x^3\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] - (4*c*atan((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) -
 d^(1/2)))))/(e^2)^(1/2) - ((b/d + (b*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*
x)^(1/2)) - ((d - e*x)^(1/2)*(a/(3*d) + (2*a*e^2*x^2)/(3*d^3) + (2*a*e^3*x^
3)/(3*d^4) + (a*e*x)/(3*d^2)))/(x^3*(d + e*x)^(1/2))
```

3.143 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=160

$$-\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{(5bd^2+4ae^2)(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(15cd^4+10bd^2e^2+8ae^4)(d^2-e^2x^2)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/5*a*(-e^2*x^2+d^2)/d^2/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(4*a*e^2+5*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(8*a*e^4+10
*b*d^2*e^2+15*c*d^4)*(-e^2*x^2+d^2)/d^6/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {534, 1279, 464, 270}

$$-\frac{(d^2-e^2x^2)(8ae^4+10bd^2e^2+15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(4ae^2+5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/5*(a*(d^2 - e^2*x^2))/(d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((5*b*d^2
+ 4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((15
*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2))/(15*d^6*x*Sqrt[d - e*x]*S
qrt[d + e*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
```

```
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^6\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}\\ &= -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-5bd^2-4ae^2-5cd^2x^2}{x^4\sqrt{d^2-e^2x^2}}\,dx}{5d^2\sqrt{d-ex}\sqrt{d+ex}}\\ &= -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5bd^2+4ae^2)(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\Big((15cd^4-2e^2(-\cdots}{\cdots}\\ &= -\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(5bd^2+4ae^2)(d^2-e^2x^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{(15cd^4+10bd^2e^2\cdots}{15d^6x\sqrt{d-\cdots}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 87, normalized size = 0.54

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\,(15cd^4x^4+5bd^2x^2(d^2+2e^2x^2)+a(3d^4+4d^2e^2x^2+8e^4x^4))}{15d^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^2
*x^2) + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(d^6*x^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.15, size = 86, normalized size = 0.54

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3d^4a)}{15x^5d^6}$	82
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3d^4a)}{15x^5d^6}$	82
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}(e)^2(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3d^4a)}{15d^6x^5}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/15*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*csgn(e)^2/d^6*(8*a*e^4*x^4+10*b*d^2*e^2*
x^4+15*c*d^4*x^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5
```

Maxima [A]
time = 0.49, size = 139, normalized size = 0.87

$$-\frac{\sqrt{-x^2e^2+d^2}\,c}{d^2x}-\frac{2\,\sqrt{-x^2e^2+d^2}\,be^2}{3\,d^4x}-\frac{\sqrt{-x^2e^2+d^2}\,b}{3\,d^2x^3}-\frac{8\,\sqrt{-x^2e^2+d^2}\,ae^4}{15\,d^6x}-\frac{4\,\sqrt{-x^2e^2+d^2}\,ae^2}{15\,d^4x^3}-\frac{\sqrt{-x^2e^2+d^2}\,a}{5\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -sqrt(-x^2*e^2 + d^2)*c/(d^2*x) - 2/3*sqrt(-x^2*e^2 + d^2)*b*e^2/(d^4*x) -
1/3*sqrt(-x^2*e^2 + d^2)*b/(d^2*x^3) - 8/15*sqrt(-x^2*e^2 + d^2)*a*e^4/(d^6
*x) - 4/15*sqrt(-x^2*e^2 + d^2)*a*e^2/(d^4*x^3) - 1/5*sqrt(-x^2*e^2 + d^2)*
a/(d^2*x^5)
```

Fricas [A]
time = 0.37, size = 76, normalized size = 0.48

$$-\frac{(3\,ad^4+(15\,cd^4+10\,bd^2e^2+8\,ae^4)x^4+(5\,bd^4+4\,ad^2e^2)x^2)\sqrt{ex+d}\,\sqrt{-ex+d}}{15\,d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*d
^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(145) = 290$.
time = 3.51, size = 1103, normalized size = 6.89

$$-\frac{4\left(15\,cd^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^9e^2+15\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^9e^4-240\,cd^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^7e^2+15\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^9e^6-160\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^7e^4+1440\,cd^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^5e^2-80\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^7e^6+800\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^5e^4-3840\,cd^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^3e^2+928\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^5e^6-2560\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^3e^4+3840\,cd^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)e^2-1280\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^3e^6+3840\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)e^4+3840\,a\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)e^6\right)e^{(-1)}}{15\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^2-4\right)^5d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/15*(15*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*
e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^2 + 15*b*d^2*((sqrt(2)*sqrt(
d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(
-x*e + d)))^9*e^4 - 240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e
+ d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^2 + 15*a*((sqr
t(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(
d) - sqrt(-x*e + d)))^9*e^6 - 160*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 +
1440*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^2 - 80*a*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))^7*e^6 + 800*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sq
rt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 3840*c*d^4*((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^3*e^2 + 928*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x
*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 2560*b*
d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt
(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4 + 3840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(
-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))
)*e^2 - 1280*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e
 + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 3840*b*d^2*((sqrt(2)*sqrt
(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt
(-x*e + d)))*e^4 + 3840*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d)
 - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^6)*e^(-1)/((((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^2 - 4)^5*d^6)
```

Mupad [B]
time = 1.73, size = 146, normalized size = 0.91

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{5\,d}+\frac{x^4\,\left(15\,c\,d^5+10\,b\,d^3\,e^2+8\,a\,d\,e^4\right)}{15\,d^6}+\frac{x^5\,\left(15\,c\,d^4\,e+10\,b\,d^2\,e^3+8\,a\,e^5\right)}{15\,d^6}+\frac{x^2\,\left(5\,b\,d^5+4\,a\,d^3\,e^2\right)}{15\,d^6}+\frac{x^3\,\left(5\,b\,d^4\,e+4\,a\,d^2\,e^3\right)}{15\,d^6}+\frac{a\,e\,x}{5\,d^2}\right)}{x^5\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(5*d) + (x^4*(15*c*d^5 + 10*b*d^3*e^2 + 8*a*d*e^4))/(1
5*d^6) + (x^5*(8*a*e^5 + 10*b*d^2*e^3 + 15*c*d^4*e))/(15*d^6) + (x^2*(5*b*d
^5 + 4*a*d^3*e^2))/(15*d^6) + (x^3*(4*a*d^2*e^3 + 5*b*d^4*e))/(15*d^6) + (a
*e*x)/(5*d^2)))/(x^5*(d + e*x)^(1/2))
```

3.144 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=226

$$-\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{(35cd^4+28bd^2e^2+24ae^4)(d^2-e^2x^2)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{2e^2(35cd^4}{10}$$

```
[Out] -1/7*a*(-e^2*x^2+d^2)/d^2/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/35*(6*a*e^2+7*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(24*a*e^4+
28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^6/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-
2/105*e^2*(24*a*e^4+28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^8/x/(-e*x+d)^(1
/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {534, 1279, 464, 277, 270}

$$-\frac{2e^2(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(6ae^2+7bd^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/7*(a*(d^2 - e^2*x^2))/(d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((7*b*d^2
+ 6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((35
*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^3*Sqrt[d - e*
x]*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*
x^2))/(105*d^8*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx &= \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^8\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-7bd^2-6ae^2-7cd^2x^2}{x^6\sqrt{d^2-e^2x^2}}\,dx}{7d^2\sqrt{d-ex}\sqrt{d+ex}} \\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left((35cd^4-4e^2(-7\right.}{} \\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(35cd^4+28bd^2e^2}{105d^6x^3\sqrt{d}} \\
&= -\frac{a(d^2-e^2x^2)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{(7bd^2+6ae^2)(d^2-e^2x^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{(35cd^4+28bd^2e^2}{105d^6x^3\sqrt{d}}
\end{aligned}$$

Mathematica [A]

time = 0.19, size = 124, normalized size = 0.55

$$-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,(35cd^4x^4(d^2+2e^2x^2)+7b(3d^6x^2+4d^4e^2x^4+8d^2e^4x^6)+3a(5d^6+6d^4e^2x^2+8d^2e^4x^4+16e^6x^6))}{105d^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*(
3*d^6*x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 + 8
*d^2*e^4*x^4 + 16*e^6*x^6)))/(d^8*x^7)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.17, size = 122, normalized size = 0.54

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(48a\,e^6x^6+56b\,d^2e^4x^6+70c\,d^4e^2x^6+24a\,d^2e^4x^4+28b\,d^4e^2x^4+35c\,d^6x^4+18a\,d^4e^2x^2+21b\,d^6x^2+15a}{105x^7d^8}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(48a\,e^6x^6+56b\,d^2e^4x^6+70c\,d^4e^2x^6+24a\,d^2e^4x^4+28b\,d^4e^2x^4+35c\,d^6x^4+18a\,d^4e^2x^2+21b\,d^6x^2+15a}{105x^7d^8}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\mathrm{csgn}(e)^2(48a\,e^6x^6+56b\,d^2e^4x^6+70c\,d^4e^2x^6+24a\,d^2e^4x^4+28b\,d^4e^2x^4+35c\,d^6x^4+18a\,d^4e^2x^2+21b\,d}{105d^8x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/105*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*csgn(e)^2/d^8*(48*a*e^6*x^6+56*b*d^2*e^
4*x^6+70*c*d^4*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*
d^4*e^2*x^2+21*b*d^6*x^2+15*a*d^6)/x^7
```

Maxima [A]
time = 0.51, size = 211, normalized size = 0.93

$$-\frac{2\sqrt{-x^2e^2+d^2}\,ce^2}{3d^4x}-\frac{\sqrt{-x^2e^2+d^2}\,c}{3d^2x^3}-\frac{8\sqrt{-x^2e^2+d^2}\,be^4}{15d^6x}-\frac{4\sqrt{-x^2e^2+d^2}\,be^2}{15d^4x^3}-\frac{\sqrt{-x^2e^2+d^2}\,b}{5d^2x^5}-\frac{16\sqrt{-x^2e^2+d^2}\,ae^6}{35d^8x}-\frac{8\sqrt{-x^2e^2+d^2}\,ae^4}{35d^6x^3}-\frac{6\sqrt{-x^2e^2+d^2}\,ae^2}{35d^4x^5}-\frac{\sqrt{-x^2e^2+d^2}\,a}{7d^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -2/3*sqrt(-x^2*e^2 + d^2)*c*e^2/(d^4*x) - 1/3*sqrt(-x^2*e^2 + d^2)*c/(d^2*x
^3) - 8/15*sqrt(-x^2*e^2 + d^2)*b*e^4/(d^6*x) - 4/15*sqrt(-x^2*e^2 + d^2)*b
*e^2/(d^4*x^3) - 1/5*sqrt(-x^2*e^2 + d^2)*b/(d^2*x^5) - 16/35*sqrt(-x^2*e^2
 + d^2)*a*e^6/(d^8*x) - 8/35*sqrt(-x^2*e^2 + d^2)*a*e^4/(d^6*x^3) - 6/35*sq
rt(-x^2*e^2 + d^2)*a*e^2/(d^4*x^5) - 1/7*sqrt(-x^2*e^2 + d^2)*a/(d^2*x^7)
```

Fricas [**A**]
time = 0.40, size = 110, normalized size = 0.49

$$-\frac{(15\,ad^6+2\,(35\,cd^4e^2+28\,bd^2e^4+24\,ae^6)x^6+(35\,cd^6+28\,bd^4e^2+24\,ad^2e^4)x^4+3\,(7\,bd^6+6\,ad^4e^2)x^2)\sqrt{ex+d}\,\sqrt{-ex+d}}{105\,d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*d
^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sqrt
(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. 2(204) = 408.
time = 6.41, size = 1517, normalized size = 6.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/105*(105*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^4 + 105*b*d^2*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^13*e^6 - 1960*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqr
t(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^4 + 105
*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8 - 1400*b*d^2*((sqrt(2)*sqrt(d) - sqrt(
-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))
)^11*e^6 + 16240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^4 - 840*a*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^11*e^8 + 12656*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sq
```

```
rt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 806
40*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/
(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 + 14448*a*((sqrt(2)*sqrt(d) - sqr
t(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d
)))^9*e^8 - 69888*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
 sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 259840*c*d^4*((s
qrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqr
t(d) - sqrt(-x*e + d)))^5*e^4 - 40704*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 +
202496*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e +
 d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 501760*c*d^4*((sqrt(2)*sqrt
(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt
(-x*e + d)))^3*e^4 + 231168*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e
+ d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 358400*b*d
^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 430080*c*d^4*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))*e^4 - 215040*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 430080*b*d^2*((sqrt(2)
*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) -
 sqrt(-x*e + d)))*e^6 + 430080*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x
*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8)*e^(-1)/(((
(sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*s
qrt(d) - sqrt(-x*e + d)))^2 - 4)^7*d^8)
```

Mupad [B]
time = 1.82, size = 218, normalized size = 0.96

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{7\,d}+\frac{x^2\,\left(21\,b\,d^7+18\,a\,d^5\,e^2\right)}{105\,d^8}+\frac{x^4\,\left(35\,c\,d^7+28\,b\,d^5\,e^2+24\,a\,d^3\,e^4\right)}{105\,d^8}+\frac{x^7\,\left(70\,c\,d^4\,e^3+56\,b\,d^2\,e^5+48\,a\,e^7\right)}{105\,d^8}+\frac{x^3\,\left(21\,b\,d^6\,e+18\,a\,d^4\,e^3\right)}{105\,d^8}+\frac{x^5\,\left(35\,c\,d^6\,e+28\,b\,d^4\,e^3+24\,a\,d^2\,e^5\right)}{105\,d^8}+\frac{x^6\,\left(70\,c\,d^5\,e^2+56\,b\,d^3\,e^4+48\,a\,d\,e^6\right)}{105\,d^8}+\frac{a\,e\,x}{7\,d^2}\right)}{x^7\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(7*d) + (x^2*(21*b*d^7 + 18*a*d^5*e^2))/(105*d^8) + (x
^4*(35*c*d^7 + 24*a*d^3*e^4 + 28*b*d^5*e^2))/(105*d^8) + (x^7*(48*a*e^7 + 5
6*b*d^2*e^5 + 70*c*d^4*e^3))/(105*d^8) + (x^3*(18*a*d^4*e^3 + 21*b*d^6*e))/
(105*d^8) + (x^5*(24*a*d^2*e^5 + 28*b*d^4*e^3 + 35*c*d^6*e))/(105*d^8) + (x
^6*(56*b*d^3*e^4 + 70*c*d^5*e^2 + 48*a*d*e^6))/(105*d^8) + (a*e*x)/(7*d^2))
)/(x^7*(d + e*x)^(1/2))
```

3.145 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=292

$$-\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}}-\frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{(21cd^4+18bd^2e^2+16ae^4)(d^2-e^2x^2)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2(21cd^4}{315}$$

```
[Out] -1/9*a*(-e^2*x^2+d^2)/d^2/x^9/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/63*(8*a*e^2+9*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(16*a*e^4+
18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^6/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-
4/315*e^2*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^8/x^3/(-e*x+d)^
(1/2)/(e*x+d)^(1/2)-8/315*e^4*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^
2)/d^10/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {534, 1279, 464, 277, 270}

$$-\frac{8e^4(d^2-e^2x^2)(16ae^4+18bd^2e^2+21cd^4)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2(d^2-e^2x^2)(16ae^4+18bd^2e^2+21cd^4)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(16ae^4+18bd^2e^2+21cd^4)}{105d^6x^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(8ae^2+9bd^2)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/9*(a*(d^2 - e^2*x^2))/(d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((9*b*d^2
+ 8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((21
*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^5*Sqrt[d - e*
x]*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*
x^2))/(315*d^8*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d
^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x]
)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\,\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-9bd^2-8ae^2-9cd^2x^2}{x^8\sqrt{d^2-e^2x^2}}\,dx}{9d^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{\Big((63cd^4-6e^2(-\cdots}{\cdots}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2\cdots}{105d^6x^5\sqrt{d\cdots}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2\cdots}{105d^6x^5\sqrt{d\cdots}}$$

$$= -\frac{a(d^2-e^2x^2)}{9d^2x^9\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(9bd^2+8ae^2)(d^2-e^2x^2)}{63d^4x^7\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{(21cd^4+18bd^2e^2\cdots}{105d^6x^5\sqrt{d\cdots}}$$

Mathematica [A]
time = 0.24, size = 158, normalized size = 0.54

$$-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,\left(21cd^4x^4(3d^4+4d^2e^2x^2+8e^4x^4)+9b(5d^8x^2+6d^6e^2x^4+8d^4e^4x^6+16d^2e^6x^8)+a(35d^8+40d^6e^2x^2+48d^4e^4x^4+64d^2e^6x^6+128e^8x^8)\right)}{315d^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 +
8*e^4*x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*x^
8) + a*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*e^8
*x^8)))/(d^10*x^9)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.16, size = 158, normalized size = 0.54

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x^4+54b\,d^6e^2x^{\cdots}}{315x^9d^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x^4+54b\,d^6e^2x^{\cdots}}{315x^9d^{10}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\mathrm{csgn}(e)^2(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x^4+5\cdots}{315d^{10}x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/315*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*csgn(e)^2/d^10*(128*a*e^8*x^8+144*b*d^2
*e^6*x^8+168*c*d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x
^6+48*a*d^4*e^4*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8
*x^2+35*a*d^8)/x^9
```

Maxima [A]
time = 0.49, size = 283, normalized size = 0.97

$$-\frac{8\sqrt{-x^2e^2+d^2}\,ce^4}{15\,d^6x}-\frac{4\sqrt{-x^2e^2+d^2}\,ce^2}{15\,d^4x^3}-\frac{\sqrt{-x^2e^2+d^2}\,c}{5\,d^2x^5}-\frac{16\sqrt{-x^2e^2+d^2}\,be^6}{35\,d^8x}-\frac{8\sqrt{-x^2e^2+d^2}\,be^4}{35\,d^6x^3}-\frac{6\sqrt{-x^2e^2+d^2}\,be^2}{35\,d^4x^5}-\frac{\sqrt{-x^2e^2+d^2}\,b}{7\,d^2x^7}-\frac{128\sqrt{-x^2e^2+d^2}\,ae^8}{315\,d^{10}x}-\frac{64\sqrt{-x^2e^2+d^2}\,ae^6}{315\,d^8x^3}-\frac{16\sqrt{-x^2e^2+d^2}\,ae^4}{105\,d^6x^5}-\frac{8\sqrt{-x^2e^2+d^2}\,ae^2}{63\,d^4x^7}-\frac{\sqrt{-x^2e^2+d^2}\,a}{9\,d^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -8/15*sqrt(-x^2*e^2 + d^2)*c*e^4/(d^6*x) - 4/15*sqrt(-x^2*e^2 + d^2)*c*e^2/
(d^4*x^3) - 1/5*sqrt(-x^2*e^2 + d^2)*c/(d^2*x^5) - 16/35*sqrt(-x^2*e^2 + d^
2)*b*e^6/(d^8*x) - 8/35*sqrt(-x^2*e^2 + d^2)*b*e^4/(d^6*x^3) - 6/35*sqrt(-x
^2*e^2 + d^2)*b*e^2/(d^4*x^5) - 1/7*sqrt(-x^2*e^2 + d^2)*b/(d^2*x^7) - 128/
315*sqrt(-x^2*e^2 + d^2)*a*e^8/(d^10*x) - 64/315*sqrt(-x^2*e^2 + d^2)*a*e^6
/(d^8*x^3) - 16/105*sqrt(-x^2*e^2 + d^2)*a*e^4/(d^6*x^5) - 8/63*sqrt(-x^2*e
^2 + d^2)*a*e^2/(d^4*x^7) - 1/9*sqrt(-x^2*e^2 + d^2)*a/(d^2*x^9)
```

Fricas [A]
time = 0.44, size = 144, normalized size = 0.49

$$-\frac{\left(35\,ad^8+8\,(21\,cd^4e^4+18\,bd^2e^6+16\,ae^8)x^8+4\,(21\,cd^6e^2+18\,bd^4e^4+16\,ad^2e^6)x^6+3\,(21\,cd^8+18\,bd^6e^2+16\,ad^4e^4)x^4+5\,(9\,bd^8+8\,ad^6e^2)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{315\,d^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*c
*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2 +
16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x
+ d)/(d^10*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1931 vs. $2(263) = 526$.
time = 7.39, size = 1931, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4/315*(315*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^17*e^6 + 315*b*d^2*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^17*e^8 - 6720*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqr
t(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^15*e^6 + 315
*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^17*e^10 - 5040*b*d^2*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))^15*e^8 + 76608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
 sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^6 - 3360*a*((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^15*e^10 + 68544*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d)
)/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8
- 580608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e
 + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^6 + 76608*a*((sqrt(2)*sqrt(d
) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-
x*e + d)))^13*e^10 - 509184*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(
x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^8 + 28922
88*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/
(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 327168*a*((sqrt(2)*sqrt(d) - sq
rt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e +
d)))^11*e^10 + 2363904*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e +
 d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^8 - 9289728*c*d
^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 2728448*a*((sqrt(2)*sqrt(d) - sqrt(-x
*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^
9*e^10 - 8146944*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 + 19611648*c*d^4*((
sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sq
rt(d) - sqrt(-x*e + d)))^5*e^6 - 5234688*a*((sqrt(2)*sqrt(d) - sqrt(-x*e +
```

```
d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^1
0 + 17547264*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt
(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 27525120*c*d^4*((sqrt
(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d
) - sqrt(-x*e + d)))^3*e^6 + 19611648*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^10 -
 20643840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*
e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 20643840*c*d^4*((sqrt(2)
*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) -
 sqrt(-x*e + d)))*e^6 - 13762560*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt
(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^10 + 2064
3840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8 + 20643840*a*((sqrt(2)*sqrt(d) -
sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e
+ d)))*e^10)*e^(-1)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - s
qrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^9*d^10)
```

Mupad [B]
time = 1.87, size = 290, normalized size = 0.99

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{9\,d}+\frac{x^2\,\left(45\,b\,d^9+40\,a\,d^7\,e^2\right)}{315\,d^{10}}+\frac{x^6\,\left(84\,c\,d^7\,e^2+72\,b\,d^5\,e^4+64\,a\,d^3\,e^6\right)}{315\,d^{10}}+\frac{x^7\,\left(84\,c\,d^6\,e^3+72\,b\,d^4\,e^5+64\,a\,d^2\,e^7\right)}{315\,d^{10}}+\frac{x^4\,\left(63\,c\,d^9+54\,b\,d^7\,e^2+48\,a\,d^5\,e^4\right)}{315\,d^{10}}+\frac{x^9\,\left(168\,c\,d^4\,e^5+144\,b\,d^2\,e^7+128\,a\,e^9\right)}{315\,d^{10}}+\frac{x^3\,\left(45\,b\,d^8\,e+40\,a\,d^6\,e^3\right)}{315\,d^{10}}+\frac{x^5\,\left(63\,c\,d^8\,e+54\,b\,d^6\,e^3+48\,a\,d^4\,e^5\right)}{315\,d^{10}}+\frac{x^8\,\left(168\,c\,d^5\,e^4+144\,b\,d^3\,e^6+128\,a\,d\,e^8\right)}{315\,d^{10}}+\frac{a\,e\,x}{9\,d^2}\right)}{x^9\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^10*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(9*d) + (x^2*(45*b*d^9 + 40*a*d^7*e^2))/(315*d^10) + (
x^6*(64*a*d^3*e^6 + 72*b*d^5*e^4 + 84*c*d^7*e^2))/(315*d^10) + (x^7*(64*a*d
^2*e^7 + 72*b*d^4*e^5 + 84*c*d^6*e^3))/(315*d^10) + (x^4*(63*c*d^9 + 48*a*d
^5*e^4 + 54*b*d^7*e^2))/(315*d^10) + (x^9*(128*a*e^9 + 144*b*d^2*e^7 + 168*
c*d^4*e^5))/(315*d^10) + (x^3*(40*a*d^6*e^3 + 45*b*d^8*e))/(315*d^10) + (x^
5*(48*a*d^4*e^5 + 54*b*d^6*e^3 + 63*c*d^8*e))/(315*d^10) + (x^8*(144*b*d^3*
e^6 + 168*c*d^5*e^4 + 128*a*d*e^8))/(315*d^10) + (a*e*x)/(9*d^2)))/(x^9*(d
+ e*x)^(1/2))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:  #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```